The Tritium is dead, long live the Tritium!

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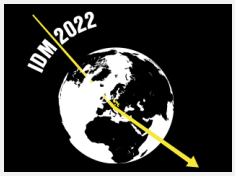

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Abstract

Detecting relic neutrinos is a longstanding goal in fundamental physics. Experimentally, this goal is extremely challenging as the required energy resolution is defined by the tiny neutrino masses ($\sim 10\,\mathrm{meV}$). The current consensus is that sufficient statistics together with the clean spectrum could only be achieved if beta decayers are attached to a solid state substrate. However, this inevitably imposes irreducible intrinsic limitations on the energy resolution coming from the Heisenberg's uncertainty principle. This limitation appears to be critical for the currently accepted decayer - Tritium. Here we analyze ways of mitigation of this limitation that are known at the moment and provide an up-to-date conclusion regarding the viability of using the Tritium for the relic neutrino detection.

Contents

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1 Introduction

Standard Model cosmology predicts that today's Universe should be filled with a cold, $T = 1.95\,\mathrm{K}$, background of almost free relic neutrinos produced in the first second following the Big Bang [1]. It is believed that this cosmic neutrino background (CνB) contains invaluable information about the early seconds of the Universe. Moreover, so-called sterile neutrinos are considered as a candidate of the dark matter particle [2, 3]. This makes the detection of the relic neutrinos and measurement of their mass a strategic goal for fundamental physics. Indirect evidence for the existence of the relic neutrinos was found in the observed CMB [4], however, due to the extreme weakness of the interactions between neutrinos and other forms of matter, direct detection of the CνB remains a major experimental challenge.

Today it is widely accepted that the most practicable route to the direct detection of the CνB lies through the measurement of the fine structure of the β-spectrum of a radioactive element [5–9]. Among the challenges of such a measurement are: the weakness of the signal which can be only compensated by the large amounts of the radioactive atoms (at least 100 g in order to achieve ten events per year in the case of atomic Tritium) and the need in the extraordinary high energy resolution (50 meV or better) of the experiment.

While the first problem can be practically by-passed by considering solid state based experimental architectures where bounding the emitters to a substrate allows to reach higher emitter densities [7], the second still remains unresolved. Although nowadays it is possible to reach an outstanding energy resolution ($\sim 10\,\mathrm{meV}$) of the measurement apparatus [7], it is not the only ingredient that determines the overall energy resolution. Along with the error introduced by the measurement device there are other intrinsic sources of the uncertainty in the electron energy (and hence the smearing of the spectrum) that come from the interaction of the radioactive atom and the emitted electron with the environment.

The most simple but prominent effect, as was recently shown [10], comes from accounting for the fact that the β emitter is not free but rather bound to a substrate [10, 11]. For the currently accepted candidate for β-decayer, namely Tritium, this leads to the energy smearing of the order of $\sim 0.1\,\mathrm{eV}$ which is beyond the accepted value for the current experiment. This limitation is completely independent from the technical characteristics of the measurement device and can be only overcame by the conceptual modifications to the architecture of the experimental setup.

Different solutions to changing the setup while keeping Tritium for a role of β-emitter are discussed in a follow-up paper [12] published by PTOLEMY collaboration. In this work, we try to re-analyze them and comment on their viability. We also discuss the possibilities of changing the β-emitter to a heavier one. We make an update on the list of possible β-emitter candidates that may be suitable for the relic neutrino detection including ^{63}Ni and ^{147}Pm to the previously discussed list [13].

2 β-decay on the surface

Along with the requirement in the extreme energy resolution (order of 10 meV), another major challenge in relic neutrino detection lies in the weakness of the signal. A naive estimate for the neutrino capture cross section is $(\sigma v)_\nu \simeq (\tau Q^3)^{-1}$ [8], where Q is the energy released in the β-decay and τ is the lifetime of the β emitter. A lower bound on the lifetime comes from the need to have enough time to assemble the experimental setup: $\tau \gtrapprox 1\mathrm{yr}$. From the other hand, all the viable emitters have Q that does not go lower then $\sim 10\,\mathrm{keV}$. From this, we have an upper bound of $(\sigma v)_\nu$ suggesting that in order to have at least one neutrino capture event per year we need large amounts of the radioactive atoms (at least 100 g in order to achieve

ten events per year in the case of atomic Tritium).

