Design of a Majorana trijunction

Juan Daniel Torres Luna[1*], Sathish R. Kuppuswamy[2], Anton R. Akhmerov[2†].

[1] QuTech, Delft University of Technology, Delft 2600 GA, The Netherlands
[2] Kavli Institute of Nanoscience, Delft University of Technology, 2600 GA Delft, The Netherlands
* jd.torres1595@gmail.com
† trijunction@antonakhmerov.org

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Abstract

Braiding of Majorana states demonstrates their non-Abelian exchange statistics. One implementation of braiding requires control of the pairwise couplings between all Majorana states in a trijunction device. In order to have adiabaticity, a trijunction device requires the desired pair coupling to be sufficently large and the undesired couplings to vanish. In this work, we design and simulate of a trijunction device in a two-dimensional electron gas with a focus on the normal region that connects three Majorana states. We use an optimisation approach to find the operational regime of the device in a multi-dimensional voltage space. Using the optimization results, we simulate a braiding experiment by adiabatically coupling different pairs of Majorana states without closing the topological gap. We then evaluate the feasibility of braiding in a trijunction device for different shapes and disorder strengths.

1 Introduction

A pair of well-separated Majorana states encodes the occupation of a single fermionic state non-locally as two zero-energy states [1] . Under exchange of two Majorana states, i.e. braiding, the protected ground state evolves via unitary operations. The discrete nature of braiding allows to implement all Clifford operations with very low error rates—a requirement for universal fault tolerant quantum computation [2]. This has brought a lot of attention to the field in the past two decades with several proposals for experimental realization [3,4] and detection [5–7] of Majorana bound states. Therefore, there are several proposals for braiding that include moving Majoranas around each other in semiconductor nanowire networks [8,9], long range coupling of Majorana islands connected by quantum dots [10–13], and networks of Josephson junctions connected by trijunctions [14,15].

Braiding in hybrid semiconductor-superconductor devices requires coupling all Majorana states via control of the electrostatic potential. Two-dimensional electron gases (2DEGs) are suitable for realising trijunction devices because they combine different ingredients such as electrostatic control and superconductivity [4] in a non-linear layout. 2DEGs are an active field of research for topological physics with experiments focused on detecting signatures of Majorana states in single nanowires [16–18], planar Josephson junctions [19,20], or in minimal realisations of the Kitaev chain [21,22]. Unambiguous detection of Majoranas requires distinguishing them from non-Majorana physics producing similar results [23,24]. The recently proposed topological gap protocol [7] establishes a first step towards fully-automated detection of Majorana states.

A braiding experiment poses additional requirements to the creation of spatially isolated Majoranas. It requires measurement of the fermion parity of Majoranas belonging to the same nanowire [25,26]. Furthermore, it also requires a trijunction—a switch that selectively couples Majoranas from three different nanowires—which is the focus of our work. The requirements for a braiding experiment are such that (i) the energy of the coupled pairs needs to be larger than the thermal broadening, (ii) the ratio of the energies of coupled pairs with the remaining Majoranas should be as large as possible to ensure adiabaticity, and (iii) the gap between the zero-energy ground state and the coupled Majoranas does not close while coupling different pairs. A trijunction device that satisfies these requirements is suitable to perform braiding.

