

Weak Ergodicity Breaking in Non-Hermitian Many-body Systems

Qianqian Chen¹, Shuai A. Chen², Zheng Zhu^{1,3*}

1 Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China

2 The Hong Kong University of Science and Technology, Hong Kong, China

3 CAS Center for Excellence in Topological Quantum Computation, University of Chinese Academy of Sciences, Beijing, 100190, China

* zhuzheng@ucas.ac.cn

September 15, 2022

Abstract

The recent discovery of persistent revivals in the Rydberg-atom quantum simulator has revealed a weakly ergodicity-breaking mechanism dubbed quantum many-body scars, which are a set of nonthermal states embedded in otherwise thermal spectra. Until now, such a mechanism has been mainly studied in Hermitian systems. Here, we establish the non-Hermitian quantum many-body scars and systematically characterize their nature from dynamic revivals, entanglement entropy, physical observables, and energy level statistics. Notably, we find the non-Hermitian quantum many-body scars exhibit significantly enhanced coherent revival dynamics when approaching the exceptional point. The signatures of non-Hermitian scars switch from the real-energy axis to the imaginary-energy axis after a real-to-complex spectrum transition driven by increasing non-Hermiticity, where an exceptional point and a quantum tricritical point emerge simultaneously. We further examine the stability of non-Hermitian quantum many-body scars against external fields, reveal the non-Hermitian quantum criticality and eventually set up the whole phase diagram. The possible connection to the open quantum many-body systems is also explored. Our findings offer insights for realizing long-lived coherent states in non-Hermitian many-body systems.

Contents

1	Introduction	2
2	Model setup and Methods	4
3	Results	5
3.1	Non-Hermitian quantum criticality	5
3.2	The periodic revivals and overlaps of specific states	7
3.3	Entanglement entropy and physical observables	7
3.4	Energy level statistics	10
3.5	Connection to open quantum many-body systems	11
4	Summary and Discussion	12

A Correspondence to a master equation of Lindblad form.	14
References	15

1 Introduction

The quantum ergodicity governed by the eigenstate thermalization hypothesis (ETH) depicts most isolated quantum many-body systems locally evolving into an equilibrium statistical ensemble [1–4], and plays a fundamental role in bridging quantum physics and statistical mechanics. Due to the quest for realizing long-lived coherent dynamics, tremendous efforts have been devoted to the ergodicity-breaking mechanisms. It is well-known that, in the presence of an extensive number of conserved quantities, such as the integrable systems [5–8] and many-body localized (MBL) phase [9], the systems fail to thermalize and strongly break ergodicity. In contrast, the quantum many-body scar (QMBS) systems [10–14], which have much fewer numbers of conserved quantities and are free of disorder, exhibit a distinct ergodicity breaking mechanism [15–28, 28–37, 37–44]. The QMBS system typically consists of both thermal and nonthermal eigenstates, and is distinguished by specific initial states experiencing periodic revivals, as first observed in an ultra-cold Rydberg atom chain [45].

Until now, the ergodicity-breaking mechanisms are mainly focused on the ideal isolated systems with Hermiticity. Nevertheless, perfect Hermiticity can be broken down [46–54]. The non-Hermiticity could either arise from the non-reciprocal process, such as the cold-atom platforms [55–62] with spontaneous decay or imaginary field, or introduced by the contact with thermal or nonthermal environments, namely the open quantum systems [63–71]. Unlike the Hermitian systems, the study of the ergodicity breaking in many-body non-Hermitian systems remains in the early stage.

The non-Hermiticity hosts many peculiar phenomena [see reviews [53, 71, 72] and reference therein] beyond the Hermitian framework, including complex-valued energy spectra, a biorthonormal basis, the non-Hermitian quantum criticality and the exceptional points (EP) with simultaneous coalescence of eigenvalues and eigenstates. In quantum many-body systems, the interplay between strong interaction and non-Hermiticity may cause unseen thermodynamic phenomena that are far from equilibrium, and thus it is of fundamental importance to ask whether the features decisive for the thermalization/non-thermalization in a Hermitian system still persist when one of the most fundamental premises in quantum mechanics, i.e., Hermiticity, is broken non-perturbatively.

In a different context, the non-Hermiticity is also closely relevant to open quantum systems. In reality, perfect isolated systems with Hermiticity hardly exist due to inevitable contact with the environmental bath. The system coupled to a thermal bath usually relaxes to a Gibbs ensemble with the temperature of bath [64–67], while the system coupled to a nonthermal bath may host distinct thermalization mechanisms [68, 69] due to the arbitrary nonunitary process. Theoretically, the open quantum system is commonly described by the Lindblad master equation [63]. Alternatively, the dynamics of open quantum systems could also be captured by a non-Hermitian Hamiltonian under certain circumstances, for instance, when quantum jumps can be neglected under the postselection [70] or in semi-classical systems [71]. Then the investigation of non-Hermitian many-body Hamiltonian is insightful for open many-body systems.

Previous studies have made considerable progress on the strong ergodicity breaking and

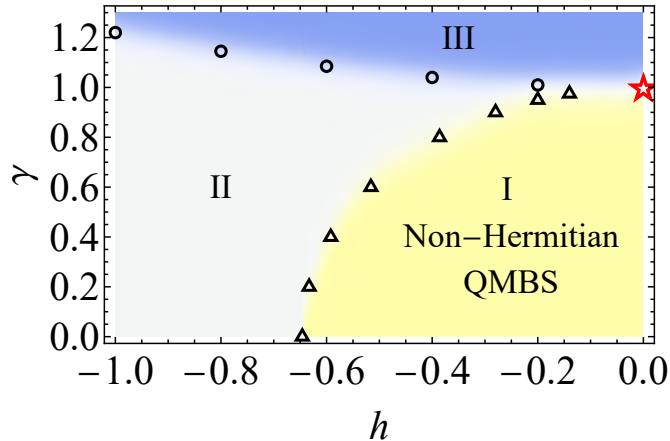


Figure 1: Ground-state phase diagram and excited-state properties. We identify three distinct phases (I, II, III) of model Hamiltonian [5] as a function of γ and h , the phase boundaries of which are determined by the derivatives of the ground-state energy and the fidelity susceptibility. The triangles (circles) are quantum critical points of continuous (first-order) quantum phase transitions driven by the external fields h (strength of non-Hermiticity γ). The red star denotes the quantum tricritical point, which is an exceptional point (EP) and also features a real-to-complex energy spectrum transition. Moreover, by probing the excited-state properties from the dynamics of quantum fidelity for $|Z_2\rangle$ (Fig. 2a), the entanglement entropy (Fig. 4a), the physical observable (Fig. 4b) and the energy level statistics (Fig. 5), we identify that the non-Hermitian many-body scarred states exist in parameter regime (I) with real energy spectra. The phase diagram is determined with data steps $\Delta h = 0.005, \Delta \gamma = 0.005$ for $N = 26$ system.

non-Hermitian ergodicity [41, 73–79]. However, whether the weak ergodicity breaking exists in the presence of non-Hermiticity is still elusive both theoretically and experimentally. Motivated by the above, in this work, we propose a weak ergodicity breaking mechanism in non-Hermitian systems from QMBS in a non-perturbative way, named the non-Hermitian QMBS, and characterize their nature via the periodic many-body revivals in dynamics, the entanglement entropy, the physical observables and the energy-level statistics using both the bi-orthogonal and self-normal eigenstates [80, 81]. We examine their stability in the presence of external fields, find their critical features when approaching the exceptional point, reveal the non-Hermitian quantum criticality, and finally establish the whole ground-state phase diagram with corresponding excited-state properties, as illustrated in Fig. 1. Interestingly, we find that the fingerprints of non-Hermitian scarred states switch from the real-energy axis to the imaginary-energy axis after a real-to-complex spectra transition, where an exceptional point and a quantum tricritical point emerge simultaneously. In particular, we find that the non-Hermitian QMBS exhibit longer revival periods and become fragile as the non-Hermiticity strength increases in the real-spectrum region, but they still exhibit substantial stability. The insights into the scarred dynamics in an open system are also discussed. Our construction of the non-Hermitian QMBS might be probed via the newly developed measurement of the non-Hermiticity or future advanced techniques in ultracold-atom platforms [45, 55–61].