Then come the question of how should we store such large quantities of radioactive material. The lower bound on the size of the experimental setup comes from a very simple consideration – it should be smaller then the mean free path of the emitted electron with respect to the collisions with the other emitters. Otherwise, the scattering processes will corrupt the energy resolution. In order to make a very crude lower estimate, let us assume that emitted electrons scatter on Tritium atoms through the hard sphere potential. The mean free path is defined by the cross-section $\sigma = R_{\text{atom}}^2$ and the concentration of the emitters $n = N/L^3$: $\lambda = (\sigma n)^{-1}$. If we fix the number of emitters such that we have 1 event per year ($N \sim 10^{24}$), we would arrive to the very rough estimate of the lower bound on the linear size of the experimental setup that is of the order of 100 m. The biggest relic neutrino detector nowadays is KATRIN that has the cross- section area of the container about 50 cm^2, so the effective mass of tritium molecules is only about 50 µg [14].

The only viable solution to the problem of the controllable handling of such a large amount of radioactive material nowadays is proposed by the PTOLEMY collaboration [7]. In this proposal, the tritium atoms are deposited on the graphene substrate which can efficiently store atomic tritium by locally binding it to carbon atoms (either by chemisorption, physisorption). Along with the high tritium storage, PTOLEMY also offers a very precise control over the emitted electrons with the help of the elaborate configuration of the electric and magnetic fields that "guide" the electrons to the detector. An overall energy resolution of 10 meV is achieved.

However, the possibility to store large quantities of ^{3}H comes at a price. The presence of the environment (in this case graphene) distorts the spectrum by introducing additional intrinsic energy uncertainty to it. The general form of the β-spectrum with the presence of the environment $d\Gamma/dE_e$ (see Eq. 1) differs from the one in vacuum $d\Gamma^{(0)}/dE_e$ (see Eq. 2) by the fact that the energy conservation no longer holds (derivation is presented in appendix A). The delta-function responsible for it gets substituted by some function $\mathcal{F}$ the form of which depends on the environment and is generally unknown.

$$\frac{d\Gamma}{dE_e} = \frac{4E_e p(E_e)}{(2\pi)^4 \hbar} \int E_\nu k(E_\nu) dE_\nu \left| \int dx\, j^\mu_{\text{lept}}(x, E_e, E_\nu) J^{\text{nucl}}_\mu(x) \right|^2 \mathcal{F}(E_e + E_\nu - E_0) \tag{1}$$

as compared with the spectrum for the β-decay in the vacuum

$$\frac{d\Gamma^{(0)}}{dE_e} = \frac{4E_e p(E_e)}{(2\pi)^4 \hbar} \int E_\nu k(E_\nu) dE_\nu \left| \int dx\, j^\mu_{\text{lept}}(x, E_e, E_\nu) J^{\text{nucl}}_\mu(x) \right|^2 \delta(E_e + E_\nu - E_0). \tag{2}$$

Eq. 1 accounts for all types of the interactions of the emitter with the substrate. Among them (the list is not by any means exhaustive): zero-point motion of the emitter [10], finite lifetime of the daughter ion due to redistribution of the charges on its shells and tunneling to graphene, breakdown of the angular momentum conservation due to the presence of the substrate, X-ray edge anomaly leading to a gamma-shaped broadening of the emission peak [15], creation of vibrational excitations of the lattice, emission of plasmons and surface polaritons, inhomogeneous broadening due to any kind of inhomogeneities in the emitter arrangement [1].

Already the first effect in the above-mentioned extensive list, namely zero-point motion of the emitter has dramatic effect on the spectrum [10]. It rests on the Heisenberg's uncertainty principle saying that an atom restricted to some finite region in space by the bonding potential

[1] However, it does not include the interaction of the emitted electron with the substrate that can also manifest itself through many different mechanisms such as: screening of the daughter atom by the charges in graphene, creation of shock wave emission due to the motion of the emitted electron at grazing angles at speeds exceeding the Fermi velocity, etc.

cannot be exactly at rest. This means that the uncertainty in the velocity of center of mass of the atom will be transmitted to the electron leading to the uncertainty in its energy that is measured in the laboratory reference frame. The value for this energy uncertainty is defined by two factors: properties of the β-emitter: $\Delta E \sim \left(Q^2/m^3_{\text{nucl}}\right)^{1/4}$ where Q is the energy released during the decay, and on the properties of the binding potential: $\Delta E \sim \kappa^{1/4}$, where κ is the stifness of the binding potential [10].