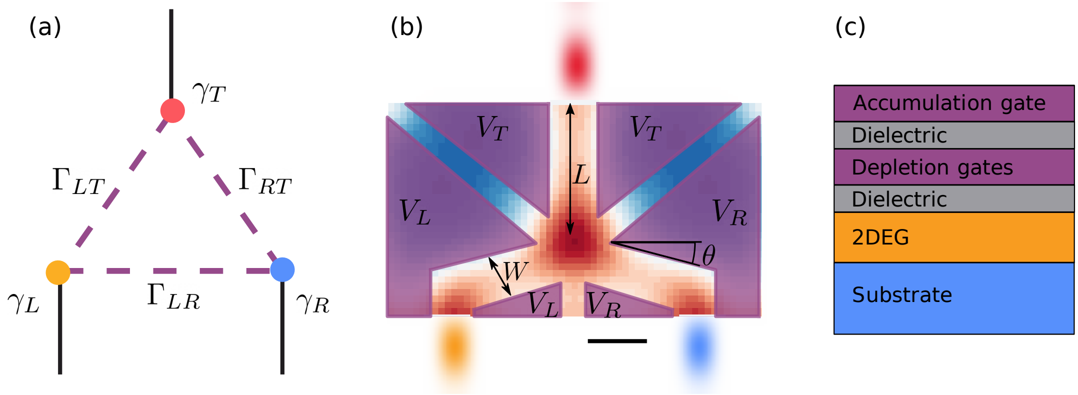

Figure 1: A trijunction device. (a) Minimal illustration of a trijunction device with three Majorana states (red, blue and yellow) and their pairwise couplings. (b) Illustration of our trijunction device deisgn. The shape of the depletion gates (purple) is parametrized by L, W, and θ. The potential induced by the gates is shown in the background. Blue regions are depleted and red regions are accumulated. Scalebar is 100 nm in all following device plots. (c) Heterostructure configuration.

In order to evaluate feasibility of a braiding experiment, we design and simulate a trijunction device as shown in Fig. 1. In order to find the operational regime of the device, we use an optimisation approach using an effective Hamiltonian in the basis of decoupled Majorana states. Then, we illustrate the device operation by simulating the braiding protocol from Ref. [15] where we switch the coupling between different pairs of Majorana states while preserving the energy gap. We define quality metrics relevant for braiding and systematically compare the performance of different trijunction device geometries. We highlight the geometries that are suitable for braiding and investigate their resilience to increasing concentration of electrostatic disorder that is unavoidable in this system [7].

2 Device layout

A braiding protocol [15,27] requires time-dependent manipulation of the pair couplings between three Majorana states shown in Fig. 1 (a). The computational subspace—one Majorana in the trijunction and three Majoranas in the far nanowires' ends—is protected as long as the number of zero-energy modes remains constant. In other words, the computation is protected as long as two out of six Majorana states are always coupled. The full braiding protocol requires coupling Majoranas from the same wire via a transmon [26] or flux qubit [25], which is outside the scope of this work. It also requires to move one Majorana state between three different wires by coupling different pairs of Majoranas via a trijunction. By combining these two procedures, it is possible to perform a braiding experiment where two Majorana states exchange positions.

Detailed modelling of Majorana nanowires is outside the scope of our study. Therefore, we consider an idealised model of topological nanowires. We simulate clean nanowires of size $W_{NW} = 70\,\text{nm}$ and $L_{NW} = 1.5\,\mu\text{m}$ such that the Majoranas are well-separated. The nanowires are parallel to a homogeneous magnetic field, which drives them into the topological phase simultaneously. We connect the nanowires to the trijunction formed in the central normal region as shown in Fig. 1 (b). We use one layer of depletion gates shown in Fig. 1 (b) to form the trijunction and a second layer for a global accumulation gate to control the electron density. We parameterize the shape of the device using channel length L, channel width W and the angle θ between the x-axis and the arms. We use the materials from Ref. [28] for the substrate, dielectric and gate electrodes.

We simulate the three dimensional device configuration shown in Fig. 1 (b-c). We use the electrostatic solver of Ref. [29] to numerically solve the Poisson's equation

$$\nabla \cdot [\epsilon_r(\mathbf{r})\nabla U(\mathbf{r})] = -\frac{\rho(\mathbf{r})}{\epsilon_0}, \tag{1}$$

where ρ_r is the charge density, ϵ_0 is the vacuum permittivity and ϵ_r is the relative permittivity. Because the 2DEG has a low electron density, we neglect the potential induced by charges in the 2DEG. We express U as a linear combination of the potential induced by each gate electrode

$$U(\mathbf{r}) = \sum_i V_i U_i(\mathbf{r}) + U_0(\mathbf{r}), \tag{2}$$

where $U_0(\mathbf{r})$ is the potential induced by dielectric impurities when $\mathbf{V} = 0$, and V_i are the elements of $\mathbf{V} = (V_L, V_R, V_T, V_{\text{global}})$. In order to reduce the number of control parameters, we apply the same voltages to the depletion gates closest to a channel shown in Fig. 1 (b).