2 Model setup and Methods

Before establishing the non-Hermitian quantum many-body scar, we first introduce the Hermitian counterparts realized in a Rydberg-atom quantum simulator [45, 82] with the Hamiltonian

$$H_{\text{Ryd}} = \sum_{i=1}^N \left(\frac{\Omega}{2} \sigma_i^x + \Delta n_i \right) + \sum_{i<j} V_{i,j} n_i n_j . \quad (1)$$

Here, $\sigma_i^x = |g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|$ couples an atom between the ground state $|g_i\rangle$ and the Rydberg excited state $|r_i\rangle$ at position i , which is realized by a two-photon transition and driven at Rabi frequency Ω . Δ denotes the strength of the laser detuning and $n_i = |r_i\rangle\langle r_i|$. The potential $V_{i,j} \propto 1/R_{i,j}^6$ characterizes the van der Waals interaction between atoms in Rydberg states at a distance $R_{i,j}$. In the limit of strong nearest-neighbor interactions $V_{i,i+1} \gg \Omega$, the system can be effectively described by

$$H_{\text{PXP}} = \sum_i^N P_{i-1} \sigma_i^x P_{i+1} , \quad (2)$$

without simultaneous Rydberg excitations of nearest neighbors [10,11], where $P_i = (1 - \sigma_i^z)/2$ is a projection operator with $\sigma_i^z = |r_i\rangle\langle r_i| - |g_i\rangle\langle g_i|$. Here we introduce the non-Hermiticity by generalizing the symmetric coupling between $|r_i\rangle$ and $|g_i\rangle$ to be non-reciprocal, i.e.,

$$|g_i\rangle\langle r_i| + |r_i\rangle\langle g_i| \rightarrow (1 - \gamma) |g_i\rangle\langle r_i| + (1 + \gamma) |r_i\rangle\langle g_i| , \quad (3)$$

so as to yield a non-Hermitian many-body Hamiltonian,

$$H_{\text{nH-PXP}} = \sum_j^N P_{j-1} \left(\sigma_j^x + i\gamma \sigma_j^y \right) P_{j+1} , \quad (4)$$

where the parameter $\gamma \in \mathbb{R}$ denotes the strength of non-Hermiticity. Incidentally, the non-Hermitian term in Eq. (4) could also be regarded as the application of an imaginary magnetic field [62]. In the following, besides establishing and characterizing the non-Hermitian QMBS states, we will also examine their stability against an external magnetic field $h \in \mathbb{R}$. The whole Hamiltonian can be written as

$$H = H_{\text{nH-PXP}} + \sum_j^N h \sigma_j^z . \quad (5)$$

To examine the whole spectrum of the model Hamiltonian (5) as a function of γ, h , we use the exact diagonalization (ED) approach. We utilize both the translational symmetry and the spatial inversion symmetry under the periodic boundary condition (PBC), which enables us to fully diagonalize the Hamiltonian up to $N = 32$ sites. The quantum number of the momentum is labelled as k ($k = 2\pi m/N$ with $m = -N/2, \dots, N/2$), while the inversion symmetric or anti-symmetric sector is denoted as I ($I = +, -$). The quantum many-body scarred states are located in the $(k, I) = (0, +)$ and $(k, I) = (\pi, -)$ sectors, both of which give similar results, we therefore mainly focus on the $(k, I) = (0, +)$ sector with Hilbert space dimension up to $D = 77, 436$ for $N = 32$ system.

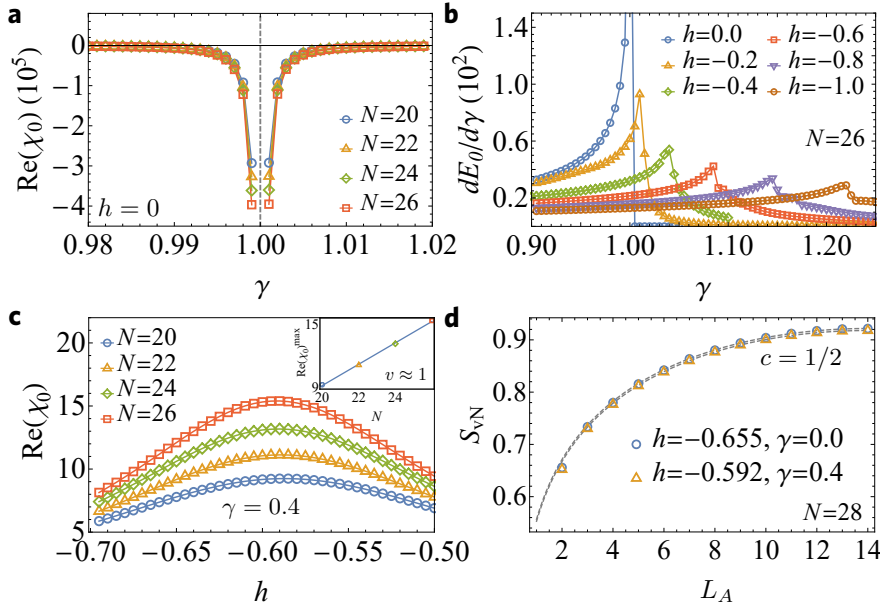


Figure 2: Exceptional points and quantum critical points. **a** The real part of the generalized fidelity susceptibility $\text{Re}(\chi_0)$ with respect to the strength of non-Hermiticity γ . We remark that the real parts of the eigenenergies are all zero for $\gamma > 1$ and $h = 0$. Thus, to depict the behavior of the imaginary eigenenergies at such parameter region, here we take the states with the lowest imaginary parts of the eigenenergies as ground states. The divergence of $\text{Re}(\chi_0)$ towards negative infinity when approaching $\gamma_c = 1$ signifies an exceptional point (EP). **b** The first order derivative of the ground-state energy with different fixed magnetic field h and the singularity characterize the quantum phase transitions driven by tuning the strength of non-Hermiticity γ . **c** The real part of the generalized fidelity susceptibility $\text{Re}(\chi_0)$ as a function of the external fields h . The inset shows the finite-size scaling of the maxima of $\text{Re}(\chi_0)$, where linear fit suggests the critical exponent $v \approx 1$. **d** The entanglement entropy of the ground states S_{vN} as a function of the subsystem length L_A for $N = 28$ system. At the critical points, the numerical data can be fitted through $S_{vN} \sim c/3 \ln(\sin(\pi L_A/N)) + \text{const.}$ (dashed gray lines) with the central charge $c = 1/2$.

3 Results

3.1 Non-Hermitian quantum criticality

We begin with establishing the ground-state phase diagram using the bi-orthogonal eigenstates in the γ - h plane before examining the excited-state properties. As shown in Fig. 1, the black triangles and the circles denote the boundaries of phases (I, II, III). A quantum tricritical point (red star) emerges at $(\gamma, h) = (1, 0)$, which is also an exceptional point (EP) and a real-to-complex spectrum transition point simultaneously. Here, for the consistency of the definition, we define the ground state as the state with the smallest real eigenenergy like in Hermitian systems unless stated otherwise, and our main focus phase (I) has fully real eigenvalues. To examine the phase boundaries, we compute both the derivatives of the ground-state energy and the fidelity susceptibility. We adopt the generalized fidelity \mathcal{F} ,

$$\mathcal{F}(\lambda, \delta\lambda) = \langle \psi_L(\lambda) | \psi_R(\lambda + \delta\lambda) \rangle \langle \psi_L(\lambda + \delta\lambda) | \psi_R(\lambda) \rangle, \quad (6)$$

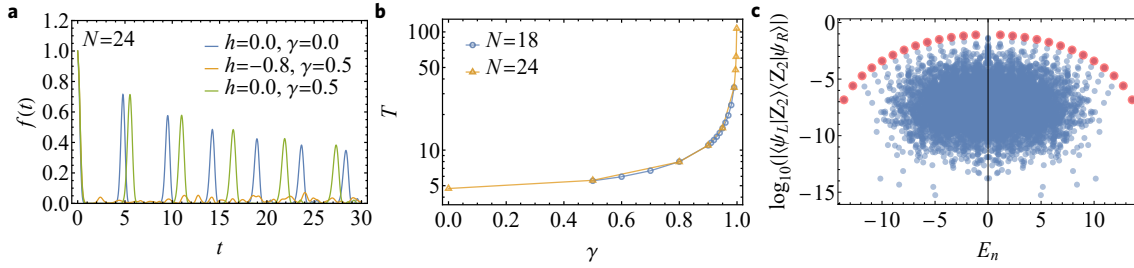


Figure 3: The quantum fidelity in dynamics. **a** The fidelity dynamics of initial $|Z_2\rangle$ state $f(t) = |\langle Z_2|Z_2(t)\rangle|^2$ exhibits coherent periodic revivals with period T in phase (I) (green) but collapses abruptly in phase (II) (yellow). The Hermitian case (blue) is depicted for comparison. **b** The revival period T as a function of the non-Hermiticity strength γ at $h = 0$, which exhibits divergent behavior when approaching the exceptional point $(\gamma, h) = (1, 0)$. **c** The overlap between eigenstates and the $|Z_2\rangle$ state with respect to eigenenergies. Red dots denote non-Hermitian quantum many-body scarred states with approximate equal energy spacing $\Delta E = 2\pi/T$. Data shown are for $N = 26$ and $\gamma = 0.5, h = 0$ in the zero and π -momentum sector.

where $|\psi_L\rangle$ ($|\psi_R\rangle$) is the bi-orthogonal left (right) eigenvectors satisfying

$$H|\psi_R\rangle = E|\psi_R\rangle, \quad H^\dagger|\psi_L\rangle = E^*|\psi_L\rangle. \quad (7)$$

The corresponding fidelity susceptibility χ can be obtained via

$$\chi(\lambda) \approx (1 - \mathcal{F})/\delta\lambda^2. \quad (8)$$

Physically, the divergence of χ towards negative infinity signifies EPs [83–85].

