SciPost Physics Submission

Dark matter as a QCD effect in an Anti de Sitter Geometry

(Cosmogonic implications of de Sitter, Anti de Sitter and Poincaré symmetries)

G. Cohen-Tannoudji[1], J.-P. Gazeau[2⋆]

1 Laboratoire de recherche sur les sciences de la matière, LARSIM CEA, Université Paris-Saclay, F-91190 Saint-Aubin, France
2 Université Paris Cité, CNRS, Astroparticule et Cosmologie, F-75013 Paris, France
* Corresponding Author gazeau@apc.in2p3.fr

December 15, 2022

34th International Colloquium on Group Theoretical Methods in Physics
Strasbourg, 18-22 July 2022
doi:10.21468/SciPostPhysProc.?

Abstract

The ΛCDM standard model of cosmology involves two dark components of the universe, dark energy and dark matter. Whereas dark energy is usually associated with the (positive) cosmological constant Λ associated with a de Sitter geometry, we propose to explain dark matter as a pure QCD effect, namely a gluonic Bose Einstein condensate with the status of a *Cosmic Gluonic Background (CGB)*. This effect is due to the trace anomaly viewed as an effective negative cosmological constant determining an Anti de Sitter geometry and accompanying baryonic matter at the hadronization transition from the quark gluon plasma phase to the colorless hadronic phase. Our approach also allows to assume a ratio Dark/Visible equal to 11/2.

Contents

1 Introduction

Let us start out this group theoretical oriented contribution with three motivating quotes. The first one from Newton and Wigner (1949) [1] is about the concept of elementary system.

> *The concept of an "elementary system" requires that all states of the system be obtainable from the relativistic transforms of any state by superpositions. In other words, there must be no relativistically invariant distinction between the various states of the system which would allow for the principle of superposition. This condition is often referred to as irreducibility condition ...*
> *The concept of an elementary system (...) is a description of a set of states which forms, in mathematical language, an irreducible representation space for the inhomogeneous Lorentz (≃ Poincaré) group*

The second one from Fronsdal (1965) [2] is about curvature versus flatness of space-time.

> *A physical theory that treats spacetime as Minkowskian flat must be obtainable as a well-defined limit of a more general physical theory, for which the assumption of flatness is not essential.*

The third one from Sakharov [3], quoted by Adler in [4].

> *The presence of the action*

$$S_{\text{grav}} = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}(R-2\Lambda)\,. \tag{1}$$

> *leads to a metrical elasticity of space, i.e.,, to generalized forces which oppose the curving of space. Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum if space is curved.*

These statements are the leitmotiv guiding our interpretation of dark matter, as it will be exposed in the sequel. Section 2 is devoted to the description of three fundamental space-time symmetries, Poincaré group and its two deformations, de Sitter (dS) and Anti de Sitter (AdS) groups, and their respective significance in terms of invariants, spin, mass, and "energy at rest". Cosmology chronology is put in perspective in Section 3 with regard to our interpretation [5] of the dark matter as a gluonic Bose-Einstein condensate emerging at the end of the so-called quark period (see also [7–9] about the genesis of our common work). Following the short conclusion (Section 4), we give in Appendix A some insight in relation with the ΛCDM standard model.

2 Three maximal symmetries, Poincaré, dS, AdS

2.1 The place of the cosmological constant

Firstly let us observe that there exist two standpoints about the Einstein equation of general relativity [10] :

- **Standpoint 1**

$$\underbrace{\mathrm{R}_{\mu\nu}-\frac{1}{2}\mathrm{R}\,g_{\mu\nu}}_{\text{geometrical content}} = \underbrace{-\kappa\,T_{\mu\nu}-\Lambda\,g_{\mu\nu}}_{\text{matter content}}, \quad \kappa=\frac{8\pi G}{c^4}\,. \tag{2}$$

Here, the fundamental state that contains the maximum number of symmetries is the Minkowskian geometry, and the cosmological term $\Lambda g_{\mu\nu}$ may be interpreted as an extra pressure, named world matter by de Sitter in his debate with Einstein:

$$\Lambda>0\sim\text{“dark energy”}\ ,\ \Lambda<0\sim\text{“dark matter”?}$$

- **Standpoint 2**

$$\underbrace{\mathrm{R}_{\mu\nu}-\frac{1}{2}\mathrm{R}\,g_{\mu\nu}+\Lambda\,g_{\mu\nu}}_{\text{geometrical content}} = \underbrace{-\kappa\,T_{\mu\nu}}_{\text{matter content}}\quad . \tag{3}$$

Here, the fundamental states that contain the maximum number of symmetries are the de-Sitter (dS) ($\Lambda\equiv\Lambda_{\mathrm{dS}}>0$) and the Anti-de-Sitter (AdS) ($\Lambda\equiv\Lambda_{\mathrm{AdS}}<0$) geometries.

Note that the split between these two standpoints should not be considered as absolute, since we could as well model situations in a mixed way:

- **Standpoint 3**

$$\underbrace{\mathrm{R}_{\mu\nu}-\frac{1}{2}\mathrm{R}\,g_{\mu\nu}+\Lambda_L\,g_{\mu\nu}}_{\text{geometrical content}} = \underbrace{-\kappa\,T_{\mu\nu}-\Lambda_R\,g_{\mu\nu}}_{\text{matter content}}. \tag{4}$$

2.2 Two unique deformations of Poincaré symmetry

From the above di-or tri-lemna let us give present some points in favor of dS/AdS studies

- dS and AdS are *maximally symmetric* (remind that in a metric space of dimension n, the maximum number of metric preserving symmetries is $n(n+1)/2$, here 10 since $n=4$).
- Their symmetries are one-parameter deformations of Minkowskian symmetry with
  - negative curvature $-\varkappa_{\mathrm{dS}}=-\sqrt{\Lambda_{\mathrm{dS}}/3}$ ($=-H/c$, H : Hubble parameter)
  - positive curvature $\varkappa_{\mathrm{AdS}}=\sqrt{|\Lambda_{\mathrm{AdS}}|/3}$

  respectively
- As soon as a constant curvature is present, we lose some of our so familiar conservation laws like energy-momentum conservation!
- Then what is the physical meaning of a scattering experiment (“space” in dS is like the sphere $\mathbb{S}^3$, let alone the fact that time is ambiguous)?
- Which relevant “physical” quantities are going to be considered as (asymptotically? contractively?) experimentally available?

