

Site-wise dynamic defects in a non-conserving exclusion process

Nikhil Bhatia^{1*} and Arvind K. Gupta^{2†}

1,2 Department of Mathematics, Indian Institute of Technology Ropar, Rupnagar-140001,
Punjab, India

* nikhil.19maz0007@iitrpr.ac.in, † akgupta@iitrpr.ac.in

Abstract

Motivated by the significant influence of the defects in the dynamics of the natural or man-made transportation systems, we propose an open, dynamically disordered, totally asymmetric simple exclusion process featuring bulk particle attachment and detachment. The site-wise dynamic defects might randomly emerge or vanish at any lattice location, and their presence slows down the motion of the particles. Using a mean-field approach, we obtain an analytical expression for both particle and defect density and validate them using Monte Carlo simulation. The study investigates the steady-state characteristics of the system, including phase transitions, analysis of boundary layers, and phase diagrams. Our approach streamlines the defect dynamics by integrating two parameters into one called the obstruction factor, which helps in determining an effective binding constant. The impact of the obstruction factor on the phase diagram is explored across various combinations of binding constants and detachment rates. A critical value of the obstruction factor is obtained, about which an infinitesimal change results in a substantial qualitative change in the structure of the phase diagrams. Further, the effect of the detachment rate is studied, and critical values along which the system observes a quantitative transition of the stationary phases are obtained as a function of the obstruction factor. Overall, the system shows stationary phases ranging from three to seven depending upon the value of the obstruction factor, the binding constant, and the detachment rate. Moreover, we scrutinized the impact of the obstruction factor on the shock dynamics and found no finite-size effect.

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1

2 Contents

3	1 Introduction	2
4	2 Model overview	4
5	3 Master equations	5
6	4 Continuum mean-field approximations	6
7	5 Analytical solution at stationary state	8
8	5.1 Analysis for $K^* = 1$	8

9	5.1.1 Existence of stationary phases	9
10	5.2 $K^* \neq 1$	11
11	6 Results & Discussion	13
12	6.1 System behavior for $K = 1$	14
13	6.2 System behavior for $K > 1$	15
14	6.3 System behavior for $K < 1$	17
15	7 Shock analysis & Finite-size effect	21
16	8 Conclusion	21
17	9 Acknowledgements	23
18	A Monte Carlo simulations	23
19	B Numerical scheme	23
20	References	24
21		
22		

23 1 Introduction

24 Transport has an indispensable role in our everyday lives and over the decades, there has
 25 been a surge of interest to explore stochastic transport phenomena of various complex non-
 26 equilibrium systems ranging from natural to man-made such as vehicular traffic flow, pedes-
 27 trian motion [1–5]. In eukaryotic cells, vehicles are molecular motors that proceed along
 28 intracellular filaments or DNA/mRNA strands, or ions migrating through ion channels [6–8].
 29 One of the characteristics of all such non-equilibrium systems is a non-zero current in a steady
 30 state. In contrast to the thermodynamically balanced systems, there is no overarching theoret-
 31 ical framework to figure out the characteristics of the aforementioned systems. The stochastic
 32 transport in such situations is captured by the paradigmatic model totally asymmetric simple
 33 exclusion process (TASEP) [9, 10]. In its simplest incarnation, TASEP was proposed to model
 34 biopolymerization, such as the synthesis of RNA on DNA templates [11, 12]. It captures the
 35 collective non-equilibrium dynamics of active species represented by particles traveling across
 36 a one-dimensional lattice. In an open TASEP, the particles are allowed to enter and depart at
 37 the extreme ends of a lattice and hop along a preferred direction within the bulk while taking
 38 the hard-core exclusion principle into account. From a theoretical standpoint, TASEP has been
 39 extensively studied as an archetype model of jamming, helped by the property that it is exactly
 40 solvable and that a mean-field approach gives the same result as the exact solution [2, 13–20].

41 TASEP has undergone several generalizations that imitate different facets of transportation
 42 ranging from micro to macro. One such non-conserving model that integrates an equilibrium
 43 process, i.e., Langmuir Kinetics (LK), with the non-equilibrium process TASEP is known as
 44 TASEP-LK. The LK dynamics represent the adsorption/desorption of particles on a lattice and
 45 their rates are re-scaled while preserving the inverse proportionality to the system size in order
 46 to study the conflict between the TASEP and the LK dynamics. This model is inspired by
 47 the diffusive and directed motion along the microtubule that is alternated by the processive
 48 molecular motors [21] and encompasses several intricate aspects, including the presence of a
 49 delocalized domain wall resulting in a phase of coexistence between low and high densities

50 [22–24].

51 The existence of a disorder that slows or momentarily obstructs particle movement is one
52 of the important aspects that are visible in almost all transport systems. For instance, a vehicle
53 on the road may be stopped or slowed down by other vehicles or periodically switching traffic
54 lights or during gene transcription; the molecular traffic is often "roadblocked" by histones that
55 form the core of nucleosomes or by microtubule-binding proteins, etc. [25, 26]. These obsta-
56 cles (or defects) can either be static or dynamic, leading to position-dependent hopping rates
57 (site-wise disorder) and, thus, have a significant influence on the system dynamics. The de-
58 fects have been extensively studied in the context of TASEP. Earlier, TASEP with static obstacles
59 has been studied extensively. These defects permanently reside at a location called a specific
60 site, and these sites were assigned hopping rates that were distinct from the others. Examples
61 include the investigation of the role of single local inhomogeneity or quenched site-wise inho-
62 mogeneity, a random distribution of spatially varying hopping rates [27–31]. Dynamic defects,
63 on the other hand, are more pertinent to research due to their ability to replicate the dynamics
64 of several natural and realistic transport systems. Stochastic dynamic defects, alternatively
65 known as dynamic defects, can emerge or disappear randomly at specific sites, altering the
66 hopping rate compared to unaffected sites. This variation may impede particle movement,
67 but particles move at their regular hopping rate in unobstructed regions. Previously, studies
68 have explored uncontrolled disordered systems involving a single dynamic defect that binds or
69 unbinds at a fixed location within a TASEP model with periodic boundary conditions [32] and
70 has also been studied for open boundary conditions [33]. Several other modifications, such
71 as interaction dynamics [34], non-conserving dynamics [35], reservoir crowding [36], etc.,
72 were incorporated into an open TASEP model where a single dynamic defect binds/unbinds at
73 a fixed site. Another generalization of a single dynamic defect has been proposed in a closed
74 lattice [37] where the defect diffuses as well as binds/unbinds throughout the lattice (no fixed
75 site).

76 The scenario where multiple dynamic defects appear/disappear on the lattice, also termed
77 a site-wise disorder, has been explored less. Although it seems more realistic and is capa-
78 ble of mimicking natural phenomena such as the traffic jams due to the binding/unbinding
79 of microtubule-associated proteins [38] from microtubules which are observed in several in
80 vivo [39] and in vitro, [40] experiments. In literature, the study of site-wise disorder has
81 been investigated under the framework of exclusion process [41, 42]. Some versions of TASEP
82 incorporating dynamic disorder (ddTASEP) have been investigated in a resource-constrained
83 environment [43] whereas in [44], the model has additional feedback (the particle-defect in-
84 teraction) where defects are removed by particles. Further, recently an effort has been made
85 to numerically study a generalization of an open ddTASEP model that incorporates the Lang-
86 muir kinetics for particles [45]. However, it lacks three crucial aspects: (i) the role of defects
87 in the particle dynamics is not incorporated at the boundary sites which ultimately govern
88 the stationary properties of the system such as boundary-induced phase transitions; (ii) lack
89 of uniform proportionality in the affected attachment rate and affected hopping rate of par-
90 ticles due to defects and (iii) the steady-state numerical solution for density is insufficient to
91 characterize the influence of all the parameters. Therefore, in light of the above-mentioned
92 shortcomings, we propose to analyze the role of the non-conserving dynamics of the totally
93 asymmetric simple exclusion process with the dynamic disorder. In contrast to the previously
94 studied model [42, 45], we have inculcated the concept of an affected hopping rate at the
95 entry site also, which significantly impacts the system's stationary state properties, particu-
96 larly when compared to the reference [45], the obstruction due to the defects in the proposed
97 model affects the particle hopping and attachment rates in uniform proportion. To explore the
98 dynamics of the model, we approach the system theoretically via mean-field approximation,
99 and we mainly intend to address the following points: (i) What impact do site-wise dynamic

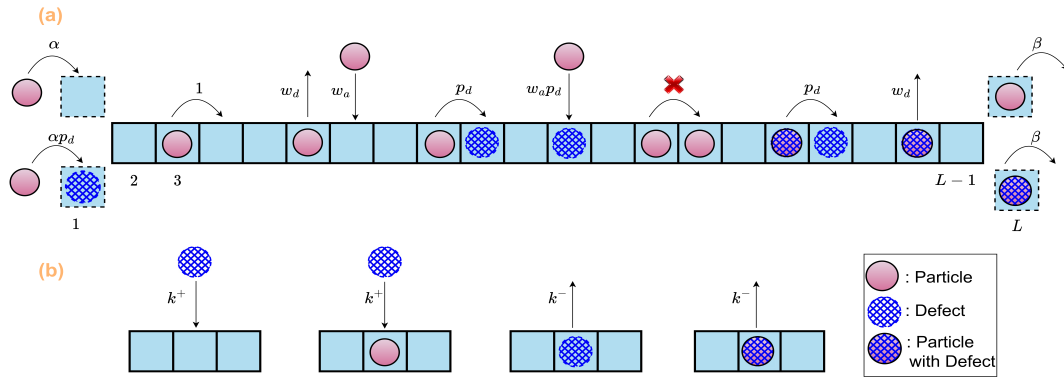


Figure 1: A diagrammatic representation of a non-conserving TASEP model depicting the dynamics of particles (highlighted in pink) and defects or obstacles (illustrated in a blue mesh). (a) Depicts different particle dynamics, including entry, exit, hopping, and attachment/detachment, along with the corresponding rates at which these events occur in the presence and absence of defects. (b) Illustrates the dynamics of defects on the lattice, including defect binding/unbinding and their corresponding rates.

100 defects have on the stationary properties of the standard TASEP-LK system, including particle
 101 flux, density profiles, and stationary phases? (ii) What factors affect the system's station-
 102 ary properties? (iii) Does the system remain symmetric with respect to particle-hole in the
 103 presence of dynamic defects? (iv) Does the domain wall remain localized in the presence of
 104 defects? If yes, what is the impact of defects on the domain wall?