We first probe the quantum phase transitions when tuning the non-Hermiticity strength γ . At $h = 0$, as shown in Fig. 2a, the real parts of the ground-state fidelity susceptibility $\text{Re}(\chi_0) \rightarrow -\infty$ with increasing system length when approaching $\gamma_c = 1$ from its two sides, which demonstrates an EP. The first-order derivative curve $dE_0/d\gamma$ also displays a singularity at γ_c (see Fig. 2b). These observations of EP are consistent with the evolution of the energy spectra, where the whole real spectra at $\gamma < 1$ develop into complex conjugate pairs at $\gamma > 1$, separated by a transition at $\gamma_c = 1$. The quantum phase transitions at other fixed fields h can also be identified similarly [see Fig. 2b], as denoted by the circles in Fig. 1.

By contrast, we find the Hamiltonian exhibits a continuous phase transition when tuning the magnetic field h at $\gamma < 1$. Figure 2c shows $\text{Re}(\chi_0)$ as a function of h for different system lengths N , where the smooth curve as well as the increasing maximal values of $\text{Re}(\chi_0)$ with N indicate the continuity of such a phase transition, which can also be confirmed by the absence of singularities in the first-order derivative curve dE_0/dh . We further analyze the critical exponent ν and the central charge c . The critical exponent ν can be directly extracted from the fidelity susceptibility via $\text{Re}(\chi_0)^{\text{max}} = L^{2/\nu-1}$ [81, 86], as shown in the inset of Fig. 2c, where the linear fitting demonstrates $\nu = 1$. The central charge can be obtained from the generalized bipartite von Neumann entanglement entropies (definition see below) with different subsystem length L_A through

$$S_{vN} \sim c/3 \ln(\sin(\pi L_A/N)) + \text{const}. \quad (9)$$

The logarithmic fitting in Fig. 2d indicates a finite central charge $c = 1/2$. Here we notice that the critical exponent ν and the central charge c are the same as the phase transition reported in the Hermitian limit (i.e., $\gamma = 0$) [87–90], indicating the same universality

class might be generalized to the non-Hermitian case. When $\gamma = 0$ and $h \rightarrow \infty$, the ground states are two-fold degenerate due to the projection constraint, both breaking the \mathbb{Z}_2 symmetry and violating thermalization. In particular, we find a quantum tricritical point when the above two phase transitions meet at the EP $(\gamma, h) = (1, 0)$.

3.2 The periodic revivals and overlaps of specific states

We have established the ground-state phase diagram in the above. Below we further examine the excited-state properties and show that the non-Hermitian QMBS states exist in the parameter regime of phase (I).

The existence of many-body scarred states in the phase (I) can be inferred by the periodic revival of the quantum fidelity for the experimentally realizable antiferromagnetic state $|Z_2\rangle \equiv |r_1 g_2 r_3 g_4 \dots\rangle$ (c.f. Fig. 3a), manifesting a long-time memory of the initial state in the non-Hermitian many-body systems. The revival period T corresponds to a characteristic energy interval ΔE of the scarred states through the relation $T = 2\pi/\Delta E$. Strikingly, we find such revival period T tends to diverge when approaching the exceptional point $(\gamma, h) = (1, 0)$, as depicted in Fig. 3b. This observation suggests the enhanced coherence time in revival dynamics with increasing the non-Hermiticity strength towards an exceptional point.

Moreover, the eigenstates that have large overlap with the initial product state $|Z_2\rangle$ in Fig. 3a are embedded in the energy spectrum with approximate same energy spacing ΔE , as depicted by red dots in Fig. 3c. The number of such states is equal to the system size N . Since the features of these eigenstates in the non-Hermitian many-body system are similar to the Hermitian QMBS, we dub them non-Hermitian QMBS. Besides exhibiting a longer revival period T compared to the Hermitian PXP model at $(\gamma, h) = (0, 0)$, the non-Hermitian QMBS in phase (I) also become more fragile against the external field h with increasing non-Hermitian strength γ . In contrast to phase (I), such revivals of the initial product state $|Z_2\rangle$ are absent in phase (II), exhibiting distinct features of excited states in these two phases.

3.3 Entanglement entropy and physical observables

Below we will show that the non-Hermitian QMBS can also be characterized by the abnormally low entanglement entropy and the eigenstate expectations of physical observables that deviate from those in the bulk spectrum.

In Hermitian many-body systems, entanglement entropy is a complementary way to examine thermalization. We generalize the von Neumann entanglement entropy to non-Hermitian many-body systems. We consider the non-Hermitian density matrix of the n th state ρ_n defined by

$$\rho_n = |\psi_{R,n}\rangle \langle \psi_{L,n}| \quad (10)$$

with the biorthonormal relation $\langle \psi_{L,n} | \psi_{R,m} \rangle = \delta_{nm}$ [80], and study a generic complex entanglement entropy

$$S_{\text{vN}} = -\text{Tr}_A (\rho_{A,n} \ln |\rho_{A,n}|) , \quad (11)$$

in the non-Hermitian many-body system [91–93]. Here, $\rho_{A,n}$ is the reduced density matrix for subsystem A after tracing out the rest of the system. Figure 4a shows one typical example of the generalized S_{vN} in the phase (I) at $\gamma = 0.1$ and $h = 0$. In contrast to the highly entangled state, there are a set of eigenstates exhibit abnormally low entanglement with approximately equal energy difference, as marked by the red dots. Here, we point out that these non-Hermitian scarred states are all located at the $k = 0, \pi$ momentum sectors, which also resemble the Hermitian counterparts. Notably, we find large overlaps between