We use the 2D Hamiltonian

$$H = \left(\frac{1}{2m^*}(\partial_x^2 + \partial_y^2) - U(x,y)\right)\sigma_0\tau_z + \alpha(\partial_x\sigma_y - \partial_y\sigma_x)\tau_z + E_z\sigma_y\tau_0 + \Delta(x,y)\sigma_0\tau_x, \tag{3}$$

where σ_i and τ_i are the Pauli matrices in the spin and particle-hole space, α is the spin orbit coupling strength, E_z is the Zeeman field induced by the homogeneous magnetic field, and m^* is the effective mass in the semiconductor. Using the Kwant software package [30], we discretize Eq. (3) over a 2D tight-binding square lattice with lattice constant $a = 10\,\text{nm}$ as for typical devices [31]. The electrostatic potential in the 2DEG, $U(x, y, z = 0) = U(x, y)$, is defined relative to the Fermi level in the nanowires which is set to the bottom of the lowest transverse band μ_0. The superconducting pairing is absent in the normal region, and in the nanowires it is $\Delta(x, y) = \Delta_0 e^{i\phi_j}$ where Δ_0 is the magnitude of gap and ϕ_j is the phase in the j-th nanowire. We tune the Hamiltonian to be in the topological phase for the lowest subband, i.e. $E_z > \sqrt{\mu_0^2 + \Delta^2}$. The induced gap in the nanowires is Δ_t.

3 Device tuning

We numerically compute the six lowest energy modes $|\phi_i\rangle$ of the depleted trijunction, which are linear combinations of decoupled Majorana states $|\gamma_i\rangle$. In order to obtain a basis of individual Majorana states, we use $|\gamma_i\rangle = \hat{W}|\phi_i\rangle$ where $\hat{W}$ is the matrix that simultaneously approximately diagonalizes the projected position operators $\hat{\mathbf{P}}_x = \langle\phi_i|\,\hat{\mathbf{X}}\,|\phi_j\rangle$ and $\hat{\mathbf{P}}_y = \langle\phi_i|\,\hat{\mathbf{Y}}\,|\phi_j\rangle$. The Majoranas in the maximally localized basis are shown in Fig. 2 (a). For an arbitrary voltage configuration, the three Majorana states close to the junction

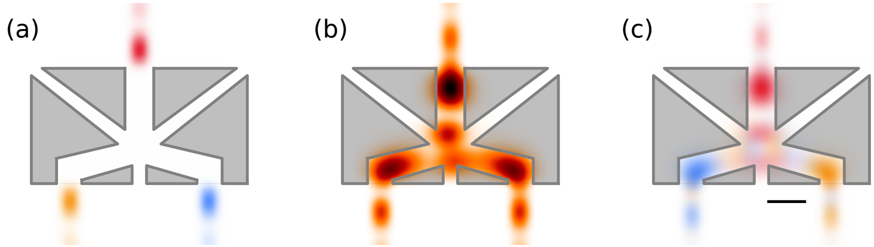

Figure 2: Representation of coupled Majoranas in the basis of localized states. (a) Densities of 3 decoupled Majorana states. (b) Wavefunction of coupled Majoranas. (c) Decomposition of coupled wavefunction into decoupled states.

interact, while the three far Majoranas remain decoupled. Our goal is to design a device that separately couples multiple pairs of Majorana states by tuning the gate voltages. We use the overlap between the coupled and decoupled Majoranas $S_{ij} = \langle\psi_i|\gamma_j\rangle$ to heuristically determine the coupling between Majoranas originating from different arms. We apply a singular value decomposition, $S = UDV^\dagger$, where U and $V^\dagger$ are unitary and D is positive diagonal. The approximate transformation is the unitary part of the SVD decomposition, i.e. $S' = UV^\dagger$. This transformation approximates the coupled Majorana wavefunction in Fig. 2 (b) as a linear combination of decoupled Majorana wavefunctions shown in Fig. 2 (c). The low-energy effective Hamiltonian is

$$H_{\text{eff}} = S'\text{diag}(E_0, E_1, E_2)S'^\dagger = i\sum_{i\neq j}\Gamma_{ij}|i\rangle\langle j|, \tag{4}$$

where Γ_{ij} is the coupling between Majoranas γ_i and γ_j, and E_k are the three lowest eigenvalues of the exact Hamiltonian. When coupling a single pair of Majorana states, the effective coupling always corresponds to the first non-zero eigenvalue, i.e. $\Gamma_{ij} = E_2$, however, when there are multiple pairs of coupled Majoranas, interpretation of the effective couplings is ambiguous.