105 2 Model overview

106 In actual transportation scenarios, obstacles frequently impede movement. On highways, these
 107 obstacles might be intersections or traffic signals, while in the microscopic domain, molecular
 108 traffic is often obstructed by proteins that are bound or temporary alterations to the 'lanes'
 109 through which traffic flows. Motivated by these stochastic disorders, we propose a model rep-
 110 resenting an open, dynamically disordered TASEP with LK dynamics. It is represented through
 111 a one-dimensional discrete lattice comprising L sites, each labeled from $j = 1$ to L . Here, par-
 112 ticles enter through the initial site ($j = 1$), traverse both horizontally and vertically within the
 113 bulk ($2 \leq j \leq L - 1$), and exit via the final site ($j = L$). Particles moving horizontally only
 114 exhibit a unidirectional horizontal movement (left to right). Moreover, the adsorption/ des-
 115 orption of particles also pertains to the lattice, where particles can also join or leave the lattice
 116 by a vertical movement from all sites other than the first and last sites. The lattice also includes
 117 a different type of entity known as defects (or obstacles), which introduce dynamic disorder
 118 and impede the movement of particles throughout the lattice. In contrast to particle move-
 119 ment, the defects only exhibit vertical movement and can randomly bind/unbind on every
 120 lattice site. Individually, both particles and defects adhere to the hard-core exclusion princi-
 121 ple. Therefore, each site can only accommodate a single particle, defect, or a combination of
 122 the two. As depicted in FIG. 1, the events showcasing possible particles and defects dynam-
 123 ics on the lattice, along with their corresponding occurrence probabilities, are illustrated as
 124 follows:

- 125 1. **Particle dynamics:** The dynamics of particles are significantly influenced by defect oc-
 126 currence; hence, these dynamics at various lattice locations are characterized as follows:

- 127 (a) **At entry:** If the first site has no particle, a particle can enter the lattice through this
 128 site with a rate α if it has no defect or with a rate αp_d ($p_d < 1$) otherwise. In case
 129 the first site is particle-occupied and its immediate right neighbor is particle-vacant,
 130 the particle can move to this neighboring site at a unit rate if the arrival site has no
 131 defect or with a rate of p_d otherwise.
- 132 (b) **At bulk:** If a particle occupies a bulk site, it first attempts detachment at a rate
 133 of w_d . If detachment fails and its immediate right neighbor is particle-vacant, the
 134 particle moves to the neighboring site with a unit rate if no defect is present at
 135 the arrival site or with a rate of p_d otherwise. At a bulk site without a particle, a
 136 particle can attach at a rate of w_a if no defect is present at the arrival site or at a
 137 rate of $w_a p_d$ otherwise.
- 138 (c) **At exit:** A particle present at the last lattice site can leave the lattice with a rate β .
- 139 2. **Defect dynamics:** A defect can randomly bind (unbind) at a site without (with) a defect
 140 with a rate k^+ (k^-). Note that the particle's presence on the arrival site has no effect on
 141 the dynamics of defects, but the converse is not true.

142 An event such as hopping of the particle, attachment/detachment of the particle, or bind-
 143 ing/unbinding of the defect is selected depending on the probability proportional to their
 144 corresponding rates.

145 Notably, the proposed model is distinctive from the ref. [42] in the sense that the attach-
 146 ment and detachment of particles are considered to make it more realistic. Moreover, this
 147 study not only addresses the dynamics of defects (binding/unbinding) at the boundary sites
 148 but also examines its impact on particle dynamics through modified rates at the boundaries, a
 149 consideration which was absent in the references [42, 45]. In the later part, we will explicitly
 150 discuss that these considerations will produce a non-trivial effect on the stationary-state char-
 151 acteristics of the model. In the subsequent section, we will offer mathematical underpinning
 152 by formulating master equations that depict the temporal evolution of the average particle and
 153 defect density, elucidating the process involved, and obtaining the stationary-state solution by
 154 solving them in the thermodynamic limit.

155 3 Master equations

156 Individually, both particles and defects obey the hard-core exclusion principle; therefore, we
 157 introduce two binary random variables σ_j and ν_j each denoting the occupancy of the particle
 158 and defect on the lattice, respectively. The random variable σ_j (or ν_j) = 0/1 signifies the
 159 absence/presence of particle (or defect) at j^{th} lattice site. Now, these variables are employed
 160 to formulate the master equation showcasing the evolution of the average occupation number
 161 for each entity, starting with the particles. The particle density in the bulk of the lattice evolves
 162 as follows:

$$\frac{d\langle\sigma_j\rangle}{dt} = J_{j-1,j} + w_a\langle(1-\nu_j)(1-\sigma_j)\rangle + w_a p_d\langle\nu_j(1-\sigma_j)\rangle - J_{j,j+1} - w_d\langle\sigma_j\rangle, \quad (1)$$

163 where,

$$J_{j-1,j} = \langle\sigma_{j-1}(1-\nu_j)(1-\sigma_j)\rangle + p_d\langle\sigma_{j-1}\nu_j(1-\sigma_j)\rangle \quad (2)$$

164 $\langle\cdots\rangle$ denotes the statistical average and $J_{j-1,j}$ is the particle-flux from $j-1^{\text{th}}$ site to j^{th} site.
 165 The equation governing the evolution of particle density at both the left-lattice and right-lattice
 166 boundaries is formulated as:

167

$$\frac{d\langle\sigma_1\rangle}{dt} = \alpha\langle(1-\sigma_1)(1-\nu_1)\rangle + \alpha p_d\langle(1-\sigma_1)\nu_1\rangle - J_{1,2}, \quad \text{and}, \quad (3)$$

$$\frac{d\langle\sigma_L\rangle}{dt} = J_{L-1,L} - \beta\langle\sigma_L\rangle, \quad (4)$$

168 respectively. Lastly, the master equation dictating the evolution of the average defect density
169 within the lattice is provided as follows:

$$\frac{d\langle\nu_j\rangle}{dt} = k^+\langle 1 - \nu_j \rangle - k^-\langle \nu_j \rangle, \quad 1 \leq j \leq L. \quad (5)$$

170 In order to comprehend the stationary-state dynamics of the system, the aforementioned equa-
171 tions require a solution. However, solving them in their current state poses a challenge due
172 to the presence of one-, two-, and three-point correlators. Therefore, in the subsequent sec-
173 tion, mean-field approximations are applied to these equations in an attempt to elucidate
174 stationary-state attributes such as density profile, potential stationary phases, phase transi-
175 tions, and current.

176 4 Continuum mean-field approximations

177 By employing mean-field approximations, all potential particle-particle and particle-defect
178 correlations are disregarded within the aforementioned system of master equations, namely
179 $\langle\sigma_j\sigma_{j+1}\rangle = \langle\sigma_j\rangle\langle\sigma_{j+1}\rangle$ and $\langle\sigma_j\nu_{j+1}\rangle = \langle\sigma_j\rangle\langle\nu_{j+1}\rangle$. Additionally, we introduce the nota-
180 tions $\rho_j = \langle\sigma_j\rangle$ and $\rho_{d,j} = \langle\nu_j\rangle$ to represent the average particle density and defect density,
181 respectively, at site j . This simplification results in reducing Eq. (1) to:

$$\frac{d\rho_j}{dt} = J_{j-1,j} + w_a((1-\rho_{d,j})(1-\rho_j) + p_d\rho_{d,j}(1-\rho_j)) - J_{j,j+1} - w_d\rho_j, \quad (6)$$

182 where,

$$J_{j-1,j} = \rho_{j-1}(1-\rho_j)(1-\rho_{d,j} + p_d\rho_{d,j}). \quad (7)$$

183 The evolution equations for average particle density at the left and right boundaries are refor-
184 mulated as:

$$\frac{d\rho_1}{dt} = \alpha(1-\rho_1)(1-\rho_{d,1} + p_d\rho_{d,1}) - J_{1,2}, \quad \text{and}, \quad (8)$$

$$\frac{d\rho_L}{dt} = J_{L-1,L} - \beta\rho_L, \quad (9)$$

185 respectively. Furthermore, the evolution of average defect density within the lattice follows
186 the subsequent equation:

$$\frac{d\rho_{d,j}}{dt} = k^+(1-\rho_{d,j}) - k^-\rho_{d,j}, \quad 1 \leq j \leq L. \quad (10)$$

187 To obtain the continuum version of the model, we coarse-grain the lattice by introducing
188 $\mathbf{x} = \epsilon j \in [0, 1]$ as the quasi-continuous space variable and $\epsilon = \frac{1}{L}$ as the lattice constant. Then,
189 the terms up to the first order of ϵ are retained in the Taylor series expansion of $\rho_{j\pm 1} \approx \rho(\mathbf{x} \pm \epsilon)$
190 in Eq. (6) to get the reformulation of Eq. (6) and Eq. (10) as:

$$\frac{\partial \rho}{\partial t'} + \frac{\partial J}{\partial x} = \Omega_a(1-\rho_d + p_d\rho_d)(1-\rho) - \Omega_d\rho, \quad (11a)$$

$$\frac{\partial \rho_d}{\partial t} = k^+(1-\rho_d) - k^-\rho_d \quad (11b)$$

191 respectively. Here, $t' = \frac{t}{L}$ is the re-scaled time variable, and $\Omega_a = w_a L, \Omega_d = w_d L$ are the
 192 modified Langmuir kinetic rates. Furthermore, the subscript j is also omitted, considering the
 193 spatial homogeneity of the lattice.

194 It is essential to utilize a modified detachment rate that is constant for $L \rightarrow \infty$ (in large
 195 systems) because the discrepancy between bulk and boundary dynamics becomes apparent
 196 only if particles remain on the lattice for a sufficient duration before detachment. A similar
 197 rationale justifies the adjusted attachment rate. The average particle current within the lattice
 198 bulk, considering a finite ϵ , is expressed as $J = (1 - \rho_d + p_d \rho_d) \left(-\frac{\epsilon}{2} \frac{\partial \rho}{\partial x} + \rho(1 - \rho) \right)$ whereas
 199 in the thermodynamic limit ($\epsilon \rightarrow 0^+$), it becomes $J = (1 - \rho_d + p_d \rho_d) \rho(1 - \rho)$. The right-
 200 hand side of the Eq. (11a) can also be expressed as $\Omega_d (K^* + 1) \left(\frac{K^*}{K^* + 1} - \rho \right)$. This suggests that
 201 the density governed by the Langmuir isotherm (ρ_l) defined as $\frac{K^*}{K^* + 1}$ will exhibit either an
 202 attracting or a repelling behavior with respect to the nonlinear relationship between current
 203 and density because this net source term is positive or negative, depending on whether the
 204 density ρ is below or above ρ_l where $K^* = K(1 - \rho_d + p_d \rho_d)$ and $K = \frac{\Omega_a}{\Omega_d}$ is the binding
 205 constant. This will prove to be a crucial concept while discussing density profiles in subsequent
 206 sections. If the density at the left end dips below the Langmuir isotherm and the current-
 207 density relation's slope is positive, $\frac{\partial J}{\partial \rho} > 0$, then the particles will accumulate into the bulk
 208 of the lattice through Langmuir kinetics. Consequently, the density increases towards ρ_l as
 209 one progresses away from the left boundary. Conversely, with a negative slope ($\frac{\partial J}{\partial \rho} < 0$),
 210 indicating densities greater than $1/2$, the density profiles diverge from the Langmuir isotherm
 211 as one moves away from the left boundary [24].