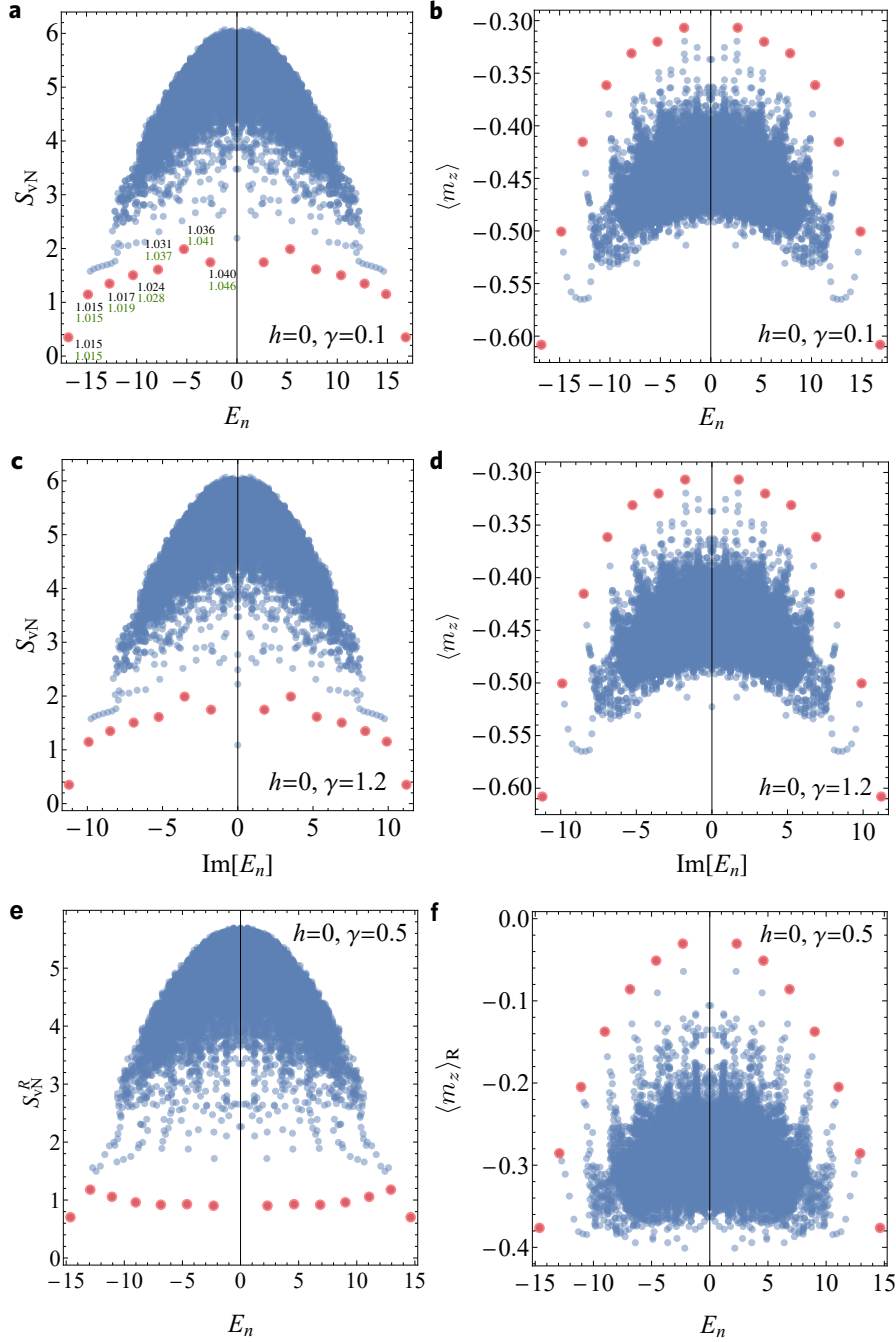


Figure 4: Entanglement entropies and physical observables. Panels (a, c, e) show the generalized bipartite von Neumann entanglement entropies S_{vN} (a, c) and S_{vN}^R (e) with respect to eigenenergies for all eigenstates. The non-Hermitian quantum many-body scarred states exhibit lower entanglement entropy and are highlighted by red dots. The values of overlaps between scar states at $\gamma = 0$ and the corresponding left (right) scar states at $\gamma = 0.1$ are written in black (green) color in a alongside the associated states. Panels (b, d, f) show the generalized expectations of local observables $\langle m_z \rangle$ (b, d) and $\langle m_z \rangle_R$ (f), the scarred states (red dots) are clearly distinct from other states and are exactly the ones identified from the entanglement entropy. All the calculations presented here are for the systems with length $N = 28$ in the momentum sector $k = 0$ at $h = 0$.

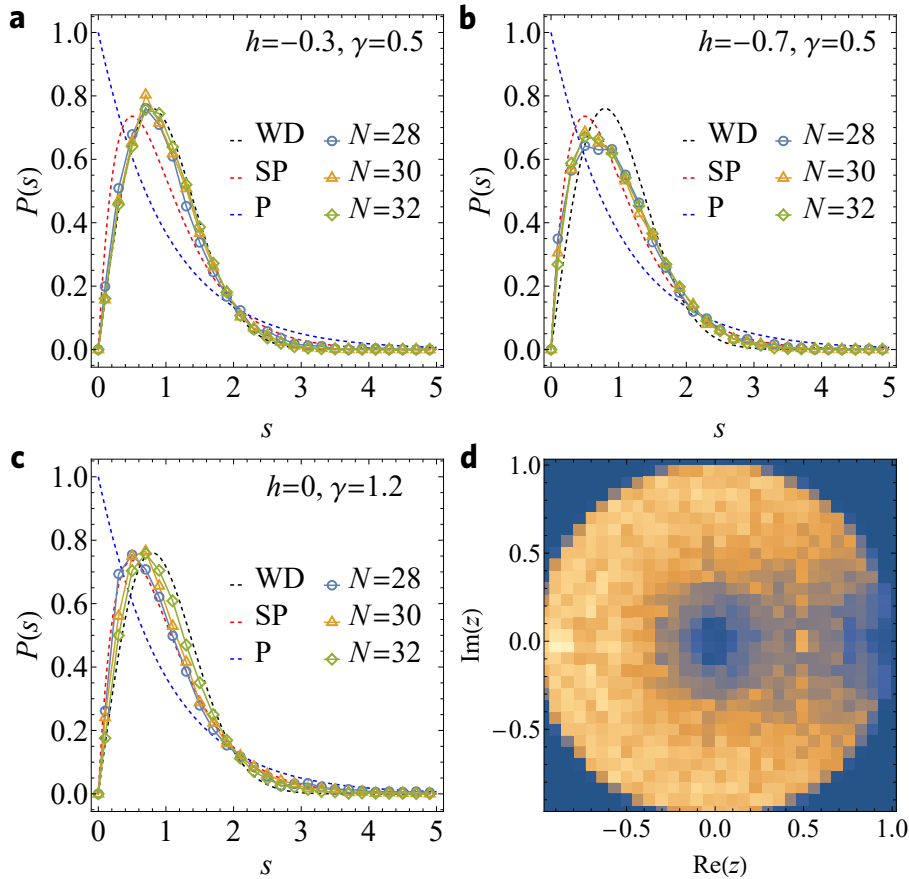


Figure 5: Typical examples of energy level spacing statistics for three phases. **a** Phase (I) with $h = -0.3, \gamma = 0.5$. **b** Phase (II) with $h = -0.7, \gamma = 0.5$ **c** Phase (III) with $h = 0, \gamma = 1.2$. As comparison, Wigner-Dyson (WD) statistics of the Gaussian orthogonal ensemble (GOE) $\pi s/2e^{-\pi s^2/4}$ (dashed black), semi-Poisson (SP) statistics $4s^{-2s}$ (dashed red) and Poisson (P) statistics e^{-s} (dashed blue) are plotted. Here, we consider the zero-momentum inversion-symmetric $(k, I) = (0, +)$ sector under periodic boundary condition (PBC), for which we eliminate 20% of the eigenenergies found at the spectrum's edges and perform the spectrum unfolding. **d** Density plot of the complex ratio z at $h = -0.3, \gamma = 1.2$ in the complex plane for the systems with length $N = 32$, the suppressed ratio density around the origin and for small angles of z suggests the significant level repulsion.

the scarred states at $\gamma = 0$ and the corresponding left (right) scarred states at $\gamma = 0.1$, the values of which are indicated in black (green) color in Fig. 4a. When approaching the EP $(\gamma, h) = (1, 0)$ along $h = 0$, such typical entanglement entropy structure of non-Hermitian QMBS still persists. As for the self-normal assumption $\langle \psi_{R,n} | \psi_{R,n} \rangle = 1$, Fig. 4e shows the self-normal entanglement entropy

$$S_{vN}^R = -\text{Tr}_A (\rho_{A,n}^R \ln |\rho_{A,n}^R|) , \quad (12)$$

with $\rho_n^R = |\psi_{R,n}\rangle\langle\psi_{R,n}|$ at $(h, \gamma) = (0, 0.5)$, where the typical low entanglement outliers with nearly equal energy difference (marked by red dots) are still existent. These results illustrate that the features of QMBS states are relatively robust over the non-Hermitian parameter space with both biorthonormal and self-normal assumption.

Physical observables can also identify the violation of the thermalization for these non-Hermitian scarred states. We examine the expectation value of the magnetization $m_z \equiv \sum_i \sigma_i^z / N$ of all eigenstates. Here, both the expectation value with the biorthonormal

Table 1: Averaged $\langle r \rangle$ and $\langle \cos \theta \rangle$ for different system sizes N . Typical values of Ginibre and Poisson distribution are also given. The averaged values converge to Ginibre distribution with an increasing N .

	$N = 32$	$N = 30$	$N = 28$	Ginibre	Poisson
$\langle r \rangle$	0.737	0.753	0.737	0.74	$\frac{2}{3} \approx 0.667$
$\langle \cos \theta \rangle$	-0.181	-0.150	-0.119	-0.24	0

eigenvectors $\langle m_z \rangle \equiv \langle \psi_L | m_z | \psi_R \rangle$ and self-normal eigenvectors $\langle m_z \rangle_R \equiv \langle \psi_R | m_z | \psi_R \rangle$ are considered. Due to the translation symmetry, we have $\langle m_z \rangle = \langle \psi_L | \sigma_1^z | \psi_R \rangle$ and $\langle m_z \rangle_R = \langle \psi_R | \sigma_1^z | \psi_R \rangle$. As shown in Fig. 4b, f, we find that a series of states (denoted by red dots) with maximal $\langle m_z \rangle$ and $\langle m_z \rangle_R$ have approximately equal energy-level spacing, and are distinct from other states clearly, similar to the violation of ETH in Hermitian systems. In particular, these special states are exactly the non-Hermitian QMBS states characterized by the low entanglement entropy in Fig. 4a,e. Thus, these eigenstate characters decisive for the weak ergodicity breaking in a Hermitian system still exist in non-Hermitian many-body physics.

Interestingly, in case of complex energy spectra at $\gamma > 1$, $h = 0$, we also find a set of eigenstates (marked as red dots in Fig. 4c,d) with low entanglement entropy and maximal expectation values of $\langle m_z \rangle$ but with respect to the imaginary part of eigenenergy. These states are analogous to the non-Hermitian scarred states at $\gamma < 1$.

3.4 Energy level statistics

Besides the above-mentioned characteristics of the non-Hermitian QMBS, in this section, we further reveal their chaotic nature by the eigenenergy level-spacing distributions and ratios.

Figures 5a-c show the nearest-level-spacing distribution $P(s)$ of the statistics parameter $s_n = |E_{n+1} - E_n|$ for real or imaginary spectrum, where the E_n are increasingly ordered according to the real or imaginary part, respectively. In phase (I) with non-Hermitian QMBS, as shown in Fig. 5a, although the scarred states have low entanglement entropy, the bulk of the unfolded level statistics displays a Wigner-Dyson (WD) distribution [94,95], indicating a prominent feature of the quantum chaos, which rules out the previously known examples of non-ergodic cases in non-Hermitian systems [73]. Here, we remark that a chaotic non-Hermitian system is expected to follow Ginibre statistics [75]. However, when the complex spectra become totally real, one may still expect the WD distribution for a chaotic system [75], as in our phase (I) case.

