The $SU(2)_D$ lepton portals for muon $g-2$, W boson mass and dark matter

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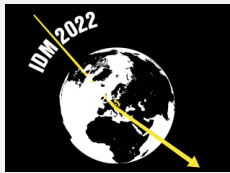
Abstract

We propose a novel model which extends the Standard Model (SM) by introducing a $SU(2)_D$ gauge symmetry. In this model, a dark $SU(2)_D$ Higgs doublet and a Higgs bi-doublet can contribute to the muon g-2 anomaly and the W boson mass, remaining in tune with the recent experimental results. At the same time, the isospin charged gauge boson of $SU(2)_D$ becomes a plausible candidate for Dark Matter(DM). We find that the resulting parameter space can fit the muon g-2, the W boson mass and the DM constraints simultaneously.

1   Introduction

Despite its success, the Standard Model (SM) still leaves us with some unanswered questions. One example is the well-stablished muon $g − 2$ anomaly from Brookhaven E821 [1], which has been recently confirmed by E989 [2]. Another one is the latest measurement of the W boson mass at Tevatron CDFII [3], which deviates with high significance from the SM prediction [4]. On top of this, we have puzzling questions regarding Dark Matter(DM). Although we have a large amount of indirect evidence for DM, such as galaxy rotation curves and the CMB, we still don't understand its nature neither its origin. The only hint we have is its total energy density, $\Omega h^2 \simeq 0.1186$, as measured by Planck [12]. A connection between these puzzles could shed some light into the future experimental and theoretical steps to take.

2   The model

In this work, we extend the SM by adding a new local $SU(2)_D$ symmetry. We introduced an $SU(2)_D$ double vector-like lepton Ψ, an $SU(2)_D$ doublet Higgs ϕ_D and a Higgs bi-doublet H'. The particle content of the model and their charges under the Z_2 parity are described in detail in reference [13].

   The Lagrangian of our theory can be contains a DM and a Yukawa part including the vector-like leptons,

$$\mathcal{L}_{\text{DM}} \quad = \quad -\frac{1}{2}\text{Tr}\Big(V_{\mu\nu}V^{\mu\nu}\Big) + i\bar{\Psi}\gamma^\mu D_\mu\Psi + |D_\mu\Phi_D|^2 + \text{Tr}\Big(|D_\mu H'|^2\Big) - V(\Phi_D, H', H) \tag{1}$$

$$\begin{aligned}
\mathcal{L}_{\text{VLSM}} \quad = \quad & -y_d\bar{q}_L d_R H - y_u\bar{q}_L u_R\tilde{H} - y_l\bar{l}_L e_R H - y_\nu\bar{l}_L\nu_R\tilde{H} - M_R\overline{\nu_R^c}\nu_R \\
& -M_E\bar{\Psi}\Psi - \lambda_E\bar{\Psi}_L\Phi_D e_R - y_E\bar{l}_L H'\Psi_R + \text{h.c.},
\end{aligned} \tag{2}$$

with the scalar potential given by

$$\begin{aligned}
V(\Phi_D, H, H') \quad = \quad & \mu_1^2 H^\dagger H + \mu_2^2\text{Tr}(H'^\dagger H') - (\mu_3 H^\dagger H'\Phi_D + \text{h.c.}) \\
& + \lambda_1(H^\dagger H)^2 + \lambda_2(\text{Tr}H'^\dagger H')^2 + \lambda_3(H^\dagger H)\text{Tr}(H'^\dagger H') \\
& + \mu_\phi^2\Phi_D^\dagger\Phi_D + \lambda_\phi(\Phi_D^\dagger\Phi_D)^2 + \lambda_{H\Phi}H^\dagger H\Phi_D^\dagger\Phi_D + \lambda_{H'\Phi}\text{Tr}(H'^\dagger H')\Phi_D^\dagger\Phi_D.
\end{aligned} \tag{3}$$

Finally, the mass Lagrangian for the lepton sector is

$$\mathcal{L}_{L,\text{mass}} = -M_E\bar{E}E - M_E\bar{E}'E' - m_0\bar{e}e - (m_R\bar{E}_L e_R + m_L\bar{e}_L E_R + \text{h.c.}). \tag{4}$$

Diagonalizing the Lagrangian in equation (4), the vector-like lepton contributions are found to be naturally small [6, 7], consistently with a simultaneous symmetry breaking of both the electroweak and the $SU(2)_D$ symmetries.

3   Muon $g − 2$ and W mass constraints

In this section, we briefly summarize the contributions from the new particles in our model to the muon $g − 2$ and the W mass.