212 The hindrance to particle movement within the lattice is directly proportional to the num-
 213 ber of defects present on the lattice, or equivalently ρ_d , and inversely proportional to the
 214 affected hopping rate p_d . Consequently, we have introduced an obstruction factor that ratio-
 215 nalizes the role of defects in impeding particle movement and reduces the model's parameter
 216 space. This simplification will facilitate the focused study of defects on the stationary-state
 217 characteristics of the system in subsequent sections. It is defined as:

$$z = \rho_d(1 - p_d), \quad (12)$$

218 Utilizing the Eq. (12), the expression for the stationary-state current in the bulk of the lattice
 219 reduces to:

$$J = (1 - z)\rho(1 - \rho), \quad (13)$$

220 The above-obtained expression for the particle current indicates that the proposed model can
 221 be perceived as a generalization of the standard TASEP model or a model with static localized
 222 defects, where the effective hopping rate of particles is $1 - z$ [32]. Note that the obstruction
 223 factor, being a function of ρ_d and p_d , remains confined within the range of 0 and 1, as both
 224 parameters are bounded in the same range. The obstruction on the lattice diminishes to zero
 225 either when there are no defects on the lattice ($\rho_d = 0$) or when the affected hopping rate due
 226 to defects attains the standard unit hopping rate ($p_d = 1$). For this case, the expression for
 227 the current in Eq. (13) shows that the model reduces to that of a standard open TASEP with
 228 LK dynamics [24]. Conversely, the particle faces maximum hindrance when all lattice sites
 229 are entirely occupied by defects, i.e., $\rho_d = 1$, and simultaneously, the defects prevent particle
 230 hopping in their presence, indicated by $p_d = 0$. For this case, the particle current vanishes
 231 from the lattice and can be easily validated from Eq. (13).

232 In the next section, we will obtain a stationary state analytical solution to the derived
 233 continuum equations for the particle as well as defect density and compare it to simulation
 234 results.

235 5 Analytical solution at stationary state

236 Theoretical defect density at the stationary state can be readily computed from Eq. (11b) as:

$$\rho_d = \frac{k^+}{k^+ + k^-}. \quad (14)$$

237 At stationary state, the nonlinear differential Eq. (11a) in the limit $\epsilon \rightarrow 0$, reduces to a first
238 order differential equation,

$$\frac{\partial J}{\partial x} = \Omega_d(K^* + 1) \left(\frac{K^*}{K^* + 1} - \rho \right), \quad (15)$$

239 Next, we will elucidate in detail how one can analytically solve the continuum equation, Eq.
240 (11a), in the steady state. This discussion will lead to a categorization of the potential solutions
241 based on the entry rate (α), exit rate (β), the effective binding constant ($K^* = K(1-z)$), and
242 the detachment rate (Ω_d).

243 One can easily verify that the Eq. (11a) of the system remains invariant under the following
244 transformations: $\rho(x) \leftrightarrow 1 - \rho(1-x)$, $w_a(1-z) \leftrightarrow w_d$. This implies that $K^* \leftrightarrow 1/K^*$
245 and hence this symmetry with respect to K^* allows us to restrict our choices to values with
246 $K^* \geq 1$. Then, the two scenarios that need to be distinguished are $K^* = 1$ and $K^* > 1$. The
247 scenario where $K^* = 1$ is somewhat hypothetical and requires careful manipulation of the
248 binding constant and obstruction factor, but it is technically more straightforward to analyze.
249 Therefore, we will address this case first. Additionally, we will compare these results with the
250 outcomes obtained from Monte Carlo simulations.

251 5.1 Analysis for $K^* = 1$

252 Theoretical computation of the average particle density becomes mathematically simplified
253 when $K^* = 1$, as Eq. (15) factorizes to:

$$(2\rho - 1) \left((1-z) \frac{\partial \rho}{\partial x} - \Omega_d \right) = 0. \quad (16)$$

254 Upon solving Eq. (16), we retrieve two different solutions: a constant density $\rho_{MC}(x) = \frac{1}{2}$
255 associated with a maximal-current (MC) phase, and a linear profile $\rho(x) = \frac{\Omega_d}{1-z}x + C$. These
256 solutions are similar to the case of TASEP-LK without dynamic defects [24] except for the nor-
257 malization of the coefficient of x in the linear solution. To ascertain the value of the integration
258 constant C in the linear density profile, we first determine the estimate to boundary densities
259 ρ_1 and ρ_L utilizing Eqs. (8) and (9) as:

$$\rho_1 = \alpha \quad \text{and} \quad \rho_L = 1 - \beta^*, \quad (17)$$

260 where $\beta^* = \frac{\beta}{1-z}$. Now, the linear density profile yields two solutions: an entry-dominated
261 one, corresponding to the low-density (LD) phase, achieved by matching the linear solution
262 with the left boundary; and another exit-dominated, corresponding to the high-density (HD)
263 phase, obtained by matching the linear solution with the right boundary. These solutions are
264 as follows:

$$\begin{aligned} \rho_\alpha(x) &= \frac{\Omega_d}{1-z}x + \alpha, \\ \rho_\beta(x) &= \frac{\Omega_d}{1-z}(x-1) + 1 - \beta^*. \end{aligned} \quad (18)$$

265 Since we have the density solution for the standard stationary phases, we can derive a general
266 density profile $\rho(x)$ by combining three possible solutions: ρ_α , ρ_β , and ρ_{MC} . Firstly, the

267 position separating the low-density profile $\rho_\alpha(x)$ from the density profile $\rho_{MC}(x)$ is computed
 268 as $x_\alpha = \frac{(1-2\alpha)(1-z)}{2\Omega_d}$. Additionally, we compute the position $x_\beta = \frac{2\beta+2\Omega_d+z-1}{2\Omega_d}$ that separates
 269 the high-density profile $\rho_\beta(x)$ from the density profile $\rho_{MC}(x)$. Depending on the relative
 270 ordering of the x_α and x_β , the density profiles are obtained as follows: Various scenarios arise
 271 depending on the relative ordering of x_α and x_β , and the corresponding density profiles for
 272 these situations are provided as follows:

273 1. If $x_\alpha \leq x_\beta$, the continuous and piecewise linear density profile exhibiting the co-existence
 274 of three phases is given by:

$$\rho(x) = \begin{cases} \frac{\Omega_d}{1-z}x + \alpha; & 0 \leq x \leq x_\alpha, \\ \frac{1}{2}; & x_\alpha \leq x \leq x_\beta, \\ \frac{\Omega_d}{1-z}(x-1) + 1 - \beta^*; & x_\beta \leq x \leq 1. \end{cases} \quad (19)$$

275 2. If $x_\alpha > x_\beta$, a jump discontinuity between the densities $\rho_\alpha(x)$ and $\rho_\beta(x)$, arises at a
 276 point x_w in the form of a shock. The density profile exhibiting the co-existence of two
 277 phases is given by:

$$\rho(x) = \begin{cases} \frac{\Omega_d}{1-z}x + \alpha; & 0 \leq x \leq x_w, \\ \frac{\Omega_d}{1-z}(x-1) + 1 - \beta^*; & x_w \leq x \leq 1. \end{cases} \quad (20)$$

278 where the position of the shock $x_w = \frac{\beta-\alpha(1-z)+\Omega_d}{2\Omega_d}$ is obtained by utilizing the current-
 279 continuity principle at the discontinuity x_w . For $x_w \in (0, 1)$, the shock is to be visible
 280 in the bulk of the lattice. Moreover, for $x_w \leq 0$ ($x_w \geq 1$), the shock or the LD-HD
 281 co-existence phase exits from the left (right) end of the lattice leading to the LD (HD)
 282 phase whose density profile is given by $\rho_\beta(x)$ ($\rho_\alpha(x)$). The height of the shock Δ is
 283 given by,

$$\Delta = \rho_\beta(x_w) - \rho_\alpha(x_w) = 1 - (\alpha + \beta) - \frac{\Omega_d}{1-z}. \quad (21)$$

284 In the limit $z \rightarrow 0$, all the above-obtained results match that of an open TASEP with LK [24]
 285 whereas in the limit $\Omega_d \rightarrow 0^+$, the LK dynamics begin to vanish from the lattice and the
 286 stationary state density profiles converge to that of an open TASEP with site-wise dynamic
 287 defects [43].

288 5.1.1 Existence of stationary phases

289 We briefly review the stationary properties of the homogeneous open TASEP extensively stud-
 290 ied through mean-field analysis. It was observed that the system could exist in one of three
 291 phases depending on the entry and exit rates: entry-dominated low density (LD), exit-dominated
 292 high density (HD), and bulk-dominated maximal current (MC). The transition from both LD
 293 and HD phases to the MC phase occurs as a second-order transition concerning density. How-
 294 ever, the phase transition from LD to HD is first-order. In this regard, when the entry rate
 295 equals the exit rate, an LD-HD coexistence phase (Shock (S) phase) emerges, characterized
 296 by a delocalized shock traversing the lattice. Upon the incorporation of Langmuir Kinetics,
 297 the shock becomes anchored (localized shock) and extends beyond a line, encompassing a
 298 region. Furthermore, we observe various combinations of the primary phases LD, MC, and
 299 HD [23, 24].

300 In our proposed model, the lattice can possess a maximum of 21 different combinations of
 301 key phases LD, HD, and MC. However, not all of them may exist for any parameter value. Now
 302 we discuss in detail the existence of the probable stationary phases and theoretically derive
 303 their existential conditions.