In phase (II), we find the level-spacing statistics resemble semi-Poisson (SP) statistics near the criticality [c.f. Fig. 5b], at least for the largest system we have reached. As non-universal statistics, SP distribution displays the intermediate statistics between Poisson and WD, and it is typical of the pseudointegrable systems [96]. Here the slight deviation from SP might be induced by the finite size effect. Remarkably, when the whole spectra become imaginary at $\gamma > 1$, $h = 0$ in phase (III), Fig. 5c shows that the level statistics tend to approach the WD distribution, exhibiting the same feature as the phase (I) with non-Hermitian QMBS.

The energy spectra become complex at finite external field h . To avoid the ambiguity of unfolding complex spectrum, we instead consider the complex level-spacing ratios [97]

$$z_n = \frac{E_n^{\text{NN}} - E_n}{E_n^{\text{NNN}} - E_n}. \quad (13)$$

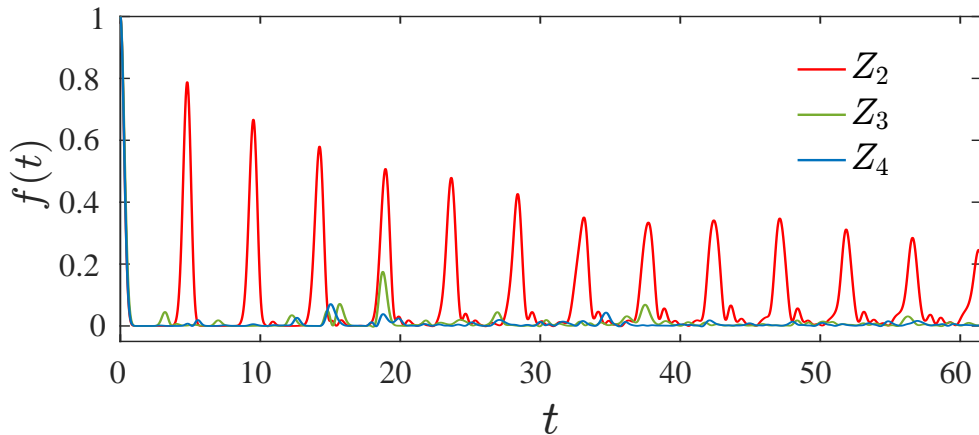


Figure 6: Quantum fidelity by the Lindblad master equation. Evolution of the quantum fidelity $f(t)$ exhibits pronounced revivals for initial product state $|Z_2\rangle$, similar to the periodic revivals obtained by the non-Hermitian model in Eq. (5). By contrast, other initial states like $|Z_3\rangle$ and $|Z_4\rangle$ show a complete absence of quantum revivals. Here, parameters are $\gamma = 0.5$ and $h = 0$ for $N = 14$ system with periodic boundary condition.

Here, E_n is referred to as the n th real or complex eigenenergy, of which the nearest neighbor and next-to-nearest neighbor in the complex plane are E_n^{NN} and E_n^{NNN} respectively. Moreover, when the distribution of z is anisotropic, we also consider the radial and angular marginal distribution with the relation $z \equiv re^{i\theta}$. We calculate the radial and angular marginal distributions of the complex ratio z based on

$$P(r) = \int d\theta r P(r, \theta), \quad P(\theta) = \int dr r P(r, \theta) \quad (14)$$

to get the means

$$\langle r \rangle = \int dr P(r) r, \quad \langle \cos \theta \rangle = \int d\theta P(\theta) \cos \theta. \quad (15)$$

Here, statistics of the complex ratio z are taken from eigenenergies lying within 80% of the real and imaginary parts from the middle of the spectrum in the zero-momentum inversion-symmetric $(0, +)$ sector. As shown in Fig. 5d for $(h, \gamma) = (-0.3, 1.2)$, we find strongly suppressed ratio density around the origin and for small angles of z , demonstrating the significant level repulsion in this phase. We further compare the averaged $\langle r \rangle$ and $\langle \cos \theta \rangle$ with Ginibre and Poisson distributions. As shown in Table 1, there is an apparent convergence of our results to the average values of Ginibre distribution with the increase of N . Our results of the complex level spacing ratios numerically demonstrate that the energy spectrum in phase III shows Ginibre distribution, a key feature of quantum chaos in systems with complex spectra.

3.5 Connection to open quantum many-body systems

Now we discuss the potential connection of the non-Hermitian QMBS to the open quantum many-body systems. We will show that, the weak ergodicity breaking in our non-Hermitian model is insightful for slow relaxation dynamics of certain initial states in open quantum many-body systems [98] under certain circumstance.

Generally, an open quantum many-body system can be described by the Lindblad master equation [63, 70]

$$\dot{\rho} = -i(H_{\text{NH}}\rho - \rho H_{\text{NH}}^\dagger) + \mathcal{D}(\rho), \quad (16)$$

where $\mathcal{D}(\rho) = \sum_j L_j \rho L_j^\dagger$ denotes quantum jumps and H_{nH} corresponds to an effective non-Hermitian Hamiltonian. The Lindblad operators L_i are related to the non-Hermitian terms in H_{nH} via (details see Appendix)

$$-\frac{i}{2}L_j^\dagger L_j = i\gamma \left(g - P_{j-1} \sigma_j^y P_{j+1} \right), \quad (17)$$

where g is a purely imaginary constant to ensure the semi-positive definiteness. We compute the evolution of quantum fidelity for certain initial states by the Lindblad master equation and compare their relaxation dynamics with those obtained from our non-Hermitian model in Eq. (5). Remarkably, as shown in Fig. 6 for $\gamma = 0.5$, the experimentally realizable $|Z_2\rangle$ initial product state [45, 82] exhibits long-time periodic revivals when a constant shift g is large, similar to the behavior of quantum many-body scars shown in Fig. 3a. By contrast, for other initial states like $|Z_3\rangle$ and $|Z_4\rangle$, we do not find any pronounced revivals in the fidelity. To understand such observations, we provide an alternative interpretation based on a perturbative analysis in the large g limit, where the quantum jumps and non-Hermiticity get suppressed (see Appendix for details), leading to an effective description of open quantum systems in the weak non-Hermiticity limit of Eq. (5).

4 Summary and Discussion

In this work, we study the exemplary mechanism of the weak ergodicity breaking in non-Hermitian many-body systems, named as non-Hermitian QMBS, in a non-perturbative manner. Using both the bi-orthogonal and self-normal eigenstates, we systematically characterize the non-Hermitian QMBS from the perspectives of the quantum revivals in dynamics, the eigenstate entanglement entropy in quantum information, the physical observables measurable in quantum simulation, and quantum chaos in statistical physics. Moreover, we also illustrate the robustness of non-Hermitian QMBS both against external field and the influence of EP, reveal the non-Hermitian quantum criticality and establish the whole ground-state phase diagram. Notably, in contrast to the Hermitian QMBS, we find the enhanced coherent revival dynamics of non-Hermitian QMBS near the exceptional point. Finally, the instructive insights on scarred quantum dynamics in open quantum many-body systems are provided and the connection is analyzed in the perturbative limit.

For the Hermitian QMBS realized in the Rydberg-atom quantum simulator [45], the symmetric coupling between the ground state and the Rydberg state is induced by a two-photon process via an intermediate level. Recently, much effort has been devoted to realizing the non-Hermiticity in cold-atom platforms [55–61], notably the Rydberg atoms [58, 59]. By tuning the coupling non-symmetrically [61], or by applying the imaginary magnetic field [62], our results are likely to be verified in recent experimental platforms or in future developed platforms. The non-Hermitian QMBS proposed in this work may also stimulate more experimental activities to realize long-lived coherent states storing the initial quantum information in open quantum many-body systems.

Our work might serve as a starting point to investigate the weak ergodicity breaking mechanism in the absence of Hermiticity. It would be interesting to study the quantum many-body scars in other non-Hermitian many-body systems or models, which may have distinct mechanisms from the non-Hermitian PXP model. Further investigations on the connections of non-Hermitian QMBS to open systems beyond the perturbative limit is also fundamentally important to deepen our understanding. In addition, it may also stimulate future studies on the interplay among thermalization, the non-Hermiticity, and the strong correlations in many-body systems.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No. 12074375), the Innovation Program for Quantum Science and Technology (Grant No. 2-6) and the Strategic Priority Research Program of CAS (Grant No.XDB33000000).

In this appendix, we give details for the correspondence between the non-Hermitian Hamiltonian and a master equation of Lindblad form

A Correspondence to a master equation of Lindblad form.

We present the explanatory details on the correspondence between the non-Hermitian model in Eq. (5) and an open system whose dynamics can be described by a master equation of the Lindblad form.