In addition to the previous observations, we should insist on the fact that dS and AdS symmetries are the two unique deformations of the Poincaré symmetry. They occupy the extreme vertex of the cubic diagram in Figure 1 showing the eleven kinematics classified by Bacry & Levy-Leblond (1968) [11]. More precisely, under the assumptions that space is isotropic (rotation invariance), parity and time-reversal are automorphisms of the kinematical groups, and inertial transformations in any given direction form a noncompact subgroup then there are eight types of Lie algebras for kinematical groups corresponding to eleven possible kinematics. These algebras are [11]:

$R1$ The two de Sitter Lie algebras isomorphic, respectively, to the Lie algebras of SO(4,1) and SO(3,2);

$R2$ The Poincaré Lie algebra;

$R3$ Two "para-Poincaré" Lie algebras, of which one is isomorphic to the ordinary Poincaré Lie algebra but physically different and the other is the Lie algebra of an inhomogeneous SO(4) group;

$R4$ The Carroll Lie algebra;

$A1$ The two "nonrelativistic cosmological" Lie algebras;

$A2$ The Galilei Lie algebra;

$A3$ The "para-Galilei" Lie algebra;

$A4$ The "static" Lie algebra.

While the Lie algebras of class R have no nontrivial central extensions by a one-parameter Lie algebra, those of class A each have one class of such extensions.

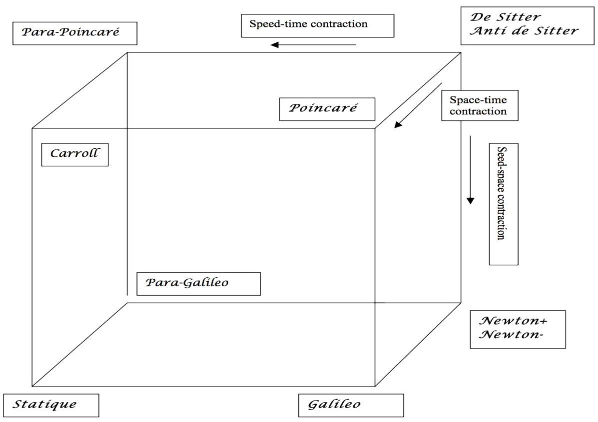

Figure 1: The eleven kinematics (Bacry & Levy-Leblond, JMP (1968)). From [12].

Hence, with the requirements of kinematical rotation, parity, and time-reversal invariance, there exists only one way to deform the proper orthochronous Poincaré group $\mathbb{R}^{1,3} \rtimes \mathrm{SO}_0(1,3)$ (or $\mathbb{R}^{1,3} \rtimes \mathrm{SL}(2,\mathbb{C})$), namely, in endowing space-time with a certain curvature. This leads to the two simple Lie groups, namely the ten-parameter de Sitter group $\mathrm{SO}_0(1,4)$ (or its universal covering $\mathrm{Sp}(2,2)$) and the ten-parameter Anti de Sitter group $\mathrm{SO}_0(2,3)$ (or its two-fold covering $\mathrm{Sp}(4,\mathbb{R})$).

2.3 de Sitter and Anti-de-Sitter Geometries

The de Sitter space may be viewed (on the left in Fig. 2) as a one-sheeted hyperboloid embedded in a five-dimensional Minkowski space with metric $\eta_{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1,-1)$ (but keep in mind that all points are physically equivalent):

$$M_{dS} \equiv \left\{ x \in \mathbb{R}^5;\; x^2 = \eta_{\alpha\beta}\, x^\alpha x^\beta = -\frac{3}{\Lambda_{\mathrm{dS}}} \right\}, \quad \alpha, \beta = 0,1,2,3,4. \tag{5}$$

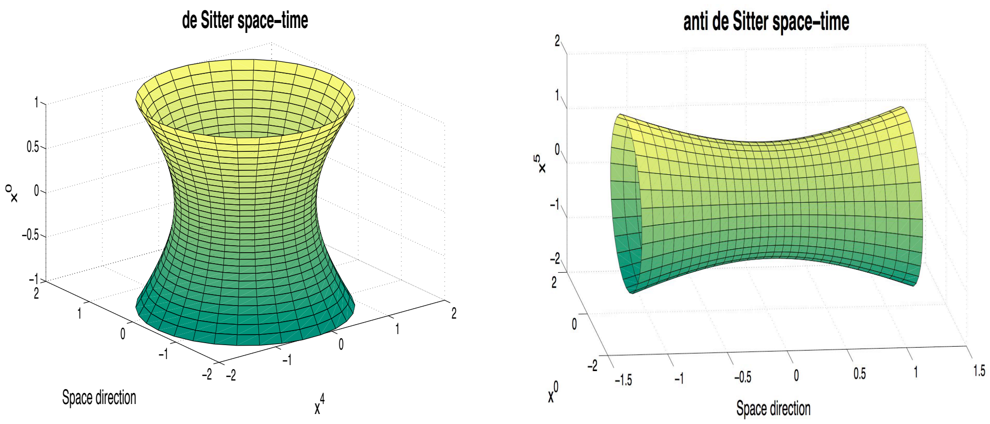

Figure 2: Left: the one-sheeted de Sitter hyperboloid as a manifold embedded in the $1+4$ Minkowski space-time. x^0 might be chosen as a time parameter, but there is no global time-like Killing vector. Right: the one-sheeted Anti de Sitter hyperboloid as a manifold embedded in the $2+3$ ambient space. Angular position along the central belt can be chosen as a local time coordinate, and it is in one-to-one correspondence with a global time-like Killing vector.

The Anti de Sitter space may as well be viewed (on the right in Fig. 2) as a one-sheeted hyperboloid embedded in another five-dimensional space with metric $\eta_{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1,1)$ (here too all points are physically equivalent) :

$$M_{AdS} \equiv \{x \in \mathbb{R}^5;\ x^2 = \eta_{\alpha\beta}\, x^\alpha x^\beta = \frac{3}{|\Lambda_{\mathrm{AdS}}|}\}, \quad \alpha,\beta = 0,1,2,3,5\,. \tag{6}$$

Note that the fifth dimension is space-like in dS whereas it is time-like in AdS.