- 304 (a) **LD phase:** In a lattice within an entry-dominated phase, the density profile is delineated
 305 by $\rho_\alpha(x)$ with a boundary layer on the right end. The phase boundaries containing the
 306 LD phase in the $\alpha - \beta$ parameter space are specified as:

$$\alpha < \min\left(\beta - \Omega_d, \frac{1-z}{2} - \Omega_d\right). \quad (22)$$

- 307 (b) **HD phase:** In a lattice characterized by an exit-dominated phase, the density profile is
 308 given by $\rho_\beta(x)$, with a boundary layer present at the left end. The phase boundaries
 309 encompassing the HD phase within the $\alpha - \beta$ parameter space are outlined as follows:

$$\beta < \min\left(\alpha(1-z) - \Omega_d, \frac{1-z}{2} - \Omega_d\right). \quad (23)$$

- 310 (c) **MC phase:** Following the expression of the current, the gradient of the current vanishes,
 311 and the maximal current is attained for $\rho = 1/2$. Hence, in this phase, the density profile
 312 in the bulk of the lattice is given by $\rho_{MC}(x) = 1/2$, along with the presence of boundary
 313 layers at both ends. This phase exists when α and β^* satisfies:

$$\alpha > \frac{1}{2} \quad \text{and} \quad \beta^* > \frac{1}{2}. \quad (24)$$

- 314 (d) **S phase:** In the shock phase, the density profile consists of a curve that is discontinuous
 315 at a point x_w , combining low and high-density profiles. The density to the left of x_w is
 316 represented by $\rho_\alpha(x)$, and to the right of x_w , it is denoted by $\rho_\beta(x)$. The conditions
 317 for the presence of this phase in the lattice are as follows:

$$\beta + \alpha(1-z) < 1 - z - \Omega_d \quad \text{and} \quad |\beta - \alpha(1-z)| < \Omega_d. \quad (25)$$

- 318 (e) **LD-MC phase:** There exists a two-phase co-existence region (or LD-MC phase) wherein
 319 the density at the left of x_α is expressed by $\rho_\alpha(x)$ and at the right of x_α is given by $1/2$
 320 with a boundary layer on the right end. The conditions for the existence of this phase in
 321 the lattice are given as:

$$\frac{1}{2} - \frac{\Omega_d}{1-z} < \alpha < \frac{1}{2} \quad \text{and} \quad \beta^* > 1/2. \quad (26)$$

- 322 (f) **MC-HD phase:** The density profile for the two-phase coexisting region (or MC-HD
 323 phase) is given by a continuous combination of two curves. To the left of x_β , the density
 324 is $1/2$, while to the right of x_β , it is represented by $\rho_\beta(x)$ with a boundary layer on
 325 the left end. The conditions for the presence of this phase in the lattice are outlined as
 326 follows:

$$\alpha > \frac{1}{2} \quad \text{and} \quad \frac{1-z}{2} - \Omega_d < \beta < \frac{1-z}{2}. \quad (27)$$

- 327 (g) **LD-MC-HD phase:** Similarly, a three-phase coexistence region (or LD-MC-HD phase)
 328 may occur. As mentioned earlier, it exists when $x_\alpha \leq x_\beta$, and the condition for its
 329 presence in the lattice is given by:

$$\beta + \alpha(1-z) > 1 - z - \Omega_d, \quad \alpha < \frac{1}{2} \quad \text{and} \quad \beta^* < \frac{1}{2}. \quad (28)$$

330 Now, we provide the argument to discard the prospect of the existence of the remain-
 331 ing fourteen phases. The existence of the three phases MC-LD, HD-LD, HD-MC can be dis-
 332 carded based on the argument that it is impossible to concatenate the density profiles for
 333 the above-discussed phases either continuously or discontinuously for $\Omega_d > 0$ while keeping
 334 $\rho_\alpha(x) < 1/2, \rho_\beta(x) > 1/2$. The rest eleven co-existing three phases involve the combination
 335 with any of the above three discarded phases and hence can be discarded following a similar
 336 argument. For example, the LD-MC-LD ceases to exist because it is a combination of the LD
 337 phase with the MC-LD phase, and the latter has already ceased to exist. Therefore, up to seven
 338 distinct stationary phases may be observed in the phase diagram when $K^* = 1$.

339 5.2 $K^* \neq 1$

340 Considering the particle-hole symmetry, we restrict our focus to the case $K^* > 1$. In contrast
 341 to the previous case i.e., $K^* = 1$, the continuum equation governing the particle density in Eq.
 342 (15) cannot be simplified, rendering the analysis considerably more intricate. For additional
 343 analysis, we transform Eq. (15) into the format of a re-scaled density σ , for which the solution
 344 is already established [24]:

$$\sigma(x) = \frac{K^* + 1}{K^* - 1}(2\rho - 1) - 1. \quad (29)$$

345 Clearly, the density $\rho(x) \in [0, 1]$ implies that the re-scaled density $\sigma(x) \in \left[\frac{-2K^*}{K^*-1}, \frac{2}{K^*-1}\right]$ and
 346 here the condition $\sigma(x) = 0$ represents the Langmuir isotherm $\rho_l = \frac{K^*}{K^*+1}$ which is similar to
 347 that in [24]. The continuum equation (15) simplifies to:

$$\left(\frac{\sigma + 1}{\sigma}\right) \frac{\partial \sigma}{\partial x} = \frac{(K^* + 1)^2 \Omega_d}{(K^* - 1)(1 - z)}. \quad (30)$$

348 Integrating the aforementioned equation results in:

$$|\sigma(x)| \exp(\sigma(x)) = Y(x), \quad (31)$$

349 where $Y(x)$ is given by:

$$Y(x) = |\sigma(x_0)| \exp\left(\frac{(K^* + 1)^2 \Omega_d}{(K^* - 1)(1 - z)}(x - x_0) + \sigma(x_0)\right), \quad (32)$$

350 and x_0 is a reference point that takes on the value of 0 or 1, as the values of $\sigma(x_0)$ are known
 351 at the boundaries, thus providing:

$$\begin{aligned} Y_\alpha(x) &= |\sigma(0)| \exp\left(\frac{(K^* + 1)^2 \Omega_d}{(K^* - 1)(1 - z)}x + \sigma(0)\right), \\ Y_\beta(x) &= |\sigma(1)| \exp\left(\frac{(K^* + 1)^2 \Omega_d}{(K^* - 1)(1 - z)}(x - 1) + \sigma(1)\right). \end{aligned} \quad (33)$$

352 Equation (31) possesses an explicit solution expressed in terms of the Lambert- W function,
 353 and can be formulated as:

$$\begin{aligned} \sigma(x) &= W(Y(x)); & \sigma(x) &\geq 0 \\ \sigma(x) &= W(-Y(x)); & \sigma(x) &< 0 \end{aligned} \quad (34)$$

354 The Lambert- W function encompasses two real-valued branches: $W_0(x)$ and $W_{-1}(x)$. De-
 355 pending on the domain and range of these branches, the solution to Eq. (34) is derived as:

$$\sigma(x) = \begin{cases} W_{-1}(-Y(x)); & \sigma < -1, \\ W_0(-Y(x)); & -1 \leq \sigma < 0, \\ W_0(Y(x)); & \sigma \geq 0. \end{cases} \quad (35)$$

356 The entry-dominated solution (σ_α) and exit-dominated solution (σ_β) can be obtained to align
 357 with the left and right boundary densities, respectively. These solutions can then be converted
 358 back to yield the solutions ρ_α and ρ_β in terms of the Lambert-W function, as follows:

$$\begin{aligned}\rho_\alpha(x) &= \frac{1}{2} \left(\frac{K^* + 1}{K^* - 1} (W_{-1}(-Y_\alpha(x)) + 1) + 1 \right), \\ \rho_\beta(x) &= \frac{1}{2} \left(\frac{K^* + 1}{K^* - 1} (\sigma_\beta(x) + 1) + 1 \right).\end{aligned}\quad (36)$$

359 where, $\sigma_\beta(x)$ is given as:

$$\sigma_\beta(x) = \begin{cases} W_0(Y_\beta(x)); & 0 \leq \beta^* \leq 1 - \rho_l, \\ 0; & \beta^* = 1 - \rho_l, \\ W_0(-Y_\beta(x)); & 1 - \rho_l \leq \beta^* \leq \frac{1}{2}. \end{cases}\quad (37)$$

360 Note that similar to the TASEP, the density solution ρ_α , associated with the low-density regime,
 361 remains stable for $\alpha < 1/2$, while the solution corresponding to the high-density regime, ρ_β ,
 362 is stable for $\beta^* \leq 1/2$.

363 Similar to the scenario with $K=1$, we now derive a comprehensive solution for the density
 364 profile by considering various feasible combinations of the solutions ρ_α and ρ_β . In the parame-
 365 ter range where $\alpha, \beta^* \leq \frac{1}{2}$, different solutions emerge depending on whether $1 - \beta^*$ surpasses,
 366 falls short of, or equals ρ_l . These solutions converge towards the Langmuir isotherm within
 367 the bulk while satisfying both boundary conditions [24]. When $\beta^* = 1 - \rho_l$, a flat profile of ρ_β
 368 is obtained, aligning with the Langmuir isotherm value ρ_l . Within this range, a domain wall
 369 emerges, characterized by a density expressed through a combination of $\rho_\alpha(x)$ and $\rho_\beta(x)$,
 370 given by:

$$\rho(x) = \begin{cases} \rho_\alpha(x); & x \leq x_w, \\ \rho_\beta(x); & x > x_w. \end{cases}\quad (38)$$

371 where x_w is the position of the domain wall that can be determined utilizing the condition
 372 $\rho_\alpha(x_w) = 1 - \rho_\beta(x_w)$. The height of the domain wall Δ is given by $\rho_\beta(x_w) - \rho_\alpha(x_w)$. If
 373 $0 < x_w < 1$, a region consisting of a shock (S) phase is formed. If $x_w > 1$ then the lattice
 374 is in the low-density regime whose bulk is characterized by the density profile $\rho_\alpha(x)$ with a
 375 boundary layer on the right end. If $x_w < 0$ then the lattice is in a high-density regime whose
 376 density profile is characterized by $\rho_\beta(x)$ with a boundary layer on the left end. In the left-
 377 region phases ($\alpha < 1/2, \beta^* < 1/2$), the phase boundaries extend for $\beta^* > 1/2$, remaining
 378 independent of the exit rate β and aligned parallel to the β -axis. When $\alpha = 1/2$, the system
 379 transitions into the High-Density (HD) phase, where the bulk profile fails to match the entry
 380 rate, resulting in a boundary layer at the left end. Further increases in α primarily affect this
 381 boundary layer at the left end. However, an increase in β^* beyond $1/2$ introduces a boundary
 382 layer at the right end. Consequently, the HD phase for $\beta^* \geq 1/2$ stands distinct from the
 383 HD phase for $\beta^* < 1/2$. In the bulk, the density profile remains unaffected by the entrance
 384 and exit rates, α and β , at the left and right boundaries. It is characterized by the extremal
 385 solution $W_0(-Y_{\beta=1/2})$ and is termed the ‘‘High-Density Meissner (HD_M)’’ phase. Hence, we
 386 deduce that a maximum of four possible stationary phases can occur in the phase diagram for
 387 $K^* > 1$ that are LD, HD, S, and HD_M phase.