A master equation of the Lindblad form describes quantum dynamics of an open system that is bathed in an environment under Markov approximation. In general, we can write the master equation as (with $\hbar = 1$)

$$\dot{\rho} = -i \left(H_{\text{nH}} \rho - \rho H_{\text{nH}}^\dagger \right) + \mathcal{D}(\rho), \quad (18)$$

where H_{nH} is a non-Hermitian Hamiltonian and is composed of two parts: the Hermitian H_0 and the non-Hermitian $-\frac{i}{2} L_j^\dagger L_j$,

$$H_{\text{nH}} = H_0 - \sum_j \frac{i}{2} L_j^\dagger L_j \quad (19)$$

with L_j being the Lindblad operators. The last term $\mathcal{D}(\rho) = \sum_j L_j \rho L_j^\dagger$ in Eq. (18) are quantum jump terms. When the quantum jump term can be neglected, the H_{nH} will play a role in dominating the dynamical processes.

The main step towards the correspondence is to construct the Lindblad operator with respect to the non-Hermitian Hamiltonian in Eq. (5) in the main text. For this purpose, we can introduce a constant shift term g to our model in Eq. (5) in the main text to ensure $g - P_{j-1} \sigma_j^y P_{j+1}$ semi-definite, and then the Lindblad operators L_j can be determined by the following equation,

$$-i\gamma \left(g - P_{j-1} \sigma_j^y P_{j+1} \right) = -\frac{i}{2} L_j^\dagger L_j \quad (20)$$

We remark that the constant shift term g imposes no influence on the eigenstates. When $g > 1$, we can express the Lindblad operators L_j perturbatively. For the leading orders, we have

$$L_j = \sqrt{2g\gamma} \left[1 - \frac{1}{2g} \left(1 + \frac{1}{8g^2} \right) P_{j-1} \sigma_j^y P_{j+1} - \frac{1}{8g^2} P_{j-1} P_{j+1} \right] + \mathcal{O} \left(\frac{\sqrt{g}}{g^3} \right). \quad (21)$$

As a directed consequence, the quantum jump terms $\mathcal{D}(\rho)$ further make extra contributions to non-Hermitian terms by observing

$$\begin{aligned} \mathcal{D}(\rho) = \gamma \sum_j \left[2g\rho - \left\{ \left(1 + \frac{1}{8g^2} \right) P_{j-1} \sigma_j^y P_{j+1} \right. \right. \\ \left. \left. + \frac{1}{4g} P_{j-1} P_{j+1}, \rho \right\} + \frac{1}{2g} P_{j-1} \sigma_j^y P_{j+1} \rho P_{j-1} \sigma_j^y P_{j+1} \right], \quad (22) \end{aligned}$$

where $\{\cdot, \cdot\}$ is an anti-communication bracket and the last term in Eq. (A5) instead is the leading effective jump term. By ignoring the new quantum jump terms in Eq. (A5), we

obtain an effective non-Hermitian Hamiltonian

$$H_{\text{eff}} = \sum_i (P_{i-1} \sigma_i^x P_{i+1} - i \frac{\gamma}{8g^2} P_{i-1} \sigma_i^y P_{i+1} - i \frac{\gamma}{4g} P_{i-1} P_{i+1} + h \sigma_j^z) . \quad (23)$$

Clearly, non-Hermiticity gets suppressed when g increases. Thus, the corresponding open system can show a significant weak-ergodicity breaking with a long-time revival of an initial state in Fig. 6 of the main text. In this sense, we conclude that the non-Hermitian Hamiltonian in Eq. (5) of the main text offers instructive insights for understanding the quantum many-body scarred dynamics in an open system based on the well-established correspondence.

References

- [1] M. Srednicki, *Chaos and quantum thermalization*, Phys. Rev. E **50**, 888 (1994), doi:10.1103/PhysRevE.50.888.
- [2] M. Rigol, V. Dunjko and M. Olshanii, *Thermalization and its mechanism for generic isolated quantum systems*, nature **452**(7189), 854 (2008), doi:10.1038/nature06838.
- [3] R. Nandkishore and D. A. Huse, *Many-body localization and thermalization in quantum statistical mechanics*, Annual Review of Condensed Matter Physics **6**, 15 (2015), doi:10.1146/annurev-conmatphys-031214-014726.
- [4] L. D'Alessio, Y. Kafri, A. Polkovnikov and M. Rigol, *From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics*, Advances in Physics **65**(3), 239 (2016), doi:10.1080/00018732.2016.1198134.
- [5] M. Rigol, *Breakdown of Thermalization in Finite One-Dimensional Systems*, Phys. Rev. L **103**(10), 100403 (2009), doi:10.1103/PhysRevLett.103.100403.
- [6] J. M. Deutsch, *Quantum statistical mechanics in a closed system*, Phys. Rev. A **43**(4), 2046 (1991), doi:10.1103/PhysRevA.43.2046.
- [7] T. Kinoshita, T. Wenger and D. S. Weiss, *A quantum Newton's cradle*, nature **440**(7086), 900 (2006), doi:10.1038/nature04693.
- [8] G. Biroli, C. Kollath and A. M. Läuchli, *Effect of Rare Fluctuations on the Thermalization of Isolated Quantum Systems*, Phys. Rev. L **105**(25), 250401 (2010), doi:10.1103/PhysRevLett.105.250401.
- [9] D. A. Abanin, E. Altman, I. Bloch and M. Serbyn, *Colloquium: Many-body localization, thermalization, and entanglement*, Rev. Mod. Phys. **91**, 021001 (2019), doi:10.1103/RevModPhys.91.021001.
- [10] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn and Z. Papić, *Weak ergodicity breaking from quantum many-body scars*, Nature Physics **14**(7), 745 (2018).
- [11] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn and Z. Papić, *Quantum scarred eigenstates in a rydberg atom chain: Entanglement, breakdown of thermalization, and stability to perturbations*, Phys. Rev. B **98**, 155134 (2018), doi:10.1103/PhysRevB.98.155134.

- [12] M. Serbyn, D. A. Abanin and Z. Papić, *Quantum many-body scars and weak breaking of ergodicity*, Nature Physics **17**(6), 675 (2021), doi:10.1038/s41567-021-01230-2.
- [13] Z. Papić, *Weak ergodicity breaking through the lens of quantum entanglement*, arXiv preprint arXiv:2108.03460 (2021).
- [14] S. Moudgalya, B. A. Bernevig and N. Regnault, *Quantum Many-Body Scars and Hilbert Space Fragmentation: A Review of Exact Results*, arXiv e-prints arXiv:2109.00548 (2021).
- [15] E. J. Heller, *Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits*, Phys. Rev. L **53**(16), 1515 (1984), doi:10.1103/PhysRevLett.53.1515.
- [16] M. van Horssen, E. Levi and J. P. Garrahan, *Dynamics of many-body localization in a translation-invariant quantum glass model*, Phys. Rev. B **92**(10), 100305 (2015), doi:10.1103/PhysRevB.92.100305.
- [17] N. Shiraishi and T. Mori, *Systematic Construction of Counterexamples to the Eigenstate Thermalization Hypothesis*, Phys. Rev. L **119**(3), 030601 (2017), doi:10.1103/PhysRevLett.119.030601.
- [18] O. Vafek, N. Regnault and B. A. Bernevig, *Entanglement of exact excited eigenstates of the Hubbard model in arbitrary dimension*, SciPost Physics **3**(6), 043 (2017), doi:10.21468/SciPostPhys.3.6.043.
- [19] C. Chen, F. Burnell and A. Chandran, *How Does a Locally Constrained Quantum System Localize?*, Phys. Rev. L **121**(8), 085701 (2018), doi:10.1103/PhysRevLett.121.085701.
- [20] Z. Lan, M. van Horssen, S. Powell and J. P. Garrahan, *Quantum Slow Relaxation and Metastability due to Dynamical Constraints*, Phys. Rev. L **121**(4), 040603 (2018), doi:10.1103/PhysRevLett.121.040603.
- [21] S. Moudgalya, S. Rachel, B. A. Bernevig and N. Regnault, *Exact excited states of nonintegrable models*, Phys. Rev. B **98**(23), 235155 (2018), doi:10.1103/PhysRevB.98.235155.
- [22] S. Moudgalya, N. Regnault and B. A. Bernevig, *Entanglement of exact excited states of Affleck-Kennedy-Lieb-Tasaki models: Exact results, many-body scars, and violation of the strong eigenstate thermalization hypothesis*, Phys. Rev. B **98**(23), 235156 (2018), doi:10.1103/PhysRevB.98.235156.
- [23] W. W. Ho, S. Choi, H. Pichler and M. D. Lukin, *Periodic Orbits, Entanglement, and Quantum Many-Body Scars in Constrained Models: Matrix Product State Approach*, Phys. Rev. L **122**(4), 040603 (2019), doi:10.1103/PhysRevLett.122.040603.
- [24] C.-J. Lin and O. I. Motrunich, *Exact Quantum Many-Body Scar States in the Rydberg-Blockaded Atom Chain*, Phys. Rev. L **122**(17), 173401 (2019), doi:10.1103/PhysRevLett.122.173401.
- [25] T. Iadecola and M. Žnidarič, *Exact Localized and Ballistic Eigenstates in Disordered Chaotic Spin Ladders and the Fermi-Hubbard Model*, Phys. Rev. L **123**(3), 036403 (2019), doi:10.1103/PhysRevLett.123.036403.