2.4 Compared classifications of Poincaré, dS and AdS UIR's for quantum elementary systems

In a given unitary irreducible representation (UIR) of dS and AdS groups, ($\sim$ elementary system in Wigner's sense) their respective generators map to self-adjoint operators in Hilbert spaces of spinor-tensor valued fields on dS and AdS respectively:

$$\mathrm{K}_{\alpha\beta} \mapsto \mathrm{L}_{\alpha\beta} = \mathrm{M}_{\alpha\beta} + \mathrm{S}_{\alpha\beta}\,, \tag{7}$$

with orbital part $\mathrm{M}_{\alpha\beta} = -\mathrm{i}(x_\alpha \partial_\beta - x_\beta \partial_\alpha)$ and spinorial part $\mathrm{S}_{\alpha\beta}$ acting on the field components.

The physically relevant UIR's of the Poincaré, dS and AdS groups are denoted by $\mathcal{P}^{>}(m,s)$ ("$>$" for positive energies), $U_{\mathrm{dS}}(\varsigma_{\mathrm{dS}},s)$, and $U_{\mathrm{AdS}}(\varsigma_{\mathrm{AdS}},s)$, respectively. These UIR's are specified by the spectral values $\langle\cdot\rangle$ of their quadratic and quartic Casimir operators. The latter define two invariants, the most basic ones being predicted by the relativity principle, namely proper mass m for Poincaré and ς_{dS}, ς_{AdS} for dS and AdS respectively, and spin s for the three cases (see [12] and references therein).

Poincaré

For Poincaré the Casimir operators are fixed as

$$\begin{aligned} \mathrm{Q}^{(1)}_{\text{Poincaré}} &= \mathrm{P}^\mu \mathrm{P}_\mu = \mathrm{P}^{0\,2} - \mathbf{P}^2 = m^2 c^2\,, \\ \mathrm{Q}^{(2)}_{\text{Poincaré}} &= \mathrm{W}^\mu \mathrm{W}_\mu = -m^2 c^2 s(s+1)\hbar^2, \quad \mathrm{W}_\mu := \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathrm{J}^{\nu\rho}\mathrm{P}^\sigma\,. \end{aligned} \tag{8}$$

de Sitter

For de Sitter,

$$\begin{aligned} \mathrm{Q}^{(1)}_{\mathrm{dS}} &= -\frac{1}{2}\mathrm{L}_{\alpha\beta}\mathrm{L}^{\alpha\beta} = \varsigma_{\mathrm{dS}}^2 - \left(s-\frac{1}{2}\right)^2 + 2 \equiv \langle \mathrm{Q}^{(1)}_{\mathrm{dS}}\rangle\,,\\ \mathrm{Q}^{(2)}_{\mathrm{dS}} &= -\mathrm{W}_\alpha \mathrm{W}^\alpha = \left(\varsigma_{\mathrm{dS}}^2 + \frac{1}{4}\right)s(s+1)\,, \quad \mathrm{W}_\alpha := -\frac{1}{8}\epsilon_{\alpha\beta\gamma\delta\eta}\mathrm{L}^{\beta\gamma}\mathrm{L}^{\delta\eta}\,. \end{aligned} \tag{9}$$

Anti-de-Sitter

For Anti-de Sitter,

$$\begin{aligned} \mathrm{Q}^{(1)}_{\mathrm{AdS}} &= -\frac{1}{2}\mathrm{L}_{\alpha\beta}\mathrm{L}^{\alpha\beta} = \varsigma_{\mathrm{AdS}}(\varsigma_{\mathrm{AdS}}-3) + s(s+1) \equiv \langle \mathrm{Q}^{(1)}_{\mathrm{AdS}}\rangle\,,\\ \mathrm{Q}^{(2)}_{\mathrm{AdS}} &= -\mathrm{W}_\alpha \mathrm{W}^\alpha = -(\varsigma_{\mathrm{AdS}}-1)(\varsigma_{\mathrm{AdS}}-2)s(s+1)\,, \quad \mathrm{W}_\alpha := -\frac{1}{8}\epsilon_{\alpha\beta\gamma\delta\eta}\mathrm{L}^{\beta\gamma}\mathrm{L}^{\delta\eta}\,. \end{aligned} \tag{10}$$

2.5 Proper mass versus "at rest" energy in de Sitter and Anti-de-Sitter

While the proper mass is identified as the at rest energy, which means the energy spectrum infimum in Minkowski, these two quantities come apart in de Sitterian/Anti-de Sitterian geometry. They have to be devised from a flat-limit viewpoint, *i.e.*, from the study of the contraction limit $\Lambda \to 0$ of these representations

Proper mass versus at rest energy in de Sitter: Garidi mass

In this respect, a mass formula for dS has been established by Garidi (2003) [13]:

$$m_{\mathrm{dS}}^2 := \frac{\hbar^2\Lambda_{\mathrm{dS}}}{3c^2}(\langle Q^{(1)}_{\mathrm{dS}}\rangle - 2) = \frac{\hbar^2\Lambda_{\mathrm{dS}}}{3c^2}\left(\varsigma_{\mathrm{dS}}^2 + \left(s-\frac{1}{2}\right)^2\right)\,. \tag{11}$$

This definition should be understood through the contraction limit of representations:

$$\mathrm{dS\,UIR} \longrightarrow \mathrm{Poincaré\,UIR}\,.$$

More precisely, with

$$\Lambda_{\mathrm{dS}} \to 0 \quad \varsigma_{\mathrm{dS}} \to \infty\,, \quad \text{while fixing} \quad \varsigma_{\mathrm{dS}}\hbar\sqrt{\Lambda_{\mathrm{dS}}}/\sqrt{3}c = m_{\mathrm{Poincaré}} \equiv m\,, \tag{12}$$

we have

$$U_{\mathrm{dS}}(\varsigma_{\mathrm{dS}}, s) \underset{\substack{\Lambda_{\mathrm{dS}}\to 0,\, |\varsigma_{\mathrm{dS}}|\to\infty \\ |\varsigma_{\mathrm{dS}}|\sqrt{\Lambda_{\mathrm{dS}}}/\sqrt{3} = \frac{mc}{\hbar}}}{\longrightarrow} c_> \mathcal{P}^>(m,s) \oplus c_< \mathcal{P}^<(m,s)\,. \tag{13}$$