388 Obtaining a generalized analytical stationary-state solution for Eq. (11a) poses a signifi-
 389 cant challenge due to the presence of complex features, including dynamic disorder and non-
 390 conserving particle dynamics within our system. Therefore, numerical techniques serve as
 391 a viable alternative for solving it, and this approach has been widely adopted in the litera-
 392 ture to approximate solutions for such intricate systems. The time derivative term is retained

393 in the system, and the steady-state solution is captured using significantly large time steps.
 394 The model equation is discretized using a finite difference scheme, employing the first-order
 395 forward difference formula for the time derivative and the second-order central difference
 396 formula for space derivatives; refer to [B](#) for further details.

397 6 Results & Discussion

398 In previous sections, the analysis has been conducted with respect to the parameter K^* , which
 399 was introduced to simplify and solve Eq. (11a), which involves two parameters that are ob-
 400 struction factor z and the binding constant K . Both z and K are of great relevance as defect
 401 dynamics are controlled by z , and it quantify the hindrance caused by the defects to the parti-
 402 cle movement, whereas K is responsible for particle dynamics and signifies the ratio of particle
 403 attachment with respect to particle detachment. Therefore, we now again introduce them to
 404 investigate the effect of each of these parameters individually and compare the results with
 405 the existing literature. We begin with the analytical construction of the phase diagrams within
 406 the $\alpha - \beta$ plane utilizing the theoretical results obtained in the last two sections in order to
 407 study the effect of z , K , and Ω_d on the system's stationary characteristics. We conduct nu-
 408 merical Monte Carlo simulations employing the Gillespie algorithm with a random sequential
 409 update rule to verify our theoretical conclusions; please refer to Sec. [A](#) in the appendix for fur-
 410 ther details. Note that if the Markovian dynamics of the Langmuir Kinetics model (adsorption
 411 or desorption processes) is visualized as a graph where different configurations of the pro-
 412 cess represent nodes, and each allowed transition is a directed edge with a rate based on the
 413 process rules, then if we describe the equilibrium distribution of LK using Boltzmann weight,
 414 then the effective Hamiltonian is evaluated as $H = -k_B T \sum_2^{L-1} \sigma_i \ln K$, where the "energy"
 415 term is expressed as the sum over the product of the logarithm of the binding constant K
 416 with the occupancy number of i^{th} site (where k_B is the Boltzmann weight, and T is the tem-
 417 perature of the system). In this distribution, the scenario where $K = 1$ presents intriguing
 418 topological consequences, where the edges in the graph structure for Langmuir Kinetics (LK)
 419 lose their directionality [[24](#)]. Hence, it is expected that K will significantly affect the topol-
 420 ogy of the phase diagram in the $\alpha - \beta$ plane. To investigate the individual impact of each of
 421 these parameters on the system's stationary properties, we initially fix the parameter K and
 422 vary the rest. It must be noted that as we discuss the role of K and z individually, unlike the
 423 ref. [[24](#)], the transformations: $\rho(x) \leftrightarrow 1 - \rho(1 - x)$, $w_a \leftrightarrow w_d$ no longer implies $K \leftrightarrow 1/K$.
 424 Therefore, the analysis is done for every choice of K , namely, $K = 1$, $K > 1$, and $K < 1$,
 425 where we further study the impact of z and Ω_d on the steady-state features in each of these
 426 cases. Furthermore, the phase diagrams are developed in each scenario specifically for faster
 427 defect dynamics ($k^+, k^- \gtrsim 1$), as the naive mean-field approximation aligns closely with the
 428 Monte Carlo results within this parameter range [[42](#)]. We initially constructed the phase di-
 429 agrams using the analytical expressions of the phase separation lines. Subsequently, to verify
 430 the proximity of these lines, we conduct Monte Carlo simulations at points near these lines,
 431 with detailed information provided in [A](#). Note that the phase boundaries determined through
 432 simulation depend on the magnitude of the defect binding/unbinding rates. The lower rates
 433 result in deviations from mean-field predictions due to system correlations, while faster de-
 434 fect dynamics align more closely with theoretically obtained results. In our model, we have
 435 considered these rates to be equal to or strictly greater than 1. Moreover, the phase bound-
 436 aries determined through simulations are calculated with an estimated error of less than 2%,
 437 and the same is being taken care of by the size of the markers representing the Monte Carlo
 438 simulations.

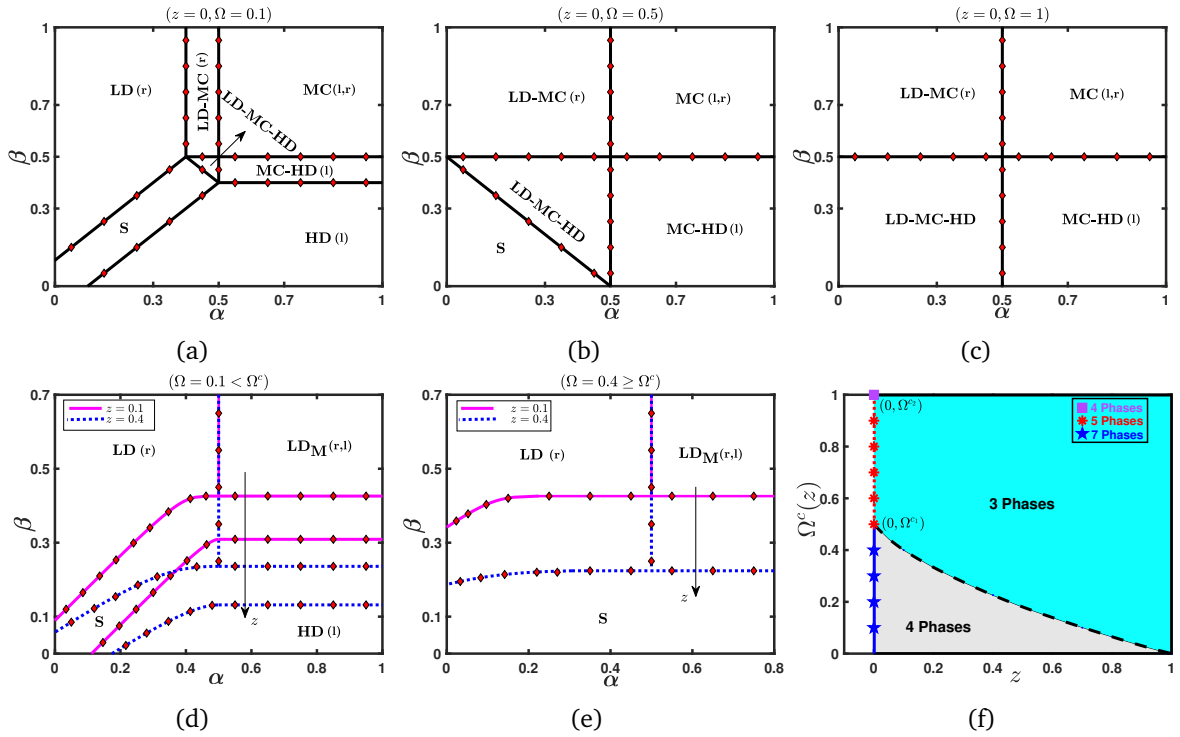


Figure 2: (a)-(e) demonstrates how the phase diagram is influenced by z and Ω when $K = 1$. Solid and dashed lines represent theoretical predictions derived from mean-field theory, while Monte Carlo simulation results are depicted with diamonds. The presence of boundary layers at the left or right end of the system is highlighted by "(l)" and "(r)" respectively. In (f), the plot illustrates Ω^c as a function of z , with different symbols (stars, circles, and squares) distinguishing the two critical values Ω^{c1} and Ω^{c2} obtained for $z = 0$.

439 6.1 System behavior for $K = 1$

440 In this context, the mathematical analysis is streamlined due to the equivalence of the at-
 441 tachment and detachment rates, denoted by $\Omega_a = \Omega_d \equiv \Omega$. Subsequently, we delve into an
 442 examination of the phase diagram's structure within the $\alpha - \beta$ parameter space, exploring its
 443 variations across different values of z and Ω . For $K = 1$, K^* is a monotonically decreasing
 444 function of z and it assumes values $K^* \leq 1$ for $z \in [0, 1)$. To assess the influence of the ob-
 445 struction factor, we generate the phase diagrams for various choices of Ω . Additionally, the
 446 effect of z is examined for a fixed Ω by varying z . The upper and lower panels of FIG. 2 depict
 447 the phase diagrams corresponding to $z = 0$ and $z \neq 0$, respectively.

448 For different values of Ω , we retrieve exactly the same phase diagrams obtained in ref.
 449 [24] in the limit $z \rightarrow 0$. We reproduce them here for the sake of comparison and analyzing
 450 the effect of z . For $\Omega < \Omega^{c1} = 0.5$, a comparatively richer phase diagram exhibiting seven
 451 stationary phases is observed, as shown in FIG. 2 (a). An increase in Ω till the critical value
 452 Ω^{c1} doesn't produce any topological changes in the phase diagram except the shifting of the
 453 phase boundaries. The boundary between the LD and LD-MC phases shifts leftward, while the
 454 boundary between the HD and MC-HD phases shifts downward. This leads to an enlargement
 455 of the LD-MC, HD-MC, and LD-MC-HD phases and a reduction of the LD, HD, and S phases,
 456 while the MC phase remains unaffected. Once Ω reaches the critical value Ω^{c1} , the LD and HD
 457 phases completely disappear from the phase diagram, and it now consists of five stationary
 458 phases only, see FIG. 2 (b). Further increasing $\Omega > \Omega^{c1}$ only affects the region $\alpha, \beta < 0.5$,

459 where the S phase shrinks and the LD-MC-HD expands whereas the LD-MC and MC-HD phases
 460 remain intact. Ultimately, at $\Omega = \Omega^c = 1$, the S phase vanishes entirely, rendering the phase
 461 diagram greatly simplified, with only four phases remaining, as depicted in FIG. 2 (c).