- [26] K. Bull, I. Martin and Z. Papić, *Systematic Construction of Scarred Many-Body Dynamics in 1D Lattice Models*, Phys. Rev. L **123**(3), 030601 (2019), doi:10.1103/PhysRevLett.123.030601.
- [27] S. Moudgalya, A. Prem, R. Nandkishore, N. Regnault and B. A. Bernevig, *Thermalization and its absence within Krylov subspaces of a constrained Hamiltonian*, arXiv e-prints arXiv:1910.14048 (2019).
- [28] V. Khemani, C. R. Laumann and A. Chandran, *Signatures of integrability in the dynamics of Rydberg-blockaded chains*, Phys. Rev. B **99**(16), 161101 (2019), doi:10.1103/PhysRevB.99.161101.
- [29] M. Schechter and T. Iadecola, *Weak Ergodicity Breaking and Quantum Many-Body Scars in Spin-1 X Y Magnets*, Phys. Rev. L **123**(14), 147201 (2019), doi:10.1103/PhysRevLett.123.147201.
- [30] S. Choi, C. J. Turner, H. Pichler, W. W. Ho, A. A. Michailidis, Z. Papić, M. Serbyn, M. D. Lukin and D. A. Abanin, *Emergent SU(2) Dynamics and Perfect Quantum Many-Body Scars*, Phys. Rev. L **122**(22), 220603 (2019), doi:10.1103/PhysRevLett.122.220603.
- [31] N. Shibata, N. Yoshioka and H. Katsura, *Onsager's Scars in Disordered Spin Chains*, Phys. Rev. L **124**(18), 180604 (2020), doi:10.1103/PhysRevLett.124.180604.
- [32] T. Iadecola and M. Schechter, *Quantum many-body scar states with emergent kinetic constraints and finite-entanglement revivals*, Phys. Rev. B **101**(2), 024306 (2020), doi:10.1103/PhysRevB.101.024306.
- [33] D. K. Mark and O. I. Motrunich, *η -pairing states as true scars in an extended Hubbard model*, Phys. Rev. B **102**(7), 075132 (2020), doi:10.1103/PhysRevB.102.075132.
- [34] A. A. Michailidis, C. J. Turner, Z. Papić, D. A. Abanin and M. Serbyn, *Stabilizing two-dimensional quantum scars by deformation and synchronization*, Physical Review Research **2**(2), 022065 (2020), doi:10.1103/PhysRevResearch.2.022065.
- [35] P. Sala, T. Rakovszky, R. Verresen, M. Knap and F. Pollmann, *Ergodicity Breaking Arising from Hilbert Space Fragmentation in Dipole-Conserving Hamiltonians*, Physical Review X **10**(1), 011047 (2020), doi:10.1103/PhysRevX.10.011047.
- [36] K. Pakrouski, P. N. Pallegar, F. K. Popov and I. R. Klebanov, *Many-Body Scars as a Group Invariant Sector of Hilbert Space*, Phys. Rev. L **125**(23), 230602 (2020), doi:10.1103/PhysRevLett.125.230602.
- [37] S. Moudgalya, N. Regnault and B. A. Bernevig, *η -pairing in Hubbard models: From spectrum generating algebras to quantum many-body scars*, Phys. Rev. B **102**(8), 085140 (2020), doi:10.1103/PhysRevB.102.085140.
- [38] V. Khemani, M. Hermele and R. Nandkishore, *Localization from Hilbert space shattering: From theory to physical realizations*, Phys. Rev. B **101**(17), 174204 (2020), doi:10.1103/PhysRevB.101.174204.
- [39] J. Ren, C. Liang and C. Fang, *Quasisymmetry Groups and Many-Body Scar Dynamics*, Phys. Rev. L **126**(12), 120604 (2021), doi:10.1103/PhysRevLett.126.120604.

- [40] P. Zhang, H. Dong, Y. Gao, L. Zhao, J. Hao, Q. Guo, J. Chen, J. Deng, B. Liu, W. Ren *et al.*, *Probing quantum many-body scars on a superconducting processor*, arXiv preprint arXiv:2201.03438 (2022).
- [41] K. Pakrouski, P. N. Pallegar, F. K. Popov and I. R. Klebanov, *Group theoretic approach to many-body scar states in fermionic lattice models*, Phys. Rev. Research **3**, 043156 (2021), doi:10.1103/PhysRevResearch.3.043156.
- [42] H. Zhao, A. Smith, F. Mintert and J. Knolle, *Orthogonal quantum many-body scars*, Phys. Rev. Lett. **127**, 150601 (2021), doi:10.1103/PhysRevLett.127.150601.
- [43] H. Zhao, J. Vovrosh, F. Mintert and J. Knolle, *Quantum many-body scars in optical lattices*, Phys. Rev. Lett. **124**, 160604 (2020), doi:10.1103/PhysRevLett.124.160604.
- [44] W.-L. You, Z. Zhao, J. Ren, G. Sun, L. Li and A. M. Oleś, *Quantum many-body scars in spin-1 kitaev chains*, Phys. Rev. Research **4**, 013103 (2022), doi:10.1103/PhysRevResearch.4.013103.
- [45] H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, M. Endres, M. Greiner *et al.*, *Probing many-body dynamics on a 51-atom quantum simulator*, Nature **551**(7682), 579 (2017).
- [46] E. P. Wigner, *Normal Form of Antiunitary Operators*, Journal of Mathematical Physics **1**(5), 409 (1960), doi:10.1063/1.1703672.
- [47] C. M. Bender and S. Boettcher, *Real spectra in non-hermitian hamiltonians having \mathcal{PT} symmetry*, Phys. Rev. Lett. **80**, 5243 (1998), doi:10.1103/PhysRevLett.80.5243.
- [48] C. M. Bender, D. C. Brody and H. F. Jones, *Complex extension of quantum mechanics*, Phys. Rev. Lett. **89**, 270401 (2002), doi:10.1103/PhysRevLett.89.270401.
- [49] A. Mostafazadeh, *Pseudo-Hermiticity versus \mathcal{PT} symmetry: The necessary condition for the reality of the spectrum of a non-Hermitian Hamiltonian*, Journal of Mathematical Physics **43**(1), 205 (2002), doi:10.1063/1.1418246, math-ph/0107001.
- [50] A. Mostafazadeh, *Pseudo-Hermiticity versus \mathcal{PT} -symmetry. II. A complete characterization of non-Hermitian Hamiltonians with a real spectrum*, Journal of Mathematical Physics **43**(5), 2814 (2002), doi:10.1063/1.1461427, math-ph/0110016.
- [51] C. M. Bender, *Making sense of non-Hermitian Hamiltonians*, Reports on Progress in Physics **70**(6), 947 (2007), doi:10.1088/0034-4885/70/6/R03, hep-th/0703096.
- [52] A. Mostafazadeh, *Pseudo-Hermitian Representation of Quantum Mechanics*, International Journal of Geometric Methods in Modern Physics **07**(7), 1191 (2010), doi:10.1142/S0219887810004816, 0810.5643.
- [53] V. V. Konotop, J. Yang and D. A. Zezyulin, *Nonlinear waves in \mathcal{PT} -symmetric systems*, Rev. Mod. Phys. **88**, 035002 (2016), doi:10.1103/RevModPhys.88.035002.
- [54] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter and D. N. Christodoulides, *Non-Hermitian physics and \mathcal{PT} symmetry*, Nature Physics **14**(1), 11 (2018), doi:10.1038/nphys4323.
- [55] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter and D. N. Christodoulides, *Non-Hermitian physics and \mathcal{PT} symmetry*, Nat. Phys. **14**(1), 11 (2018), doi:10.1038/NPHYS4323.