In order to find the operational regime of the device, we use an optimisation approach to find the optimal couplings as a function of gate voltages and phase differences. For the coupling of the i-th and j-th Majorana states, we define the desired and undesired couplings as

$$\delta_+ = \Gamma_{ij}, \quad \delta_- = \Gamma_{ik} + \Gamma_{jk}, \tag{5}$$

where k is the remaining Majorana state. The goal of our device is to maximize the energy of the coupled Majorana pair while keeping the couplings to the remaining Majorana state exponentially small. Therefore, we use a loss function

$$C_{\text{pair}} = -\delta_+ + \log(\delta_-^2 + \epsilon). \tag{6}$$

Here, $\delta_\pm$ is in units of Δ_t. We use $\epsilon = 10^{-3}$ to regularize the divergence of the logarithm.

To remove local minima of the loss function and improve the convergence, we penalize the regions in the gate voltage space where either regions under the gate are not depleted or the channels are fully depleted. We achieve this by adding the following soft-threshold terms to the loss function:

$$S(U(\mathbf{r})) = A\left(\sum_{\{\mathbf{r}_{\text{acc}}\}} U(\mathbf{r}_{\text{acc}})\Theta[U(\mathbf{r}_{\text{acc}})] + \sum_{\{\mathbf{r}_{\text{dep}}\}} (U(\mathbf{r}_{\text{dep}}) - u_0)\Theta[-U(\mathbf{r}_{\text{dep}})]\right). \tag{7}$$