462 Now, we investigate the effect of the obstruction factor on the phase diagram for differ-
 463 ent choices of attachment-detachment rates. As soon as some obstruction is introduced in
 464 the lattice, the topology of the phase diagram changes drastically and becomes much simpler,
 465 consisting of four phases, see FIG. 2 (d) in comparison to the phase diagram obtained for
 466 zero obstruction factor, see FIG. 2 (a). For $\Omega < \Omega^c(\mathbf{z})$, the phase diagram consists of LD, S
 467 and HD phases along with the emergence of a LD_M phase; see FIG. 2 (d) corresponding to
 468 $\Omega = 0.1$. Further, increasing the obstruction factor on the lattice while fixing Ω results in an
 469 expansion of the LD and LD_M phases, whereas the region consisting of the S and HD phases
 470 shrinks. This can be explained as follows: an escalation in the obstruction factor intensifies the
 471 impedance to particle movement throughout the lattice, consequently enlarging the domain
 472 encompassing both the LD phase and the LD_M phases. Likewise, augmenting Ω enlarges the
 473 area encompassing the LD and S phases while the HD phase diminishes. In instances where
 474 $\Omega \geq \Omega^c(\mathbf{z})$, we note the total absence of the HD phase, resulting in a phase diagram com-
 475 prising only three phases: LD, LD_M , and S phases, as depicted in FIG. 2 (e). In this case, the
 476 effect of increasing \mathbf{z} remains the same. The FIG. 2 (f) shows the graph of the Ω^c , which is a
 477 monotonically decreasing function of \mathbf{z} . The graph demonstrates that for $\mathbf{z} > \mathbf{0}$, there is only
 478 one critical value of Ω , beyond which the number of stationary phases appearing in the phase
 479 diagram decreases from four to three. Nevertheless, when $\mathbf{z} = \mathbf{0}$, two critical values exist:
 480 $\Omega^{c1} = 0.5$ and $\Omega^{c2} = 1$. For $\Omega \geq \Omega^{c1}$, the number of stationary phases decreases from seven to
 481 five, while for $\Omega \geq \Omega^{c2}$, it decreases from five to four.

482 The phase diagram's structure differs significantly when considering equal attachment-
 483 detachment rates and a non-zero obstruction factor compared to the results obtained in ref.
 484 [24] (refer to the top and bottom panels of FIG. 2). Clearly, the presence of defects in the
 485 proposed model for the equal attachment-detachment rate of particles has made the phase
 486 diagram much simpler, which can possess at most four stationary phases depending upon the
 487 choice of Ω and \mathbf{z} .

488 6.2 System behavior for $K > 1$

489 In general, one would anticipate $K \neq 1$ because the case $K = 1$ requires a specific adjustment
 490 between the attachment and detachment rates. Therefore, without loss of generality, we first
 491 discuss the case $K > 1$ and try to understand the effect of \mathbf{z} and Ω_d on the stationary state
 492 features of the system. Analogous to the previous case, we first examine the influence of the
 493 obstruction factor by delineating the phase diagram for various selections of Ω_d . In contrast
 494 to the prior scenario, in this case, the parameter K^* varies depending on both K and \mathbf{z} . We
 495 have three different cases corresponding to the range of \mathbf{z} according to which K^* is either > 1
 496 or $= 1$ or < 1 . The panels at the top, middle, and bottom of FIG. 3 depict the phase diagrams
 497 corresponding to \mathbf{z} values within the ranges $[0, \frac{K-1}{K})$, $\mathbf{z} = \frac{K-1}{K}$, and $(\frac{K-1}{K}, 1)$, respectively.

498 In the limit $\mathbf{z} \rightarrow \mathbf{0}$, the phase diagram for $\Omega_d < \Omega_d^c(\mathbf{z}, K)$ consists of four stationary phases:
 499 LD, S, HD, and HD_M , see FIG. 3 (a). It validates the findings of the ref. [24] corresponding
 500 to $K = 3$ and $\Omega_d = 0.1$. As soon as some obstruction is introduced in the lattice, i.e., for
 501 $\mathbf{z} \in (0, \frac{K-1}{K})$, the phase boundary between the LD and the S phase as well as the one between
 502 the HD_M and the S phase shifts towards the right resulting in shrinkage in the region consisting
 503 of HD_M and HD phase whereas an expansion of the region consisting of LD and the S phase.
 504 Unlike the scenario with $K = 1$, the inclusion of the obstruction doesn't induce significant
 505 topological alterations in the phase diagram, except for expansions and contractions in the
 506 regions encompassing stationary phases. When $\Omega_d \geq \Omega_d^c(\mathbf{z}, K)$, the boundary separating the

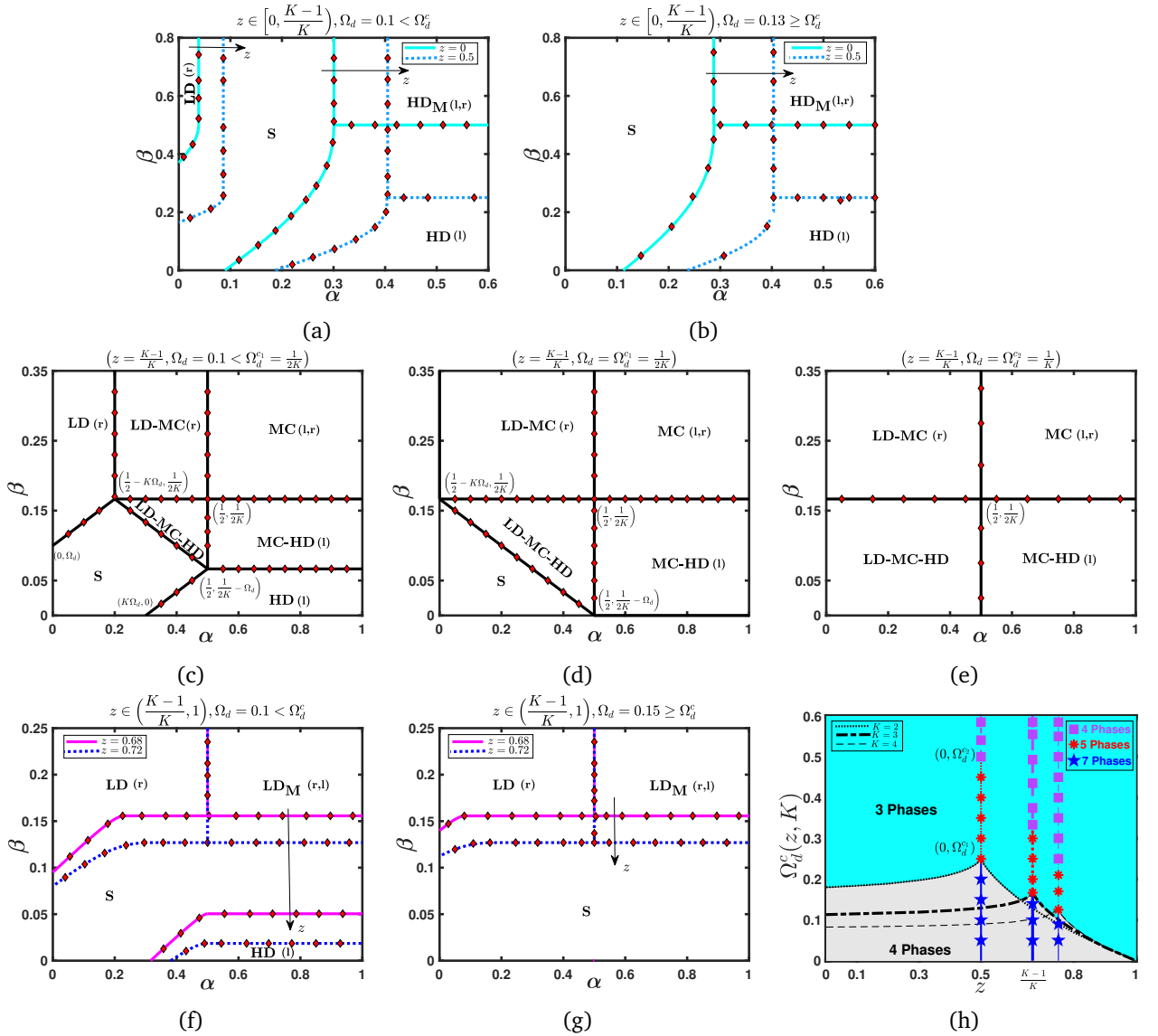


Figure 3: (a)-(g) shows the effect of z and Ω_d on the phase diagram for $K = 3$. Solid and dashed lines denote the theoretical outcomes through mean-field theory, whereas diamonds denote the Monte Carlo simulation results. In (h), the plot shows Ω_d^c as a function of z and K . Here, the symbol star, circle, and square distinguish the two critical values Ω_d^{c1} and Ω_d^{c2} obtained corresponding to $z = \frac{K-1}{K}$.

507 LD and S phases shifts leftward, leading to the total absence of the LD phase. Consequently, the
 508 phase diagram comprises only three stationary phases, as depicted in FIG. 3 (b). The impact
 509 of varying z in $(0, \frac{K-1}{K})$ remains the same for this choice of Ω_d .

510 Once the obstruction factor reaches $\frac{K-1}{K}$, as illustrated in FIG. 3 (c), FIG. 3 (d), and FIG. 3
 511 (e), the phase diagram undergoes notable topological changes. The phase diagram becomes
 512 more intricate and diverse for values of z smaller than $\frac{K-1}{K}$. It showcases seven stationary
 513 phases when $\Omega_d < \Omega_d^{c1}(K) = \frac{1}{2K}$, as depicted in FIG. 3 (c). The average density profiles
 514 for these seven stationary phases have been obtained in FIG. 4. As Ω_d increases till the criti-
 515 cal value $\Omega_d^{c1}(K)$, the phase boundary separating LD and LD-MC phases shifts to the left,
 516 whereas the phase boundary between the HD and MC-HD phases shifts downward, leading to

517 an enlargement of LD-MC, HD-MC, and LD-MC-HD phases, and a contraction of LD, HD, and S
 518 phases, while the MC phase remains unaffected. For $\Omega_d = \Omega_d^{c1}(K)$, the LD and HD phases com-
 519 pletely disappear from the phase diagram, and now it consists of only five stationary phases;
 520 see FIG. 3 (d). As Ω_d increases in the range $(\Omega_d^{c1}(K), \Omega_d^{c2}(K) = \frac{1}{K})$, the phase diagram is only
 521 affected in the region $\alpha, \frac{\beta}{1-z} < 0.5$, where the S phase shrinks and the LD-MC-HD expands,
 522 whereas the phases LD-MC, MC-HD, and MC remain intact. Finally, when Ω equals $\Omega_d^{c2}(K)$,
 523 the S phase vanishes entirely, rendering a simpler phase diagram exhibiting just four phases;
 524 refer to FIG. 3 (e).