This result was proved in [14] and discussed in [15]. Note the breaking of dS irreducibility into a direct sum of two Poincaré UIR's with positive and negative energy respectively. To some extent the choice of the factors $c_<$, $c_>$, is left to a local tangent observer. The latter will naturally fix one of these factors to 1 and so the other one is forced to vanish. This crucial dS feature originates from the dS group symmetry mapping any point $(x^0, \mathrm{P}) \in \mathrm{H}_{\mathrm{dS}}$ into its mirror image $(x^0, -\mathrm{P}) \in \mathrm{H}_{\mathrm{dS}}$ with respect to the x^0-axis. Under such a symmetry the four dS generators L_{a0}, $a = 1,2,3,4$, (and particularly L_{40} which contracts to energy operator!) transform into their respective opposite $-\mathrm{L}_{a0}$, whereas the six L_{ab}'s remain unchanged. We think that the mathematical fact (13) should be carefully revisited with regard to the inflation scenario and the breaking of the matter-antimatter symmetry [16].

Proper mass versus at rest energy in de Anti de Sitter

Concerning AdS a mass formula similar to that one for dS exists
[10, 17]:

$$\begin{aligned} m^2_{\mathrm{AdS}} &= \frac{\hbar^2|\Lambda_{\mathrm{AdS}}|}{3c^2}\left(\langle Q^{(1)}_{\mathrm{AdS}}\rangle - \langle Q^{(1)}_{\mathrm{AdS}}|_{\varsigma_{\mathrm{AdS}}=s+1}\rangle\right) \\ &= \frac{\hbar^2|\Lambda_{\mathrm{AdS}}|}{3c^2}\left[\left(\varsigma_{\mathrm{AdS}}-\frac{3}{2}\right)^2-\left(s-\frac{1}{2}\right)^2\right]. \end{aligned} \tag{14}$$

One here deals with the AdS group representations $U_{\mathrm{AdS}}(\varsigma_{\mathrm{AdS}}, s)$ with $\varsigma_{\mathrm{AdS}} \geq s+1$ (discrete series and its lowest limit), and their contraction limit holds with no ambiguity:

$$U_{\mathrm{AdS}}(\varsigma_{\mathrm{AdS}}, s) \underset{\substack{\Lambda_{\mathrm{AdS}}\to 0,\, \varsigma_{\mathrm{AdS}}\to\infty \\ \varsigma_{\mathrm{AdS}}\sqrt{|\Lambda_{\mathrm{AdS}}|/3}=\frac{mc}{\hbar}}}{\longrightarrow} \mathcal{P}^{>}(m,s). \tag{15}$$

Proper mass as an absolute invariant

Now, contraction formulae for both dS and AdS give us the freedom to write

$$m_{\mathrm{dS}} = m_{\mathrm{AdS}} = m\,,$$

This agrees with the Einstein position that the proper mass of an elementary system should be independent of the geometry of space-time, or equivalently it should not exist any difference between inertial and gravitational mass.

Rest energy of a free particle in AdS versus dS and Poincaré

Each Anti-deSitterian quantum elementary system (in the Wigner sense) has a discrete energy spectrum bounded below by its rest energy [18–20]

$$E^{\mathrm{rest}}_{\mathrm{AdS}} = \left[m^2c^4+\hbar^2c^2\frac{|\Lambda_{\mathrm{AdS}}|}{3}\left(s-\frac{1}{2}\right)^2\right]^{1/2} + \frac{3}{2}\hbar\sqrt{\frac{|\Lambda_{\mathrm{AdS}}|}{3}}\,c\,, \tag{16}$$

Hence, to the order of $\hbar$, a "massive" AdS elementary system is a deformation of both a relativistic free particle with rest energy mc^2 and a 3d isotropic quantum harmonic oscillator with ground state energy $3/2\,\hbar\sqrt{|\Lambda_{\mathrm{AdS}}|/3}\,c \equiv 3/2\,\hbar\omega_{\mathrm{AdS}}$ [21, 22].

In contrast to AdS, energy is ill-defined for dS. However a local tangent observer will naturally choose the invariant with positive sign:

$$E^{\mathrm{rest}}_{\mathrm{dS}} = \left[m^2c^4-\hbar^2c^2\frac{\Lambda_{\mathrm{dS}}}{3}\left(s-\frac{1}{2}\right)^2\right]^{1/2}. \tag{17}$$

Noticeable simplification in both AdS and dS for **fermions** $s=1/2$:

$$\text{for dS :}\quad E^{\mathrm{rest}}_{\mathrm{dS}} = mc^2\,, \tag{18}$$

$$\text{for AdS:}\quad E^{\mathrm{rest}}_{\mathrm{AdS}} = mc^2+\frac{3}{2}\hbar\omega_{\mathrm{AdS}}\,. \tag{19}$$

In the massless case and spin s, we have

$$\text{for dS :}\quad E^{\mathrm{rest}}_{\mathrm{dS}} = \pm i\hbar\sqrt{\frac{\Lambda_{\mathrm{dS}}}{3}}c\left(s-\frac{1}{2}\right), \tag{20}$$

$$\text{for AdS:}\quad E^{\mathrm{rest}}_{\mathrm{AdS}} = \hbar\sqrt{\frac{|\Lambda_{\mathrm{AdS}}|}{3}}c(s+1)\,. \tag{21}$$

Therefore, while for dS the energy at rest makes sense only for massless fermionic systems and is just zero, for AdS the energy at rest makes sense for any spin, and in particular for spin 1 massless bosons we get

$$E^{\text{rest}}_{\text{AdS}} = 2\hbar\omega_{\text{AdS}}\,. \tag{22}$$

and for scalar massless bosons

$$E^{\text{rest}}_{\text{AdS}} = \hbar\omega_{\text{AdS}}\,. \tag{23}$$

3 Dark matter from QCD: A relic of Quark Period

We now explain the rôle of the above material in our interpretation of Dark Matter.