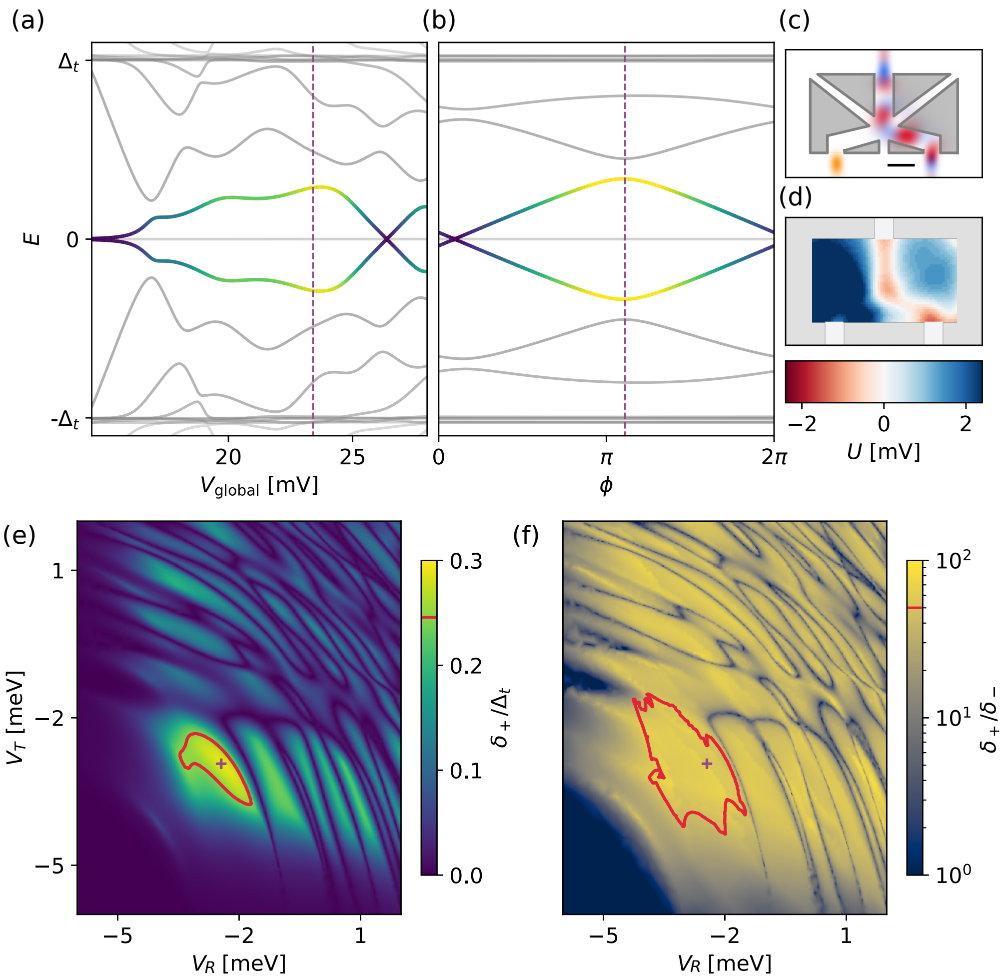

Figure 3: Spectra of a trijunction with optimally coupled right-top pair of Majoranas (colored lines) with respect to (a) global accumulation gate and (b) superconducting phase difference. Optimal point indicated by the purple dashed lines. The wavefunctions (c) and the potential (d) at the optimal point. Two dimensional scans as a function of the voltages for the top and right depletion gates of the desired coupling (e) and the ratio of desired over undesired coupling (f). The optimal point is shown as a purple cross inside of the scan. The operation range is shown as a red line with the corresponding values inside the colorbar.

Here $\Theta(x)$ is there heavy-side function. We choose $\{\mathbf{r}_{\text{acc}}\}$ and $\{\mathbf{r}_{\text{dep}}\}$ to be in the accumulated channel and in the depleted regions, respectively. We choose the scale factor $A = 10^2$, and use a threshold $u_0 \sim 1-2$meV. The total loss function is

$$L = C_{\text{pair}} + S. \tag{8}$$

Minimizing this loss function for all Majorana pairs separately yields the voltage configuration where two Majorana states are optimally coupled. The results for the right-top pair are shown in Fig 3. At the optimal point, the depletion gates form a channel between the right and top Majorana states while disconnecting the left Majorana as shown in Fig. 3 (c-d). Once the channel is formed by the depletion gates, the coupling is controlled by tuning the accumulation gate voltage V_{global} as shown in Fig. 3 (a). The phase difference between the top and right superconducting arms modulates the coupling Γ_{LR} as shown in Fig. 3 (d).

While the optimal point reaches the maximum coupling for a given pair, device operation depends on the stability of the coupling with respect to variations in gate voltages. In order to find the operational range of the device, we perform a two-dimensional scan of the gate voltages of the depletion arms corresponding to the coupled Majoranas while keeping the extra arm depleted and the global gate at the optimal point. Figures 3 (e-f) shows the operational regime of the device around the optimal point based on desired coupling magnitude and the ratio between the desired and undesired couplings, respectively. The operational regime of the device has a desired coupling comparable to the topological gap, and is exponentially larger than the undesired coupling. The area that satisfies both criteria corresponds to the operational range.

4 Braiding of Majorana states

We consider the braiding protocol from Ref. [15] that exchanges Majoranas γ_L and γ_R as shown in Fig. 4 (a). The ingredients that we require for the braiding protocol are

- coupling Majoranas within the same nanowire via charging energy [25, 26],
- coupling pairs of Majoranas via the trijunction as described in Sec. 3,
- coupling all three Majoranas in the trijunction,
- a path that connects from two to three coupled Majorana states without closing the topological gap.

In order to couple all three Majorana states, at least two pairs of Majoranas must be coupled. Because the device without disorder is symmetric around the x axis, we couple the left-top and right-top pairs simultaneously, and constrain the voltages to be symmetric, i.e. $V_L = V_R$. Furthermore, since finding the optimal path in voltage space is hard, we choose the path that linearly interpolates between the points where two and all Majorana states are coupled. Depending on the choice of the triple coupled point, the gap along this path may close. In the trijunction that we have studied, we find that the following loss function finds a triple coupled point connected by a gapped path to the pair coupled points:

$$C_{\text{triple}} = -(|\Gamma_{LT}| + |\Gamma_{RT}|) + |\Gamma_{LR}|. \tag{9}$$

The gap reaches a minimum $\lesssim 0.1 \times \Delta_t$ along the braiding path. As before, we add a soft-threshold term to ensure that all channels are formed. We obtain the optimal coupling by minimizing the loss function as in Eq. (8). The resulting spectrum of the trijunction is shown in Fig. 4 (c). The wavevunctions at the optimal points are shown in Fig. 4 (b).