For Tritium adsorbed on graphene, this estimate gives $\Delta E \sim 0.5\,\text{eV}$ [10] which is much greater than the expected neutrino mass and therefore is deadly for the relic neutrino detection measurement. The mitigation of this uncertainty is absolutely compulsory and can be done by following one of the two paths (or both): changing the β-emitter and/or changing the way it is bounded to the substrate.

3 Changing the β-emitter

One way to reduce ΔE is to choose β-emitter such that it minimizes the combination $\gamma = \left(\frac{Q^2 m_{\text{el}}}{m^3_{\text{nucl}} c^4}\right)^{1/4}$ while having sufficient neutrino capture rate and realistic lifetime. The list of all transitions that satisfy these requirements are presented in Table 1. The search was done among all existing transitions of all energy levels (not only ground states) of all elements with the help of NIST nuclear database [16].

We note, however, that another important requirement that we have to account for is that the daughter nucleus should either be stable with respect to β-decay or have Q-value smaller then the one of a parent nucleus. Otherwise, the products of the daughter decay may overlap with the initial signal that we want to measure. While ^{171}Tm and ^{63}Ni have fully stable daughter nuclei (no kinds of decay present), both ^{151}Eu (the daughter of ^{151}Sm) and ^{147}Sm (the daughter of ^{147}Pm) α-decay. Despite the fact that the lifetimes of the daughters of both nuclei is pretty big (10^{18}yr and 10^{11}yr respectively), because we have a huge amount of emitters ($N \sim 10^{25}$) there can be a significant amount of the products of the α-decay. One should do a separate detailed study about whether these products can spoil the spectrum (for example via scattering with β-electrons) or if it is harmless. At the moment we omit this question leaving both ^{151}Sm and ^{147}Pm as backup emitters in case both ^{171}Tm and ^{63}Ni are discarded due to some other reasons since they have smaller neutrino capture rate then both ^{171}Tm and ^{63}Ni [2] [13].

Parent	$\tau_{1/2}$,[yr]	Daughter	Q,[keV]	$(\sigma v)_{\text{est}}/(\sigma v)_{^3\text{H}}$ [10^{-3}]	$\gamma/\gamma_{^3\text{H}}$
^{171}Tm	1.92	^{171}Yb	96.5	45.0	0.110
^{63}Ni	101.	^{63}Cu	66.9	2.61	0.193
^{147}Pm	2.62	^{147}Sm	225.	2.67	0.188
^{151}Sm	90.0	^{151}Eu	75.9	1.99	0.107

Table 1: **List of possible candidates** for suitable β-emitter and their characteristics.

^{63}Ni undergoes so-called allowed β-decay for which the neutrino capture rate follows closely the estimate $(\sigma v)_{\text{est}} = (\tau Q^3)^{-1}$:

$$^{63}\text{Ni}: \qquad \Gamma_{\text{capt}} = 7 \cdot 10^{-28}\,\text{y}^{-1}\,\frac{\eta_\nu}{\langle \eta_\nu \rangle}\ \text{per atom}. \tag{3}$$

[2] There are many other many-body effects still to be considered.

For ^{171}Tm, ^{151}Sm and ^{147}Pm this is not the case, they all undergo a so-called 1^{st} *non-unique forbidden decays* where the information about the nuclear wave-function is also needed along with the lifetime in order to obtain the neutrino capture cross section [13]. Nevertheless, it was shown one can make an estimate of the neutrino capture cross section from measuring the end of the corresponding β decay spectrum [13]. The capture rate for the most active element of the three, ^{171}Tm that was estimated using this method is

$$^{171}\text{Tm}: \qquad \Gamma_{\text{capt}} = 1.2 \cdot 10^{-26}\,\text{y}^{-1}\,\frac{\eta_\nu}{\langle \eta_\nu \rangle} \text{ per atom.} \tag{4}$$

Here η_ν is the local cosmic neutrino number density which could be significantly larger than the average over the universe $\langle \eta_\nu \rangle \sim 53\,\text{cm}^{-3}$ due to gravitational clustering. Since the solid-state based experiments attach the emitter to the substrate atom by atom, the single event exposure per year corresponds to $1.3 \cdot 10^{27}$ atoms of ^{63}Ni or $8 \cdot 10^{25}$ atoms of ^{171}Tm. For comparison, the same number of events can be achieved with $2 \cdot 10^{24}$ atoms of ^{3}H.