3.1 Cosmology chronology: the salient stages

Let us start out with the cosmology chronology depicted in Figures 3 and 4 (see for instance [25] for a comprehensive account of early cosmology versus particle physics). In Figure 4, the cosmic evolution is schematized on the thick line, on which the cosmic time, that is proportional to the logarithm of the scale factor, is made implicit, by replacing all dimensioned quantities depending on the local time t, by "effective co-moving densities" that are scaled by the scale factor depending on a global time.

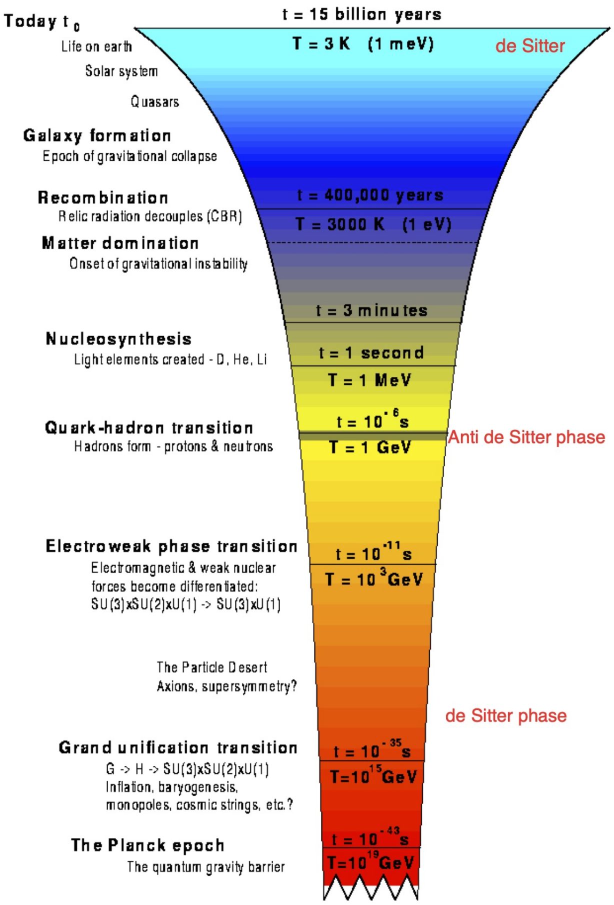

Figure 3: Cosmology chronology (from http://zebu.uoregon.edu/images/bb_history.gif).

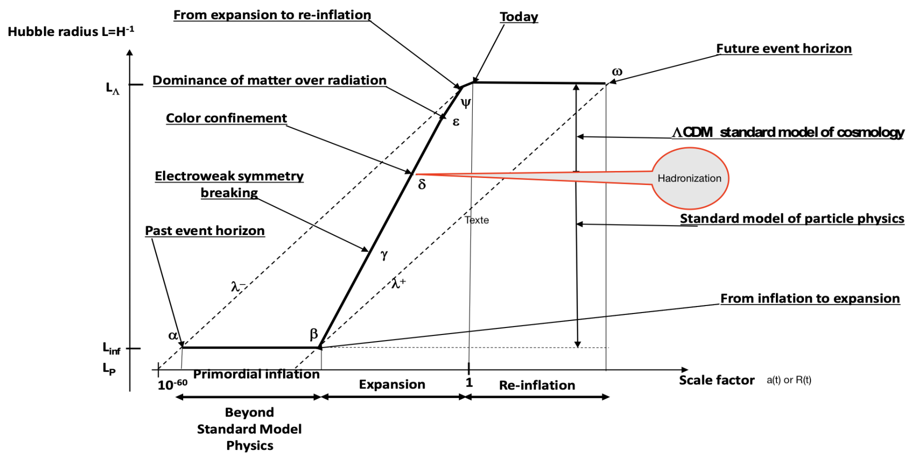

Figure 4: Cosmology chronology: Hubble radius $L(a) \equiv H^{-1}(a)$ $(c = 1)$ is plotted versus the scale factor $a(t) \equiv R(t)$ in logarithmic scale. (from [7])

In Figure 4 Greek letters represent noticeable events, to be understood as phase transitions for γ (electroweak symmetry breaking), δ (hadronization or color confinement), ϵ (dominance of matter over radiation), as Universe temperature ($\sim$ thermal time) is decreasing from the "Planck epoch" to ours. Futhermore, one should not omit the neutrino decoupling, lying between δ and ϵ, at a temperature $T \approx 1\,\text{MeV}$, as shown in Figure 3 (electroweak phase transition). Now, the cosmic microwave background (CMB) is the relic of the photon decoupling, *i.e.*, when photons started to travel freely through space rather than constantly being scattered by electrons and protons in plasma. This represents a pure QED effect, and one of its outcome is precisely that we see or experience those photons. Similarly, the cosmic neutrino background (CNB) is the relic of the neutrino decoupling when the rate of weak interactions between neutrinos and other forms of matter dropped below the rate of expansion of the universe, which produced a cosmic neutrino background of freely streaming neutrinos. In turn, this represents a pure electroweak effect, Our interpretation of Dark Matter is based on a similar scenario: gluonic component of the quark epoch (quark-gluon plasma) freely subsists after hadronization within an effective AdS environment. This represents a pure QCD effect, and we do not observe those gluons but we observe their gravitational effects. Hence, dark matter could be as well named cosmic gluonic background (CGB)... But let us tell more about dark matter.

According to the Planck 2018 analysis [26] of the CMB power spectrum, our Universe is spatially flat, accelerating, and composed of 5% baryonic matter, 27% cold dark matter (CDM, non baryonic) and 68% dark energy (Λ) [27]. (Cold) dark matter is observed by its gravitational influence on luminous, baryonic matter The dark matter mass halo and the total stellar mass are coupled through a function that varies smoothly with mass (with controversial exception(s)). One can notice that, up to now, all hypothetical particle models (WIMP, Axions, Neutrinos ...) failed direct or indirect detection tests. Similarly, alternative theories (e.g. MOND) to dark matter have failed to explain clusters and the observed pattern in the CMB, despite recurrent propitious announcements...