3.1 Muon $g-2$

The dominant contributions to the muon $g-2$ are the ones coming from vector-like leptons and gauge bosons running together with muons in the loops. The vector contributions are given by [6,7]

$$
\Delta a_\mu^{V,E} \simeq \begin{cases} \frac{g_D^2 M_E m_\mu}{16\pi^2 m_{V^0}^2}(c_V^2 - c_A^2) + \frac{g_D^2 M_E m_\mu}{32\pi^2 m_{V^0}^2}(\hat{c}_V^2 - \hat{c}_A^2), & M_E \gg m_{V^0}, \\[2ex] \frac{g_D^2 M_E m_\mu}{4\pi^2 m_{V^0}^2}(c_V^2 - c_A^2) + \frac{g_D^2 M_E m_\mu}{8\pi^2 m_{V^0}^2}(\hat{c}_V^2 - \hat{c}_A^2), & m_\mu \ll M_E \ll m_{V^0}. \end{cases} \tag{5}
$$

and

$$
\Delta a_\mu^{V^\pm, E'} = \frac{1}{2}\Delta a_l^{V^0,E}(c_V \to \hat{c}_V, c_A \to \hat{c}_A, M_E \to M_{E'}, m_{V^0} \to m_{V^\pm}), \tag{6}
$$

while the contribution coming from the scalar V_0 reads [6,7]

$$
\Delta a_\mu^{V^0,\mu} \simeq \frac{g_D^2 m_\mu^2}{12\pi^2 m_{V^0}^2}(\nu_\mu'^2 - 5a_\mu'^2). \tag{7}
$$

Here, the coefficients $c_V, c_A, \hat{c}_V, \hat{c}_A, \nu_\mu'$ and a_μ' are given in eqs. (47)-(52) in [13].

3.2 W mass

The dominant contribution for the W mass is the tree-level process arising from the mixing between the gauge neutral bosons. The contribution to the ρ parameter is then

$$
\Delta\rho_H \simeq \begin{cases} \frac{s_W^2 g_D^2}{g_Y^2}\frac{M_Z^2}{m_{V^0}^2}\sin^4\beta, & m_{V^0} \gg M_Z, \\[2ex] -\frac{s_W^2 g_D^2}{g_Y^2}\sin^4\beta, & m_{V^0} \ll M_Z. \end{cases} \tag{8}
$$

We note that one-loop contributions from E', $\tilde{\varphi}$ and $V^\pm$ are supressed by the small lepton mixing angle $\sin^2\zeta$. Consequently, they are sub-dominant compared to the tree-level effects of the $Z-V^0$ mass mixing and we can neglect them in our analysis.

We summarize our results in Fig. 1. On the left, we depict the muon $g-2$ correction as a function of m_{V^+}. The regions favored by the muon $g-2$ within $1\sigma(2\sigma)$ are shown in yellow(green). We find it necessary to set m_ϕ around the TeV scale in order to counter the $\tilde{\varphi}$ negative chirality enhanced contribution [13]. On the right, we show the correction to the W boson mass as a function of $\sin\beta$.

4 Dark matter constraints

In this section we study whether our model can reproduce the correct DM relic abundance and satisfy the stringent bounds set by direct detection experiments.

4.1 Relic Density

The gauge bosons $V^\pm$, the vector-like lepton E' and the neutral scalar $\tilde{\varphi}$ are odd under the Z_2 parity, and thus make viable DM candidates. In paticular, the $SU(2)_D$ gauge bosons are almost degenerate, but their masses receive a small positive correction proportional to the $Z-V^0$
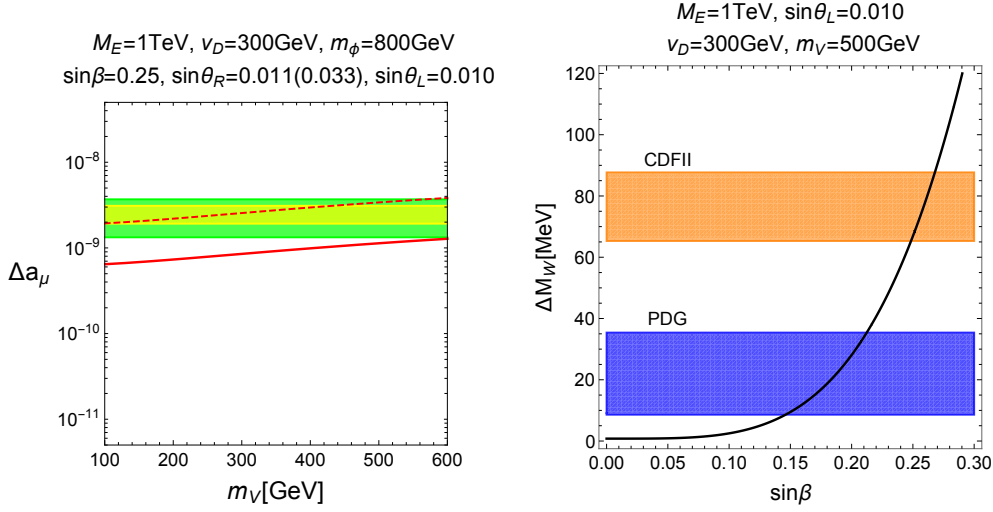
Figure 1: Parameter space for the muon $g-2$ and the W mass. These figures where first published in [13] licensed under CC BY 4.0.