Let us estimate what does it mean in terms of the scalability of the experiment. For a fully loaded graphene, the density of the radioctive atoms is about $3.8 \cdot 10^{15}\text{cm}^{-2}$. This means that in order to have 1 event per year one needs $5.3 \cdot 10^{4}\text{m}^2$ area of graphene in case of ^{3}H, $2.1 \cdot 10^{6}\text{m}^2$ for ^{171}Tm and $3 \cdot 10^{7}\text{m}^2$ for ^{63}Ni. The vertical separation of different graphene sheets is $\sim$ mm [17], so so the entire setup can fit inside

$$\begin{aligned} ^{3}\text{H}: &\qquad V \approx 1\,\text{m} \times 8\,\text{m} \times 8\,\text{m} \\ ^{171}\text{Tm}: &\qquad V \approx 10\,\text{m} \times 15\,\text{m} \times 15\,\text{m} \\ ^{63}\text{Ni}: &\qquad V \approx 31\,\text{m} \times 31\,\text{m} \times 31\,\text{m}. \end{aligned} \tag{5}$$

Furthermore, one can even find a non-planar topology to decrease the total volume of the setup by a factor of 100 [18].

One should note, however, that at large coverage the substrate may undergo a transition from the metallic to the insulating state. Such a transition will preclude maintaining all atoms at the same electrostatic potential and will thus result in a destructive homogeneous broadening of the signal. This means that the effective sizes of the experimental setups that uses Tritium will be bigger than estimated by (5), which is not the case for both ^{171}Tm (rare earth metal) and ^{63}Ni (transition metal) that are conductors. For them one can allow fully loading of graphene.

4 Modifying the binding potential

The second path to mitigate the smearing of the energy coming from the zero-point motion of the emitter is to modify the way emitters are attached to the substrate. Despite the fact that the dependence on the binding potential is much weaker then on the properties of the β-emitter, changing the latter would be much more drastic for the experimental architecture then adjusting the former. Especially, in the context of the production facilities. Therefore, before completely discarding Tritium, one needs to first fully study whether it is possible to reduce ΔE at least by an order of magnitude by changing the way how the emitters are bound to the surface.

4.1 Soft binding potential

The dependence of the energy smearing that comes from the zero-point motion of the emitter on the properties of the bounding potential (its stiffness κ) is very weak $\Delta E \sim \kappa^{1/4}$ [10].

Therefore, in order to reduce ΔE by an order of magnitude, one needs to make the potential four orders of magnitude softer.

The atom needs to be attached to the surface at least in one direction (otherwise we have the situation equivalent to the gaseous phase), while the in-plane potential can be made very soft. By increasing the in-plane mobility of the emitters we can partially restore the conservation of the momentum parallel to the surface. As example, one can attach the emitter to the interior of the carbon nanotube [12]. It is arguably possible to reach completely full mobility of the atom in the direction of the axis of the tube.

The first obstacle that arises in such a setting is that the uncertainty in the energy of the emitted electron will depend on the angle of the emission θ: it will be smaller by a factor of $\sin\theta$ compared to the case when there is a migration potential. So, if we have $\Delta E \approx 200\,\text{meV}$ for the case of fully bound Tritium and we define a threshold value for the allowed uncertainty (say $\Delta E_{\text{threshold}} = 10$ meV), we can find the corresponding threshold angle $\theta_{\text{threshold}} = \arcsin(\Delta E_{\text{threshold}}/\Delta E) \approx \arcsin(10\text{ meV}/200\text{ meV}) \approx 3°$.

In this way, electrons collected within the angle of emission $\theta < \theta_{\text{threshold}}$ will have small energy uncertainty $\Delta E < \Delta E_{\text{threshold}}$. However, the number of such electrons will be suppressed by $\approx \frac{2\pi}{\pi\theta^2} \approx 700$ times even in case of perfectly zero migration potential. Also, the capacity for the loading of the carbon nanotubes with the emitters is order of magnitude smaller then for graphene: between 10 and 20 g of Tritium per kg of material [12].

Another obstacle is that as soon as we let Tritium to move freely it would want to form molecules (which are again bounded states leading to energy smearing). A way to suppress dimerization is to spin-polarize all Tritium atoms [12]. It could be done in a low temperature ($T \sim 0.1\,\text{K}$) and high magnetic field ($B \sim 10\,\text{T}$) environment. However, even under these extreme conditions recombination still persists through three-body processes with double spin polarization [19]. In 1D three-body recombination is defined by

$$\frac{d\lambda}{dt} = -K_{1D}\lambda^3, \tag{6}$$

where λ is a linear number density. K_{1D} is a recombination rate, its estimate for Hydrogen could be found in [20] and is proportional to T^3. If we require lifetime of spin-polarized Tritium atoms $\tau \sim 100\,\text{d}$, we obtain $\lambda \sim 1/\sqrt{K_{1D}\tau} \sim 300\,\text{cm}^{-1}$ for $T = 0.1\,\text{K}$. With 5 Å spacing between nanotubes it gives us surface density of $\sim 10^{10}\text{cm}^{-2}$, what is five orders of magnitude lower than fully loaded graphene.