525 Now, we discuss the case $z > \frac{K-1}{K}$. The phase diagram again becomes topologically simpler,
 526 as shown in FIG. 3 (f) and FIG. 3 (g). For $\Omega_d < \Omega_d^c(z, K)$, the phase diagram showcases four
 527 stationary phases: HD, S, LD, and LD_M, as shown in FIG 3 (f) corresponding to $\Omega = 0.1$. For
 528 a further increase in z in the range $(\frac{K-1}{K}, 1)$, the phase boundary separating the S phase from
 529 LD as well as the LD_M phase shifts downwards, this leads to an enlargement of the LD and LD_M
 530 phases, while the S and HD phases diminish. For $\Omega_d \geq \Omega_d^c(z, K)$, the phase boundary between
 531 the HD and S phase shifts downwards, causing the complete disappearance of the HD phase,
 532 and the phase diagram exhibits only three stationary phases; see FIG. 3 (g). The influence of
 533 varying z in $(\frac{K-1}{K}, 1)$ remains the same for this choice of Ω_d .

534 In comparison to the case $K = 1$, FIG. 3 (h) shows the critical values of Ω_d^c which is a
 535 function of z as well as K . For a fixed K , $\Omega_d^c(z, k)$ is a non-monotonic function that monoton-
 536 ically increases for $z < \frac{K-1}{K}$, whereas it monotonically decreases for $z > \frac{K-1}{K}$ and attains its
 537 maximum value at $z = \frac{K-1}{K}$. Meanwhile, for a fixed value of z , it is a monotonically decreasing
 538 function of K . Clearly, for $z \neq \frac{K-1}{K}$, there exists only one critical value of Ω_d , beyond which
 539 the number of stationary phases changes from four to three. However, when $z = \frac{K-1}{K}$, two
 540 critical values emerge: $\Omega_d^{c1} = \frac{1}{2K}$ and $\Omega_d^{c2} = \frac{1}{K}$. In this scenario, if Ω exceeds Ω_d^{c1} , the count of
 541 stationary phases decreases from seven to five. Likewise, when Ω surpasses Ω_d^{c2} , the number
 542 of stationary phases decreases from five to four. It must be noted that $z^c = \frac{K-1}{K}$ is the gen-
 543 eral critical value of the obstruction factor for which the phase diagram's structure becomes
 544 intricate, featuring a maximum of seven stationary phases.

545 For an attachment rate larger than the detachment rate, the topology of the proposed
 546 model's phase diagrams remains the same compared to the ref. [24] for $z < z_c$ except for
 547 expansion or shrinkage of some phase regions (see top panel of FIG.3). But for $z \geq z_c$, the
 548 topology changes significantly in comparison to the ref. [24] (see middle and bottom panels
 549 of FIG.3). Depending on the effective binding constant K^* , the phase diagram of the proposed
 550 model can exhibit several stationary phases, including LD-MC-HD, LD-MC, MC-HD, MC, and
 551 LD_M.

552 6.3 System behavior for $K < 1$

553 Due to the defects considered in the proposed model, the particle-hole symmetry is violated
 554 with respect to K , as discussed at the beginning of this section. Therefore, the case $K < 1$ needs
 555 to be discussed separately. Similar to the previous case, we first establish the phase diagram
 556 for various Ω_d choices and analyze the influence of the obstruction factor. The parameter K^*
 557 is a function of both K and z in this case as well, and $K^* < 1$ for any possible combination of
 558 z and $K < 1$.

559 As z approaches zero, the phase diagram exhibits four stationary phases: high density
 560 (HD), shock (S), low density (LD), and LD_M, for $\Omega_d = 0.1 < \Omega_d^c(z)$, as depicted in FIG 5
 561 (a). It validates the findings of the ref. [24] corresponding to $K = 0.3$ and $\Omega_d = 0.1$. When
 562 some obstruction is introduced to the lattice, the phase boundary that separates the S phase

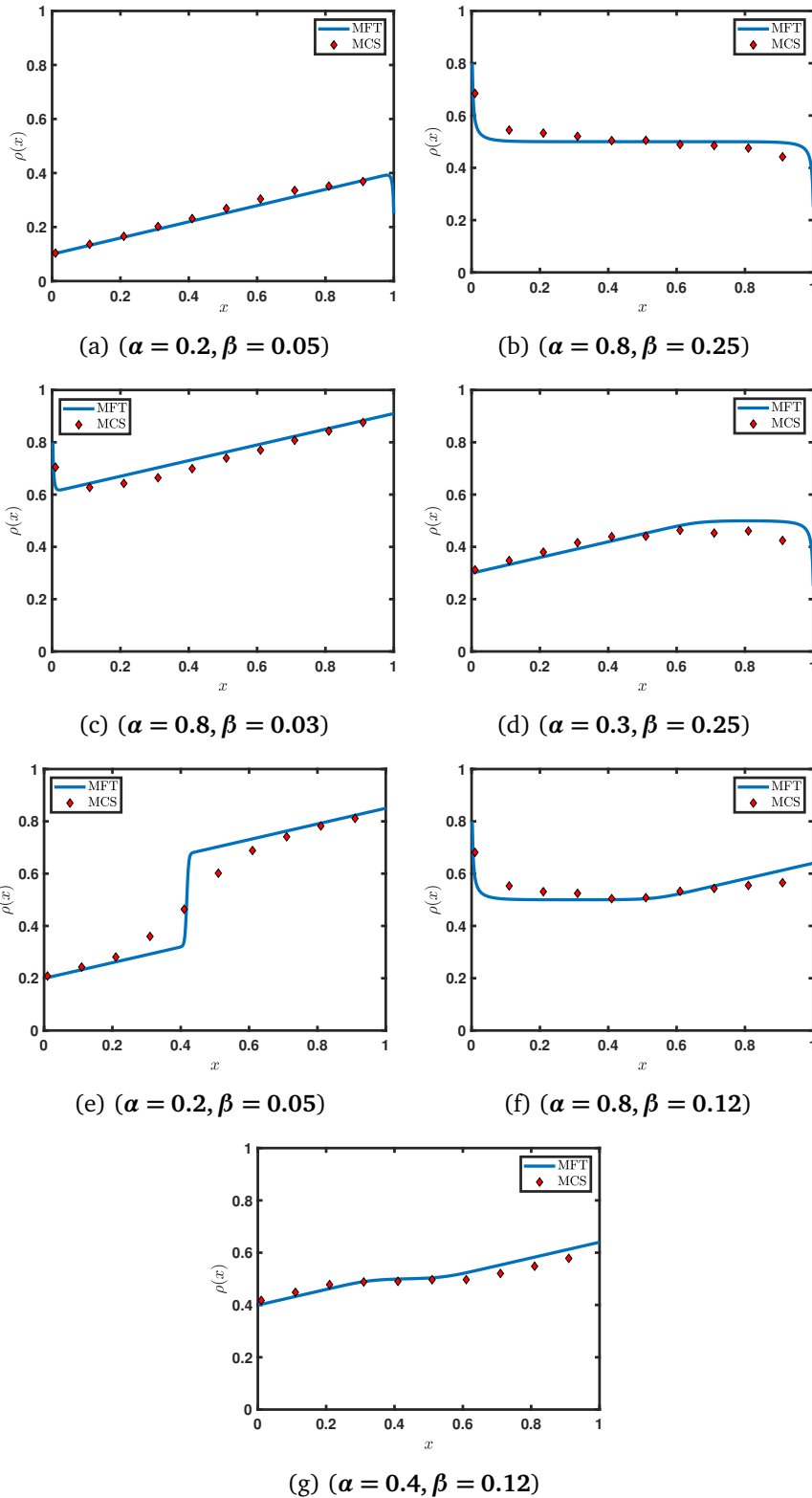


Figure 4: Average density profiles: (a) LD, (b) MC, (c) HD, (d) LD-MC, (e) LD-HD, (f) MC-HD, and (g) LD-MC-HD phases. Mean-field solutions (solid curves) contrasted with Monte Carlo simulations (markers). Parameters: $K = 3$, $\Omega_d = 0.1$, $z = \frac{K-1}{K}$. Sub-captions detail (α, β) configurations.

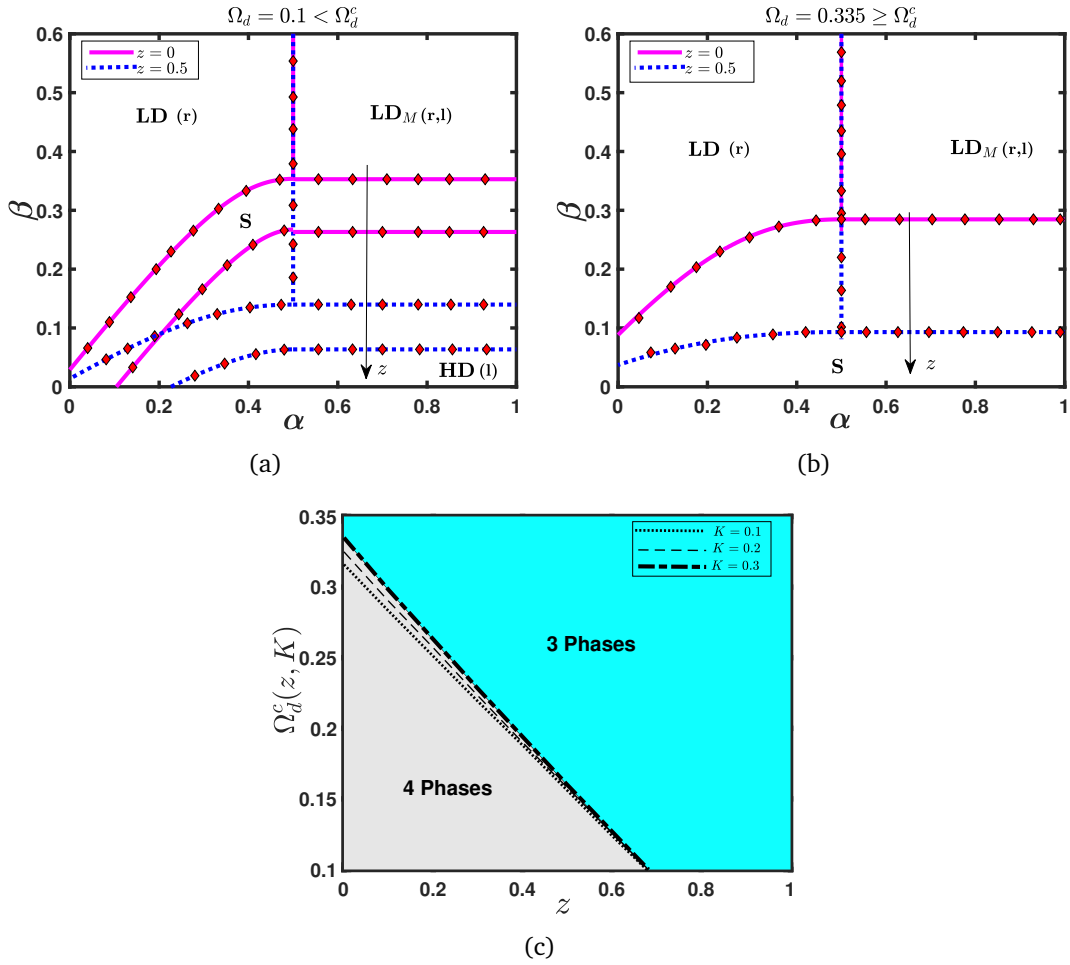


Figure 5: (a)-(b) shows the effect of z and Ω_d on the phase diagram for $K = 0.3$. Solid and dashed lines denote the theoretical outcomes through mean-field theory, whereas diamonds denote the Monte Carlo simulation results. In (c), the graph depicts Ω_d^c varying with z and K .