3.2 Quark-Gluon Plasma: experimental evidence

The main physical ingredient of our interpretation [5] is the specific state of matter Quark-Gluon Plasma (QGP), *e.g.*, see Figure 5 characteristic of the Quark Epoch quark, *i.e.* from 10^{-12}s to 10^{-6}s, with temperature $T > 10^{12}$K (point δ in Figure 4).

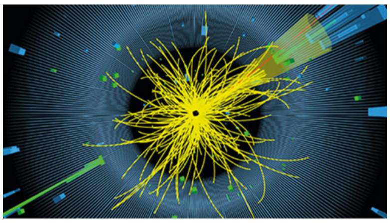

Figure 5: From Strong interactions News: *Protons probe quark-gluon plasma at CMS*, 13 January 2017.

Theories predicting the existence of quark-gluon plasma were developed in the late 1970s and early 1980s (Satz, Rafelsky, Kapusta, Müller, Letessier...), and the quark-gluon plasma was detected for the first time at CERN (2000). Lead and gold nuclei have been used for collisions yielding QGP at CERN SPS and BNL RHIC, respectively The current estimate of the hadronization temperature for light quarks is $T_{cf} = 156.5 \pm 1.5\,\text{MeV} \approx 1.8 \times 10^{12}\,\text{K}$ ("chemical freeze-out temperature"). See for instance [28–30].

3.3 Quark-Gluon Plasma and effective AdS geometry

Our scenario [5] is that the colorless gluonic component (*e.g.*, digluons) of the quark epoch which freely subsists after hadronization within an effective AdS environment (QCD effect) is the dark matter. As a matter of fact the contribution of the so-called di-gluons through what is called by Adler [4] the *gluon pairing amplitude* to the QCD trace anomaly reads as

$$\left\langle T^{\mu}_{\ \mu}\right\rangle_0 = -\frac{1}{8}\left[11N_c - 2N_f\right]\left\langle \frac{\alpha_s}{\pi}\left(F^a_{\mu\nu}F^{a\mu\nu}\right)^r\right\rangle_0 , \tag{24}$$

where N_c is the number (=3) of colors, and N_f the effective number of quark flavors which was put at 3 as a first guess, but will rather be considered as an adjustable parameter in [16], with the purpose of matching the two standard models, the one of particle phyics and the one of comology.. As asserted by G. Cohen-Tannoudji [8]

> *The minus sign in the right hand side shows that when the factor $\left(11N_c - 2N_f\right)$ is positive, all the QCD condensates contribute negatively to the energy density, which means that the QCD world-matter is globally an AdS world-matter (dominance of an AdS world-matter over a smaller dS world-matter),*

and so

- the bosonic (gluon) loops, proportional to N_c, contribute to the AdS world matter,
- the fermionic (quark) loops, proportional to N_f, contribute to the normal dS world matter.

Compare the ratio $\frac{11}{2}\frac{N_c}{N_f} \sim 5.5$ with the estimate [dark matter]/[visible matter] $\sim 27/5 = 5.4$.

3.4 Cold dark matter: Bose-Einstein condensation of (di-)gluons in effective Anti-de Sitter geometry

We know explain the mechanism which makes the remaining gluonic component the dark matter or CGB. First we remind that in an AdS geometry:

$$E^{\text{rest}}_{\text{AdS}} = \left[m^2c^4 + \hbar^2c^2\frac{|\Lambda_{\text{AdS}}|}{3}\left(s-\frac{1}{2}\right)^2\right]^{1/2} + \frac{3}{2}\hbar\sqrt{\frac{|\Lambda_{\text{AdS}}|}{3}}\,c\,. \tag{25}$$

As an assembly of N_G non-interacting (*i.e.*, colorless) scalar bosonic di-gluons with individual energies $E_n = E^{\text{rest}}_{\text{AdS}} + n\hbar\omega_{\text{AdS}}$ with $E^{\text{rest}}_{\text{AdS}}$ is mc^2 ($m = m_G$ can be zero or negligible) and degeneracy $g_n = (n+1)(n+3)/2$, those remnant components, analogous to isotropic harmonic oscillators in 3-space, are assumed to form a grand canonical Bose-Einstein ensemble whose the chemical potential μ is, at temperature T, fixed by the requirement that the sum over all occupation probabilities at temperature T yields

$$N_G = \sum_{n=0}^{\infty} \frac{g_n}{\exp\left[\frac{\hbar\omega_{\text{AdS}}}{k_B T}(n + \nu_0 - \mu)\right] - 1}, \qquad \nu_0 := \frac{E^{\text{rest}}_{\text{AdS}}}{\hbar\omega_{\text{AdS}}}. \tag{26}$$

The number N_G is very large and so the gas condensates at temperature

$$T_c \approx \frac{\hbar\omega_{\text{AdS}}}{k_B}\left(\frac{N_G}{\zeta(3)}\right)^{1/3} \qquad \zeta(3) \approx 1.2 \quad \text{(Riemann zeta function)} \tag{27}$$

to become the currently observed dark matter. The above formula involving the value $\zeta(3) \approx 1.2$ of the Riemann function is standard for all isotropic harmonic traps (see for instance [31]). Actually there is no harmonic trap here, it is the AdS geometry due to QCD trace anomaly which originates the harmonic spectrum on the quantum level. To support this scenario it is known from ultra-cold atoms physics that Bose Einstein condensation can occur in non-condensed matter but also in gas, and that this phenomenon is not linked to *interactions* but rather to the *correlations implied by quantum statistics.*

Although we do not precisely know at which stage beyond the hadronization phase transition does take place the gluonic Bose Einstein condensation, let us see if our estimate on T_c yields reasonable orders of magnitude. Take T_c equal to the current CMB temperature, $T_c = 2.78$K, and $|\Lambda_{\text{AdS}}| \approx \frac{5.5}{6.5} \times \frac{11}{24} \times \Lambda_{\text{dS}} = 0.39 \times \Lambda_{\text{dS}}$ (an estimate based on the ΛCDM model, see complements), with $\Lambda_{\text{dS}} \equiv$ present $\Lambda = 1.1 \times 10^{-52}\text{m}^{-2}$. We then get the estimate on the number of di-gluons in the condensate:

$$N_G \approx 5 \times 10^{88} \tag{28}$$

This seems reasonable since the gluons are around 10^9 times the number of baryons, and the latter is estimated to be around 10^{80}.