5 Geometry dependence

In order to evaluate the adiabaticity of the braiding protocol, we compute the desired coupling, δ_+, and the ratio between desired and undesired couplings, δ_+/δ_-, at the optimal point. Because the topological gap is small, we require the Majorana couplings to be comparable to it:

$$\delta_+ \gtrsim 0.3 \times \Delta_t. \tag{10}$$

As a minimum requirement for adiabaticity, the desired coupling should be larger than the undesired coupling:

$$\delta_+ \gtrsim 50 \times \delta_-. \tag{11}$$

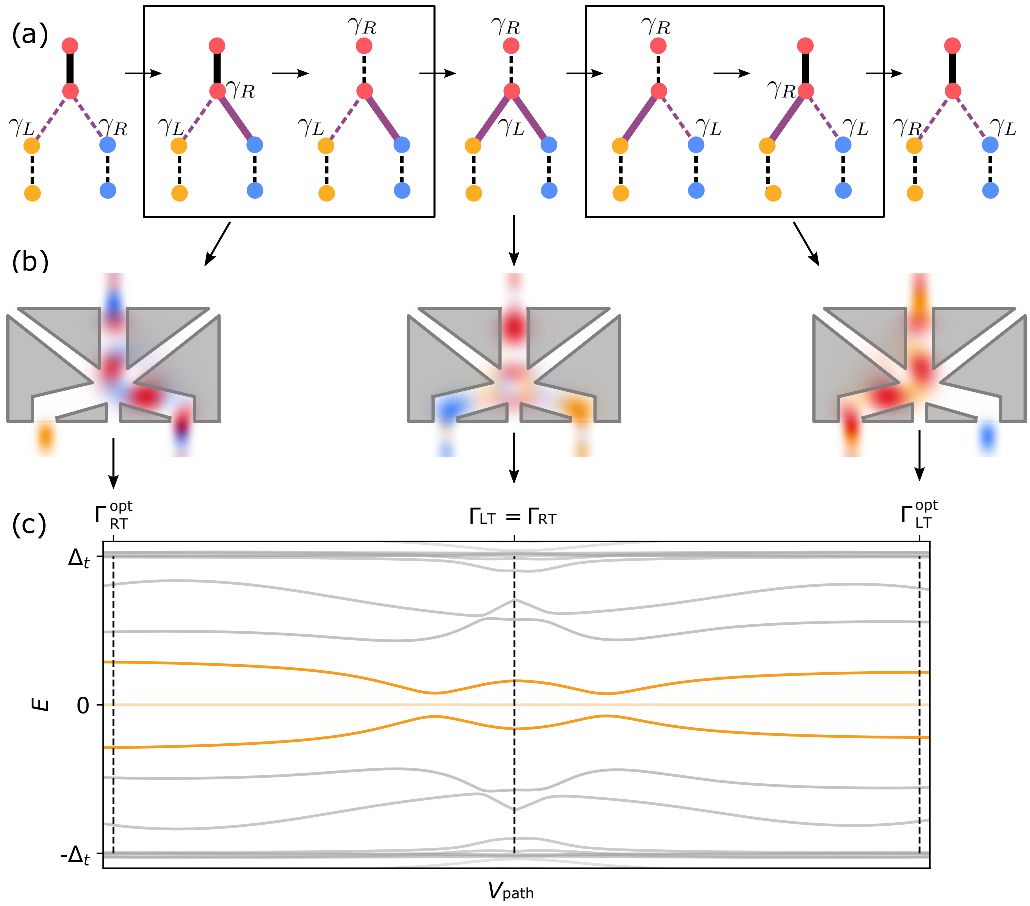

Figure 4: Exchange of two Majoranas by adiabatically coupling different pairs of Majoranas. (a) Scheme of the full braiding protocol described in Ref. [15]. Thick lines indicate coupled Majoranas via the trijunction(purple) or nanowire(black). Dashed thin lines represent decoupled Majoranas. Horizontal arrows indicate the order of operations. Vertical arrows indicate the correspondence between the steps of the braiding protocol and the results of our simulation, i.e. Majorana wavefunctions (b) and spectrum of the trijunction (c) along the voltage path.