4.2 Stiff binding potential

Along with the smeared part of the spectrum there is still a tiny signature of the CνB at the very end of the spectrum that corresponds to the recoil-less decay. The event rate for this part is, however, suppressed as $\mathcal{M} = \mathcal{M}_0 e^{-\lambda^2 k_\beta^2/4}$, where $\lambda \equiv (m_{\text{nucl}}\kappa)^{-1/4}$ is the localization length of the atom and k_β is the momentum of the emitted β-electron [12]. For the case of Tritium adsorbed on graphene, near the endpoint one has $\lambda k_\beta \approx 6$ [12] which means that the recoil-less events are extremely unlikely ($\approx 10^{-4}$ suppression). Nevertheless, this opens another leeway to avoid energy spreading – using a very stiff binding potential. This path is, however, very challenging if not impossible. The study of the vibrational frequencies of different hydrogen-based molecules shows that the stiffness of the binding potential varies within one order of magnitude. Increasing κ by one order of magnitude would not be sufficient since there still will be two orders of magnitude suppression.

5 Conclusion

We conclude that the currently existing setups for the relic neutrino detection are not able to achieve their goal and therefore need to be modified. The main limitation on the way to the relic neutrino detection arises from the zero point motion of the emitters that are bounded to the substrate. This effect rules-out the currently proposed radioactive element, namely Tritium. We argue that the most viable way to mitigate this effect is to use heavier β-emitter, namely the isotope of Thulium, ^{171}Tm. The second possible candidate is ^{63}Ni.

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A Generalized Fermi Golden Rule for β-decay in the environment

In order to understand how the spectrum will look for the β-decaying atom bounded to the substrate, let us apply the Fermi Golden Rule to the whole system containing β-decaying constituents and the substrate. Let us denote the total state of such a system (atom + environment) as $|\alpha, z\rangle$. Since the whole system is closed, the Fermi Golden Rule holds where the total energy is conserved.

$$\Gamma = \frac{2\pi}{\hbar} \sum_{\text{final states}} \left| \langle \text{final} | \hat{H}^{\beta} | \text{initial} \rangle \right|^2 \delta \left(E_{\text{in}} - E_{\text{fin}} \right), \tag{7}$$

where we know that the initial and final states have different number of protons and neutrons that is why only the Hamiltonian of the weak interaction H^{β} survives. Let us specify how initial and final states look like and what are their quantum numbers

$$\begin{aligned} |\text{initial}\rangle &= |\alpha_0, z\rangle \\ |\text{final}\rangle &= |\alpha, z+1\rangle \, |k\rangle \, |p\rangle \,, \end{aligned} \tag{8}$$

where there are 3 quantum numbers: an abstract label for the atom together with the environment state α, electron momentum k and neutrino momentum p[3]. The Hamiltonian density of the β-interaction in the full generality is

$$H^{\beta} = \frac{G_{\beta}}{\sqrt{2}} \bar{e}(x) \gamma^{\mu} (1 - \gamma_5) \nu_e(x) \hat{J}^{\text{nucl}}_{\mu}(x) + \text{h.c}, \tag{9}$$

[3] We note that in this we neglect the Coulomb interaction of the emitted electron with the nucleus and with the surroundings. Therefore, the emitted electron is a plane wave that is characterized by the momentum k. Neutrino does not interact with anything so it is generally a plane wave (specified by the momentum p).

where $e(x)$, $\nu_e(x)$ are electron and neutrino fields and $\hat{J}^{\text{nucl}}_{\mu}(x)$ is a nuclear part which depends on the atom itself and we do not specify it. We get

$$\Gamma = \frac{V^2}{\hbar}\int \frac{d^3k d^3p}{(2\pi)^6}\sum_{\alpha}\left|\langle k|\langle p|\langle \alpha, z+1|H^{\beta}|\alpha_0, z\rangle\right|^2 \int d\tau e^{i\tau(E_e+E_\nu-E_{\alpha_0}+E_\alpha)}, \tag{10}$$

where we used that $\delta(E_{\text{in}}-E_{\text{fin}}) = \int \frac{d\tau}{2\pi} e^{i\tau(E_e+E_\nu-E_{\alpha_0}+E_\alpha)}$. Expanding $|\cdot|^2$ we get