4 Conclusion

We have tentatively explained dark matter by actually asking a simple question (!): what becomes the huge amount of gluons after the transition from QGP period to hadronization? Similarly to the emergence of the two validated CMB (QED effect) and CNB (electroweak effect), we propose to consider Dark Matter, observed through its gravitational effects, as a pure QCD effect. From our viewpoint it would legitimate to replace the puzzling expression “Dark Matter” with the realistic “Cosmic Gluonic Background”.

A Complements: facts of ΛCDM standard model

Let us recall the cosmological formalism ($c = 1$) based on the Robertson metric. In an isotropic and homogeneous cosmology, the Einstein’s equation reads as

$$\mathsf{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathsf{R} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{29}$$

where the stress energy momentum stands for a perfect fluid with density ρ and isotropic pressure P, *i.e.*,

$$T_{\mu\nu} = -P g_{\mu\nu} + (P+\rho) u_\mu u_\nu \,. \tag{30}$$

Its solution is the Robertson metric:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right), \tag{31}$$

where k is the curvature index, and $R(t)$ is the time-dependent radius of the universe. It is the cosmological scale factor (also noted $a(t)$) which determines proper distances in terms of the comoving coordinates. The radial variable r is dimensionless.

The radius R, the density ρ, and the pressure P obey the Friedmann-Lemaître (FL) equations of a perfect fluid modelling the material content of the universe.

$$H^2 \equiv \left(\frac{\dot R}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3} \tag{32}$$

$$\frac{\ddot R}{R} = \frac{\Lambda}{3} - \frac{4\pi G}{3}(\rho + 3P) \tag{33}$$

$$\dot\rho = -3H(\rho + P) \qquad \text{(Conservation of the energy)}\,. \tag{34}$$

Note that the cosmological term $\Lambda g_{\mu\nu}$ is taken to the right-hand side of the Einstein's equation and may be interpreted as an extra pressure, named world matter by de Sitter in his debate with Einstein:

$$\mathsf{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\mathsf{R} = 8\pi G(P+\rho) u_\mu u_\nu + (\Lambda - 8\pi G P) g_{\mu\nu}\,. \tag{35}$$

According to the sign of this extra pressure one talks of a de Sitter world matter (Λ positive, pressure negative) or an anti-de Sitter world matter (Λ negative, pressure positive). From the first FL equation at $\Lambda \approx 0$ one derives

$$\frac{k}{R^2} = \frac{8\pi G}{3}\rho - H^2 \equiv \frac{8\pi G}{3}\rho - \frac{8\pi G}{3}\rho_{\mathrm{c}} \quad \rho_{\mathrm{c}} := \frac{3H^2}{8\pi G_{\mathrm{N}}}\,, \tag{36}$$

where ρ_{c} is the so-called critical density. Since the ($\sim$ observed) flatness rule $k = 0$ expresses the vanishing of the spatial curvature one can write

$$\rho - \rho_{\mathrm{c}} \equiv \rho_{\mathrm{vis}} + \rho_{\mathrm{DM}} + \rho_{\mathrm{DE}} - \rho_{\mathrm{c}} = 0\,, \tag{37}$$

with

$$\rho_{\mathrm{vis}} = \rho_{\mathrm{bar}} + \rho_{\mathrm{rad}}\,, \quad \rho_{\mathrm{DE}} = \frac{\Lambda}{8\pi G_N}\,. \tag{38}$$

Hence ρ_{c} is the energy density at the boundaries in the far past and in the far future of the Hubble horizon in the absence of any "integration constant" Λ and any spatial curvature ($k = 0$). Next, from the second FL equation

$$\frac{\ddot R}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3}(\rho + 3P) \equiv -\frac{4\pi G_N}{3}(\rho - 2\rho_{\mathrm{DE}} + 3P) \equiv -\frac{4\pi G_N}{3}(\rho_{\mathrm{effective}} + 3P) \tag{39}$$

one infers that at the inflection points $\ddot R = 0$ one has the "equation of state" (EoS)

$$w_{\mathrm{inflexion}} \equiv P/\rho_{\mathrm{effective}} = -1/3\,.$$

Inside the "confidence area" of the figure 6 in which $\Omega_\Lambda = \rho_{\mathrm{DE}}/\rho_{\mathrm{c}}$ is expressed versus $\Omega_{\mathrm{M}} = \rho_{\mathrm{m}}/\rho_{\mathrm{c}}$ one finds the points

- $(\Omega_{\rm DM}, \Omega_{\rm DE} + \Omega_{\rm vis})$,
- $(\Omega_{\rm m} = 1/3, \Omega_{\rm DE} = 2/3)$.

The value $\Omega_{\rm DE} = 2/3$ results from our assumption completing the flatness sum rule as which the total energy vanishes (from the Robertson metric):

$$\rho_{\rm vis} + \rho_{\rm DM} + \rho_{\rm DE} = \rho_{\rm c} = \frac{3}{2}\rho_{\rm DE}\,. \tag{40}$$

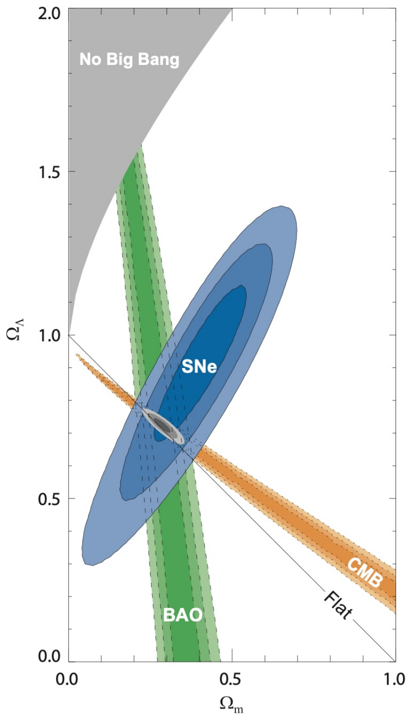

Figure 6: From [32]: 68.3 %, 95.4 % and 99.7% Confidence level contours on $\Omega_\Lambda \equiv \Omega_{DE}$ and Ω_m obtained from CMB, BAO and the Union SN set ($P/\rho \equiv w = -1$). This is “Concordance Cosmology”: The contributions of the cosmological constant Ω_Λ and of the (ordinary + dark) matter Ω_m to the ratio total density/critical density, *i.e.*, the density for which the universe is spatially flat, are yielded (modulo their uncertainty ranges) through Supernovae (SNe), Baryonic Acoustic Oscillations (BAO), and Cosmic Microwave Radiation (CMB). One sees that alternative models to Big Bang (No Nig Bang) are excluded. The straight line $\Omega_\Lambda + \Omega_{\rm m} = 1$ which is marked “flat” corresponds to a spatially flat Universe.