In order to quantify the tunability of a device, we define its operational range as the area $\mathcal{A}$ in the voltage space where $\delta_+ \gtrsim 0.85 \times \delta_{\max}$, with $\delta_{\max}$ the maximum coupling in the scan as shown in Fig. 3 (e-f).

In order to determine which geometries are suitable for braiding, we compute the quality metrics δ_+/Δ_t, δ_+/δ_-, and $\mathcal{A}$ for different L, W, and θ. We evaluate the quality metrics for the worst performing pair. We summarize the results in Fig. 5 and indicate the geometries that meet the thresholds in Eqs. (10) and (11). We find that small trijunctions have a systematically larger operational voltage range as well as larger couplings. For larger trijunctions, it is possible to find a geometry suitable for braiding, but it requires fine-tuning W. The angle θ does not affect the qualitative behaviour of the trijunction.

6 Electrostatic disorder

We compare the susceptibility to electrostatic disorder of larger and smaller geometries. For that we select two geometries and analyze their performance in the presence of disorder. We

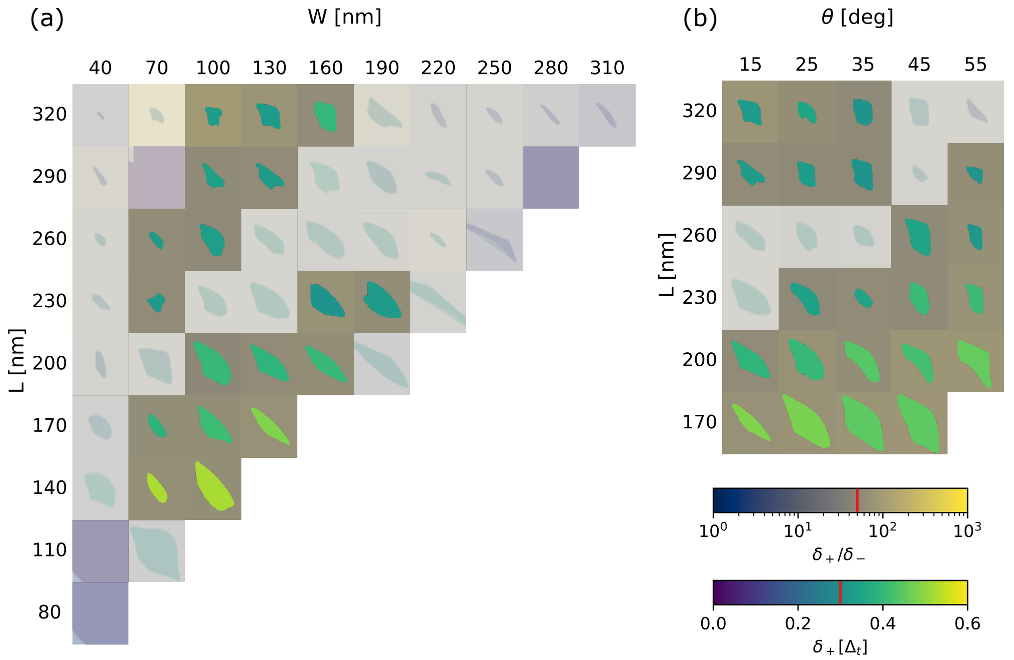

Figure 5: Analysis of quality metrics for the worst performing pair for different trijunction geometries with $\theta = 15°$ (a) and $W = 130\,\mathrm{nm}$ (b). The operation range for each geometry as identified in Fig. 3 (e) is shown inside each square and colored with δ_+ at the optimal point. The background is colored with δ_+/δ_- at the optimal point. The optimality criteria from Eqs. (10) and (11) is indicated by a red line in the respective colorbar. The geometries that do not satify this criteria have increasing transparency The squares fully covered in purple are the cases when the optimization algorithm did not find a solution.