$$\begin{aligned}\Gamma = \frac{1}{\hbar}\int \frac{d^3k d^3p}{(2\pi)^6}\int dx dx' j^{\mu}_{\text{lept}}(x,\vec{p},\vec{k}) j^{*,\nu}_{\text{lept}}(x',\vec{p},\vec{k})\times \\ \times\sum_{\alpha}\langle\alpha_0,z|\hat{J}^{\text{nucl}}_{\mu}(x)|\alpha,z+1\rangle\langle\alpha,z+1|\hat{J}^{\dagger,\text{nucl}}_{\nu}(x')|\alpha_0,z\rangle\times \\ \times\int d\tau e^{i\tau(E_e+E_\nu+E_\alpha-E_{\alpha_0})},\end{aligned} \tag{11}$$

where $j^{\mu}_{\text{lept}}(x,\vec{p},\vec{k}) = \bar{\psi}_e(x,\vec{k})\gamma^{\mu}(1-\gamma_5)\psi^c_{\nu}(x,\vec{p})$ and ψ has both functional dependence (plane wave e^{ipx} without normalization factor as we already took it into account) and spinor structure. Accounting for

$$\sum_{\alpha} e^{i\tau E_\alpha}|\alpha,z+1\rangle\langle\alpha,z+1| = e^{i\tau\hat{H}^{\alpha}_{z+1}}, \tag{12}$$

where $\hat{H}^{\alpha,z+1}$ is the Hamiltonian that describes the system that consists of the isotope of the initial atom with the charge $z+1$ and environment, we get[4]

$$\begin{aligned}\Gamma = \frac{1}{\hbar}\int \frac{d^3k d^3p}{(2\pi)^6}\left|\int dx j^{\mu}_{\text{lept}}(x,\vec{p},\vec{k})J^{\text{nucl}}_{\mu}(x)\right|^2\times \\ \times\int d\tau\langle\alpha_0,z|\hat{\chi}^{\dagger}e^{i\tau\hat{H}^{\alpha}_{z+1}}\hat{\chi}|\alpha_0,z\rangle e^{i\tau(E_e+E_\nu-E_{\alpha_0})},\end{aligned} \tag{13}$$

where $\hat{\chi}$ changes the charge of the nucleus by one and $|\alpha,z\rangle$ only has the information about the surroundings and the electron orbitals of the atom, not the nucleus itself, the latter is in $J^{\text{nucl}}(x)$. Denoting

$$\frac{1}{2\pi}\int d\tau\langle\alpha_0,z|\hat{\chi}e^{i\tau\hat{H}^{\alpha}_{z+1}}\hat{\chi}^{\dagger}|\alpha_0,z\rangle e^{i\tau\omega} = \mathcal{F}(\omega), \tag{14}$$

we get the generalized Fermi Golden rule that accounts for the interactions of the nucleus with the surroundings

$$\Gamma = \frac{1}{\hbar}\int \frac{d^3k d^3p}{(2\pi)^6}\left|\int dx j^{\mu}_{\text{lept}}(x,\vec{p},\vec{k})J^{\text{nucl}}_{\mu}(x)\right|^2\mathcal{F}(E_e+E_\nu-E_{\alpha_0}). \tag{15}$$

Or

$$\begin{aligned}\frac{d\Gamma}{dE_e} = &\frac{4E_e p(E_e)}{(2\pi)^4\hbar}\int E_\nu k(E_\nu)dE_\nu\times \\ &\times\left|\int dx j^{\mu}_{\text{lept}}(x,E_e,E_\nu)J^{\text{nucl}}_{\mu}(x)\right|^2\mathcal{F}(E_e+E_\nu-E_{\alpha_0}).\end{aligned} \tag{16}$$

[4] With the assumption that the wave function of the atom is a product of the wave function of the nucleus and the wave function of the electron shells that only depends on the charge of the nucleus.

If we compare it with the Fermi Golden Rule for the β-decay in the vacuum

$$\frac{d\Gamma^{(0)}}{dE_e} = \frac{4E_e p(E_e)}{(2\pi)^4\hbar}\int E_\nu k(E_\nu) dE_\nu \times \\ \times \left|\int dx\, j^\mu_{\text{lept}}(x, E_e, E_\nu) J^{\text{nucl}}_\mu(x)\right|^2 \delta(E_e + E_\nu - E_{\alpha_0}). \tag{17}$$

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