76   mixing[1]. This results in a forbidden channel, $V^+V^- \rightarrow V^0V^0$, which is responsible for the
77   correct relic energy density.

78   On the left plot of Fig. 2, we show the relic abundance for DM as a function $\delta \equiv m_{V^0}/m_{V^+}-1$.
79   The choice of parameters is consistent with both the muon $g-2$ and the W boson mass results
80   in the previous section. We find that the condition for a correct relic density [12] depends
81   crucially on the mass splitting δ and the DM mass m_{V^+} for a given v_D.

82   ## 4.2 Indirect Detection

83   We remark that the forbidden channel is closed for small DM velocities, namely $v_{\rm rel} \lesssim \sqrt{8\delta}$.
84   The velocity of DM in our galaxy, $v_{\rm rel} \simeq 220\,{\rm km}$ makes any signal from this channel unobserv-
85   able for $\delta \gtrsim 6 \times 10^{-7}$. However, other subdominant channels, such as $V^+V^- \rightarrow hh, V^0Z$ or
86   $V^+V^- \rightarrow {\rm SM\,SM}$, could lead to interesting signals for indirect detection, for example in the
87   CMB or in cosmic rays [14, 15].

88   ## 4.3 Direct detection

89   Decays of DM into a quark-antiquark pair, $V^+V^- \rightarrow q\bar{q}$, are subdominant for the relic den-
90   sity abundance, but they can be constrained by direct detection experiments. We find that
91   the XENON1T bound [11] on the DM-nucleon scattering cross section can be satisfied in the
92   alignment limit for the Higgs mixing angle, $\sin\theta_h \simeq -\frac{v}{\sqrt{2}v_D}\sin^2\beta$ and $m_s \gg m_h$.

93   On the right plot of Fig. 2, we show in gray the parameter space which has been ruled out
94   by XENON1T [11]. In the alignment limit and for a heavy singlet scalar with $m_s = 1.5\,{\rm TeV}$,
95   our results show that there is consistent parameter space satisfying the muon $g-2$, the W
96   boson mass and the DM constraints simultaneously.

97   # 5 Conclusions

98   In this paper, we addressed the possibility of explaining DM and flavor puzzles simultaneously.
99   With that in mind, we proposed a model that extends the SM by implementing a new $SU(2)_D$

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[1]In this sense, the mass splitting, $\delta \equiv m_{V^0}/m_{V^+} - 1$, is closely related to the W boson mass.
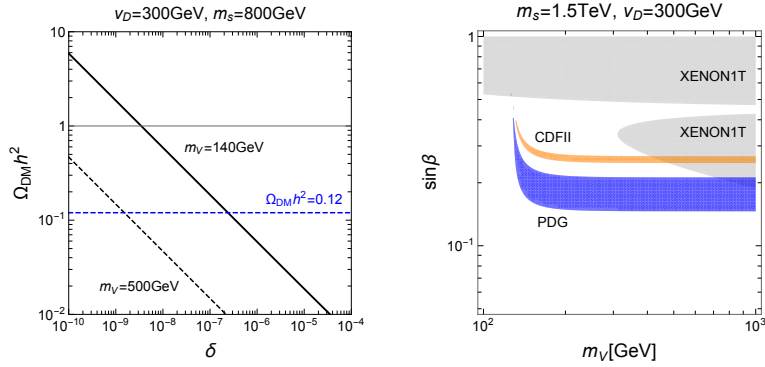
Figure 2: (Left) DM relic density as a function of $\delta \equiv m_{V^0}/m_{V^+} - 1$. (Right) Parameter space consistent with XENON1T [11]. These figures where first published in [13] licensed under CC BY 4.0.

gauge symmetry. The flavor mixing between the lepton and the vector-like lepton makes the lepton masses naturally small. Furthermore, the vector-like leptons and $SU(2)_D$ gauge bosons interactions contribute dominantly to the muon $g-2$, while the $Z - V^0$ mass mixing accounts for the deviation of the W boson mass, as recently measured by CDFII. A combination of the $U(1)_G$ global symmetry in the Higgs sector and the dark isospin symmetry leads to a Z_2 parity, allowing for suitable DM candidates. We found that the correct DM relic density can be reproduced at the same time that direct detection bounds are satisfied. Within the same parameter space we can also explain the muon $g-2$ and the W boson mass anomalies.

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