References

[1] T.D. Newton and E.P. Wigner, Localized States for Elementary Systems, Rev. Mod. Phys., **21** 400 (1949).

[2] C. Fronsdal, Elementary Particles in a Curved Space, Rev. Mod. Phys., **37** 221 (1965).

[3] A.D. Sakharov, Vacuum quantum fluctuations in curved space and the theory of gravitation, Dok. Akad. Nauk. SSSR **177** 70 (1967) [Sov. Phys. Dokl. **12** 1040 (1968)], (quoted by Adler [4].

[4] S. L. Adler, Einstein gravity as a symmetry breaking effect in quantum field theory, *Review of Modern Physics* **1982**, *54*, 729.

[5] G. Cohen-Tanoudji and J.-P. Gazeau, Cold Dark Matter: A Gluonic Bose–Einstein Condensate in Anti-de Sitter Space Time, *Universe* **2021**, *7*, 402; https://doi.org/10.3390/universe7110402

[6] G. Cohen-Tannoudji and J.-P. Gazeau, Scientific cosmogony, the time in quantum relativistic physics, in *Time and Science* (R. Lestienne and P. P. Harris, eds.), World Scientific, In press. hal-03538740v2

[7] G. Cohen-Tannoudji, Lambda, the Fifth Foundational Constant Considered by Einstein, *Metrologia* **2018**, *55* 486-498.

[8] G. Cohen-Tannoudji, The de Broglie universal substratum, the Lochak monopoles and the dark universe, *Annales de la Fondation Louis de Broglie* **2019**, *44*, 187–209 ; (https://arxiv.org/abs/1507.00460v10)

[9] J.-P. Gazeau, Mass in de Sitter and Anti-de Sitter Universes with Regard to Dark Matter, *Universe* **2020**, *6* (5), 66.

[10] J.-P. Gazeau and M. Novello, The Nature of Λ and the Mass of the Graviton: A Critical View, *Int. J. Mod. Phys. A* **2011**, *26* 3697–3720.

[11] H. Bacry and J.-M. Lévy-Leblond, Possible Kinematics, *J. Math. Phys.* **1968**, *9*, 1605.

[12] M. Enayati, J.-P. Gazeau, H. Pejhan, A. Wang, *The de Sitter Group and its Representations, An Introduction to Elementary Systems and Modeling the Dark Energy Universe*, Synthesis Lectures on Mathematics & Statistics, Springer (2022); 2201.11457 [math-ph]

[13] T. Garidi, What is mass in de Sitterian Physics? hep-th/0309104

[14] J. Mickelsson and J. Niederle, Contractions of representations of de Sitter groups, *Commun. Math. Phys.* **1972**, *27*, 167–180.

[15] T. Garidi, E. Huguet, and J. Renaud, de Sitter waves and the zero curvature limit, *Phys. Rev. D* **2003**, *67*, 124028 ; arxiv gr-qc/0304031

[16] G. Cohen-Tanoudji and J.-P. Gazeau, in progress

[17] J.-P. Gazeau and M. Novello, The question of mass in (anti-) de Sitter spacetimes, *J. Phys. A: Math. Theor.* **2008**, *41* 304008.

[18] C. Fronsdal, Elementary particles in a curved space. II, *Phys. Rev. D* **1974**, *10* 589-598.

[19] C. Fronsdal, Elementary particles in a curved space. III, *Phys. Rev. D* **1974**, *12* 3810-3818.

[20] C. Fronsdal, Elementary particles in a curved space. IV, Massless particles. *Phys. Rev. D* **1975**, *12* 3819-3830.

[21] J.-P. Gazeau and J. Renaud, Relativistic harmonic oscillator and space curvature, *Phys. Lett. A* **1993**, *179* 67.

[22] M. A. del Olmo, M. Enayati, J.-P. Gazeau, and H. Pejhan, Anti-de Sitterian elementary systems: complements to Newtonian and Minkowskian limits, in progress

[23] Binegar, B. , Fronsdal, C. , and Heidenreich, W. de Sitter QED, *Ann. Physics* **1983**, *149* 254-272.

[24] Gazeau, J.-P., Hans, M., and Murenzi, R. Invariant bilinear forms on 3 + 2 de Sitter space, *Class. Quantum. Grav.* **1989**, *6* 329-348.

[25] V. Rubakov and D. Gorbunov, *Introduction to the Theory of the Early Universe : Hot Big Bang Theory*, New Jersey: World Scientific (2018).

[26] Planck 2018 Results; VI. Cosmological parameters *A&A* **2020**, *A6*, 641. arXivhttps://arxiv.org/abs/1807.06209

[27] A. Arbey and F. Mahmoudi, Dark matter and the early Universe: A review, *Progress in Particle and Nuclear Physics* **2021**, *119*, 103865-1-41.

[28] R. Pasechnik, and M. Šumbera, Phenomenological Review on Quark-Gluon Plasma: Concepts vs. Observations, *Universe* **2017**, *3*.

[29] A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel, Decoding the phase structure of QCD via particle production at high energy, *Nature* **2018**, *561*, 321–330; DOI: 10.1038/s41586-018-0491-6

[30] J. Rafelski, Discovery of Quark-Gluon Plasma: Strangeness Diaries, *The European Physical Journal Special Topics* **2020**, *229*, 1-140.

[31] W. J. Mullin, W.J. Bose-Einstein Condensation in a Harmonic Potential, *J. Low Temp. Phys.* **1997**, *106*, 615-641.

[32] M. Kowalski et al, Improved cosmological constraints from new, old, and combined supernova data sets, *The Astrophysical Journal* **2008**, *686*, 749-778.