- [56] J. Li, A. K. Harter, J. Liu, L. de Melo, Y. N. Joglekar and L. Luo, *Observation of parity-time symmetry breaking transitions in a dissipative Floquet system of ultracold atoms*, Nature Communications **10**, 855 (2019), doi:10.1038/s41467-019-08596-1.
- [57] Z. Zhou and Z. Yu, *Interaction effects on the \mathcal{PT} -symmetry-breaking transition in atomic gases*, Phys. Rev. A **99**, 043412 (2019), doi:10.1103/PhysRevA.99.043412.
- [58] J. A. Lourenço, G. Higgins, C. Zhang, M. Hennrich and T. Macrì, *Non-hermitian dynamics and \mathcal{PT} -symmetry breaking in interacting mesoscopic rydberg platforms*, arXiv preprint arXiv:2111.04378 (2021).
- [59] C. Hang, W. Li and G. Huang, *Nonlinear light diffraction by electromagnetically induced gratings with \mathcal{PT} symmetry in a rydberg atomic gas*, Phys. Rev. A **100**, 043807 (2019), doi:10.1103/PhysRevA.100.043807.
- [60] Y.-M. Liu, F. Gao, J.-H. Wu, M. Artoni and G. C. La Rocca, *Lopsided diffractions of distinct symmetries in two-dimensional non-hermitian optical gratings*, Phys. Rev. A **100**, 043801 (2019), doi:10.1103/PhysRevA.100.043801.
- [61] T. E. Lee and C.-K. Chan, *Heralded magnetism in non-hermitian atomic systems*, Phys. Rev. X **4**, 041001 (2014), doi:10.1103/PhysRevX.4.041001.
- [62] X. Peng, H. Zhou, B.-B. Wei, J. Cui, J. Du and R.-B. Liu, *Experimental observation of lee-yang zeros*, Phys. Rev. Lett. **114**, 010601 (2015), doi:10.1103/PhysRevLett.114.010601.
- [63] G. Lindblad, *On the generators of quantum dynamical semigroups*, Communications in Mathematical Physics **48**(2), 119 (1976).
- [64] H.-P. Breuer, F. Petruccione *et al.*, *The theory of open quantum systems*, Oxford University Press on Demand (2002).
- [65] U. Weiss, *Quantum dissipative systems*, World Scientific (2012).
- [66] H. Spohn, *Kinetic equations from hamiltonian dynamics: Markovian limits*, Rev. Mod. Phys. **52**, 569 (1980), doi:10.1103/RevModPhys.52.569.
- [67] A. Mitra, S. Takei, Y. B. Kim and A. J. Millis, *Nonequilibrium quantum criticality in open electronic systems*, Phys. Rev. Lett. **97**, 236808 (2006), doi:10.1103/PhysRevLett.97.236808.
- [68] Y. Ashida, K. Saito and M. Ueda, *Thermalization and heating dynamics in open generic many-body systems*, Phys. Rev. Lett. **121**, 170402 (2018), doi:10.1103/PhysRevLett.121.170402.
- [69] T. Shirai and T. Mori, *Thermalization in open many-body systems based on eigenstate thermalization hypothesis*, Phys. Rev. E **101**, 042116 (2020), doi:10.1103/PhysRevE.101.042116.
- [70] A. J. Daley, *Quantum trajectories and open many-body quantum systems*, Advances in Physics **63**(2), 77 (2014), doi:10.1080/00018732.2014.933502, <https://doi.org/10.1080/00018732.2014.933502>.
- [71] Y. Ashida, Z. Gong and M. Ueda, *Non-Hermitian physics*, Advances in Physics **69**(3), 249 (2020), doi:10.1080/00018732.2021.1876991, 2006.01837.

- [72] N. Okuma and M. Sato, *Non-Hermitian topological phenomena: A review*, arXiv e-prints arXiv:2205.10379 (2022), 2205.10379.
- [73] R. Hamazaki, K. Kawabata and M. Ueda, *Non-hermitian many-body localization*, Phys. Rev. Lett. **123**, 090603 (2019), doi:10.1103/PhysRevLett.123.090603.
- [74] L.-J. Zhai, S. Yin and G.-Y. Huang, *Many-body localization in a non-Hermitian quasiperiodic system*, Phys. Rev. B **102**(6), 064206 (2020), doi:10.1103/PhysRevB.102.064206.
- [75] J. Ginibre, *Statistical ensembles of complex, quaternion, and real matrices*, Journal of Mathematical Physics **6**(3), 440 (1965), doi:10.1063/1.1704292.
- [76] R. Grobe, F. Haake and H.-J. Sommers, *Quantum distinction of regular and chaotic dissipative motion*, Phys. Rev. Lett. **61**, 1899 (1988), doi:10.1103/PhysRevLett.61.1899.
- [77] L. Sá, P. Ribeiro and T. c. v. Prosen, *Complex spacing ratios: A signature of dissipative quantum chaos*, Phys. Rev. X **10**, 021019 (2020), doi:10.1103/PhysRevX.10.021019.
- [78] R. Hamazaki, K. Kawabata, N. Kura and M. Ueda, *Universality classes of non-Hermitian random matrices*, Physical Review Research **2**(2), 023286 (2020), doi:10.1103/PhysRevResearch.2.023286.
- [79] J. Li, T. Prosen and A. Chan, *Spectral Statistics of Non-Hermitian Matrices and Dissipative Quantum Chaos*, Phys. Rev. L **127**(17), 170602 (2021), doi:10.1103/PhysRevLett.127.170602.
- [80] D. C. Brody, *Biorthogonal quantum mechanics*, Journal of Physics A: Mathematical and Theoretical **47**(3), 035305 (2013), doi:10.1088/1751-8113/47/3/035305.
- [81] G. Sun, J.-C. Tang and S.-P. Kou, *Biorthogonal quantum criticality in non-hermitian many-body systems*, Frontiers of Physics **17**(3), 1 (2022).
- [82] D. Bluvstein, A. Omran, H. Levine, A. Keesling, G. Semeghini, S. Ebadi, T. T. Wang, A. A. Michailidis, N. Maskara, W. W. Ho *et al.*, *Controlling quantum many-body dynamics in driven rydberg atom arrays*, Science **371**(6536), 1355 (2021).
- [83] C. M. Bender, D. C. Brody and H. F. Jones, *Complex extension of quantum mechanics*, Phys. Rev. Lett. **89**, 270401 (2002), doi:10.1103/PhysRevLett.89.270401.
- [84] C.-Y. Ju, A. Miranowicz, G.-Y. Chen and F. Nori, *Non-hermitian hamiltonians and no-go theorems in quantum information*, Phys. Rev. A **100**, 062118 (2019), doi:10.1103/PhysRevA.100.062118.
- [85] Y.-C. Tzeng, C.-Y. Ju, G.-Y. Chen and W.-M. Huang, *Hunting for the non-hermitian exceptional points with fidelity susceptibility*, Phys. Rev. Research **3**, 013015 (2021), doi:10.1103/PhysRevResearch.3.013015.
- [86] W.-L. You, Y.-W. Li and S.-J. Gu, *Fidelity, dynamic structure factor, and susceptibility in critical phenomena*, Phys. Rev. E **76**, 022101 (2007), doi:10.1103/PhysRevE.76.022101.
- [87] S. Sachdev, K. Sengupta and S. M. Girvin, *Mott insulators in strong electric fields*, Phys. Rev. B **66**, 075128 (2002), doi:10.1103/PhysRevB.66.075128.

- [88] P. Fendley, K. Sengupta and S. Sachdev, *Competing density-wave orders in a one-dimensional hard-boson model*, Phys. Rev. B **69**, 075106 (2004), doi:10.1103/PhysRevB.69.075106.
- [89] E. Rico, T. Pichler, M. Dalmonte, P. Zoller and S. Montangero, *Tensor networks for lattice gauge theories and atomic quantum simulation*, Phys. Rev. Lett. **112**, 201601 (2014), doi:10.1103/PhysRevLett.112.201601.
- [90] Z. Yao, L. Pan, S. Liu and H. Zhai, *Quantum many-body scars and quantum criticality*, arXiv preprint arXiv:2108.05113 (2021).
- [91] P.-Y. Chang, J.-S. You, X. Wen and S. Ryu, *Entanglement spectrum and entropy in topological non-hermitian systems and nonunitary conformal field theory*, Phys. Rev. Research **2**, 033069 (2020), doi:10.1103/PhysRevResearch.2.033069.
- [92] L. Herviou, N. Regnault and J. H. Bardarson, *Entanglement spectrum and symmetries in non-Hermitian fermionic non-interacting models*, SciPost Phys. **7**, 69 (2019), doi:10.21468/SciPostPhys.7.5.069.
- [93] Y.-T. Tu, Y.-C. Tzeng and P.-Y. Chang, *Rényi entropies and negative central charges in non-hermitian quantum systems*, arXiv preprint arXiv:2107.13006 (2021).
- [94] O. Bohigas, M. J. Giannoni and C. Schmit, *Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws*, Phys. Rev. L **52**(1), 1 (1984), doi:10.1103/PhysRevLett.52.1.
- [95] M. L. Mehta, *Random matrices*, Elsevier (2004).
- [96] E. B. Bogomolny, U. Gerland and C. Schmit, *Models of intermediate spectral statistics*, Phys. Rev. E **59**, R1315 (1999), doi:10.1103/PhysRevE.59.R1315.
- [97] L. Sá, P. Ribeiro and T. c. v. Prosen, *Complex spacing ratios: A signature of dissipative quantum chaos*, Phys. Rev. X **10**, 021019 (2020), doi:10.1103/PhysRevX.10.021019.
- [98] B. Buča, J. Tindall and D. Jaksch, *Non-stationary coherent quantum many-body dynamics through dissipation*, Nature Communications **10**(1), 1 (2019).