simulate disorder in the dielectric between the depletion gate layer and 2DEG by randomly positioned positive charges. Figure 6 shows that devices with an impurity concentration of $\sim 1 \times 10^{10}\,\mathrm{cm}^{-2}$ are not degraded by disorder. On the other hand, a small concentration of electrostatic disorder $\sim 1 \times 10^{11}\,\mathrm{cm}^{-2}$, which is achieved in state-of-the-art Majorana devices [7], significantly reduces the performance of a trijunction. While smaller geometries performs better, we expect that they are more susceptible to fabrication imperfections, therefore posing a tradeoff between two challenges.

7 Summary

In this work, we developed a numerical procedure to design a braiding protocol using a trijunction device—one of the ingredients for a topologically protected quantum computer—by using three dimensional electrostatic and quantum simulations. We used an optimization approach to find the voltage configurations where all different pairs of Majorana states are strongly coupled. Consequently, we discovered that a range of trijunction device geometries can be used as switches that selectively couple and decouple different Majorana states. We confirmed that trijunctions are suitable for braiding by simulating the braiding protocol from Ref. [15] without closing the gap between the ground state and the coupled Majorana states. The operation of the device is limited by the gap size, which decreases to $\lesssim 0.1 \times \Delta_t$ along the braiding protocol. We observe that state-of-the-art levels of disorder render this

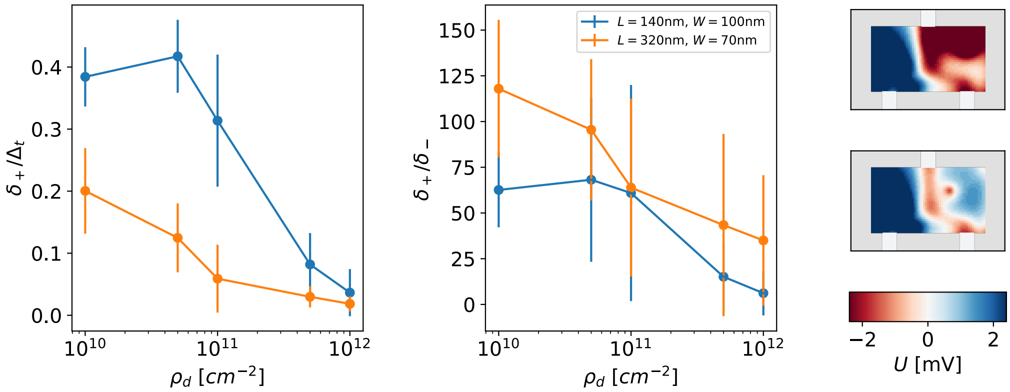

Figure 6: Impact of disorder on δ_+ (a) and δ_+/δ_- (b). We show two example disordered realisations (c) for $\rho = 1 \times 10^{10}\,\mathrm{cm}^{-2}$. We considered 10 disorder realizations for each impurity density. The error bars correspond to the standard deviation.

trijunction design inoperable because the narrow channels cannot be formed. Therefore, we expect that cleaner materials [32] or a different design would be required to resolve this problem.

The methods developed in our study are applicable to other realisations of Majorana states such as the minimal Kitaev chain [21,33]. Similarly, the optimization method that we developed is transferable to other semiconducting devices such as spin qubits [34] or planar Josephson junctions [31]. The operational regime of these devices usually lies in a region of a multidimensional space that maximises certain quantities such as the wavefunction overlap [34] or the energy gap [31]. Our work demonstrates that combining electrostatic simulations, effective Hamiltonians, and optimization routines is a powerful tool in designing and operating semiconductor devices.

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Author contributions A.R.A. defined the project goal and supervised the project. J.D.T.L. designed the trijunction device. J.D.T.L. and S.R.K. setup the simulations and obtained the results. J.D.T.L. wrote the manuscript with input from S.R.K. and A.R.A.

Data availability All code and data used in this work is available at Ref. [35].

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