563 from the LD as well as LD_M phases shifts downward, expanding the LD and LD_M phases while
 564 contracting the S and HD phases, as shown in FIG. 5 (a). However, when $\Omega_d = 0.335 \geq \Omega_d^c(z)$,
 565 the high-density (HD) phase is entirely absent, resulting in a phase diagram with only three
 566 stationary phases, as illustrated in FIG. 5(b). The influence of z within the $(0, 1)$ range remains
 567 consistent for this particular value of Ω_d .

568 The FIG. 5 (c) shows the graph of the Ω_d^c which is a linear as well as monotonically decreasing
 569 function of z and K . The graph demonstrates that for $z \in [0, 1]$ there exists only one
 570 critical value of Ω_d , beyond which the number of stationary phases appearing in the phase
 571 diagram reduces from four to three.

572 For an attachment rate smaller than the detachment rate, the obstruction factor does not
 573 change much topology of the phase diagram in comparison to the ref. [24] except for the
 574 shrinkage and expansion of the phase region. Now, we briefly revisit the link mentioned in
 575 Section 2 between the proposed model and the investigation presented in Ref. [45] to show-
 576 case several distinctions in their stationary state results. Firstly, the impact of parameters ρ_d
 577 and p_d , responsible for obstructions caused by defects, on the stationary state characteris-
 578 tics of the system, encapsulated through a single parameter z . Secondly, in the scenario of
 579 equal attachment-detachment rates and non-zero obstruction, the proposed model exhibits a

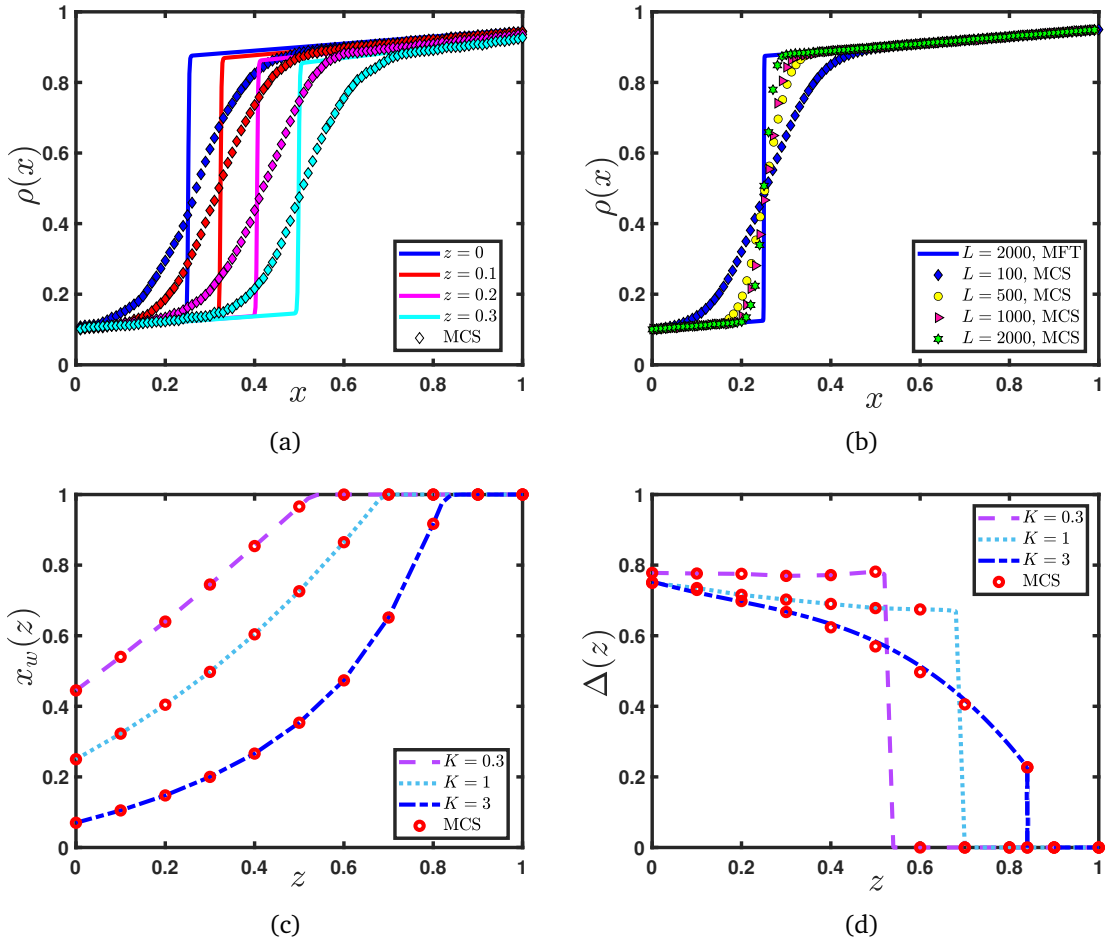


Figure 6: (a) Examining shock profiles under varying z for $\alpha = 0.1$, $\beta = 0.05$, $\Omega_d = 0.1$, and $K = 1$. (b) Finite-size effects on shock profiles at $z = 0$. Investigating (c) shock displacement and (d) shock amplitude across z .

580 maximum of four stationary phases in its phase diagrams, while in [45] the phase diagram con-
 581 sists of seven stationary phases. For this case, the phase diagram also includes a low-density
 582 Meissner (HD_M) phase, which was not observed in the [45]. Moreover, the topology of these
 583 phase diagrams differs significantly from our observations in [45]. In the case of disparate
 584 attachment-detachment rates and non-zero obstruction, the phase diagram within our pro-
 585 posed study can feature up to seven stationary phases in the phase diagram, while in [45], the
 586 system can exhibit a maximum of four stationary phases. The configurations of these phase
 587 diagrams exhibit variations compared to the findings in [45], contingent upon the selection of
 588 attachment-detachment rates and the obstruction factor. This discrepancy can be elucidated
 589 by considering the significance of boundary densities in an open system, as they strongly in-
 590 fluence the stationary properties and phase diagrams. The inclusion of the effects of defects
 591 binding/unbinding at the boundaries, which is absent in ref. [45], is a probable reason for
 592 this distinction. Lastly, unlike the model proposed in [45], our system's stationary-state results
 593 are obtained analytically, providing a comprehensive characterization of the influence of all
 594 parameters.

595 7 Shock analysis & Finite-size effect

596 One distinctive aspect of the proposed model is the emergence of the localized shock (S) phase,
 597 where the shock position remains constant over time. Typically, a qualitative examination of
 598 shock dynamics can be straightforwardly conducted using the continuity (or hydrodynamic)
 599 equation, which is expressed as:

$$\frac{\partial \rho}{\partial t'} + \frac{\partial J}{\partial x} = \omega_d L (K^* - (1 + K^*)\rho). \quad (39)$$

600 In this context, the flow-density relation, denoted by $J = (1 - z)\rho(1 - \rho)$, is well-established,
 601 allowing for the analysis of the equation above. However, the description provided by the first-
 602 order differential equation (39) becomes invalid as soon as a discontinuity arises between the
 603 densities ρ_α and ρ_β and at the intersection points of the characteristic lines corresponding to
 604 (39). This discontinuity propagates at a speed $v = \beta^* - \alpha$, determined by the balance of mass
 605 current. To establish the formation of a shock, the discontinuity must reach a position where
 606 the mass current through it is zero, thus ensuring the shock remains stationary, indicating that
 607 v must be zero.

608 Progressing further involves analyzing how the obstruction factor influences the shock
 609 profiles. A comprehensive shock profile spanning the entire system can be derived by aligning
 610 the boundary densities ρ_α and ρ_β at the location of the shock, which needs to be identified.
 611 For $K = 1$, the precise formulae for the shock position (x_w) and its height (Δ) are provided
 612 as follows:

$$x_w = \frac{\beta - \alpha(1 - z) + \Omega_d}{2\Omega_d}, \quad \& \quad \Delta = 1 - (\alpha + \beta) - \frac{\Omega_d}{1 - z}. \quad (40)$$

613 Evidently, the shock's position is consistently influenced by z , increasing as z increases, while
 614 its height shows the opposite trend, decreasing as z increases. Although obtaining explicit
 615 expressions for x_w and Δ for $K^* \neq 1$ remains challenging, their corresponding Monte Carlo
 616 results are depicted in FIG. 6 (c) and 6 (d), with fitted curves confirming their dependency
 617 on z . In FIG. 6 (a), for fixed values of α , β , and K , it's evident that the shock profile shifts
 618 from the left to the right boundaries with increasing obstruction on the lattice. This shift occurs
 619 because as z increases, particles encounter more obstructions from defects, decreasing particle
 620 density. Consequently, the HD phase's elimination and the LD phase's expansion are observed.
 621 These findings are consistent with the phase diagrams discussed in the preceding section.

622 Finite-size effects for finite L have been accounted for by incorporating second-order terms
 623 in the mean-field description. Discrepancies between the second-order mean field and the
 624 Monte Carlo results arise from shock fluctuations, which are inaccurately captured by mean-
 625 field theory and require separate treatment. Nonetheless, it's noteworthy that the shock is
 626 indeed localized, and its width grows sub-extensively, indicating sharpness as $L \rightarrow \infty$, as
 627 shown in FIG 6 (b).

628 8 Conclusion

629 We've extensively examined stochastic transportation within a one-dimensional system, incor-
 630 porating dynamic disorder in a totally asymmetric simple exclusion process alongside Lang-
 631 muir kinetics dynamics. The dynamic defects represent disorder that stochastically binds/unbinds
 632 throughout the lattice and hinders particle movement. The particle movement has been sub-
 633 ject to these dynamic defects and hops with an affected hopping rate p_d . The model is inspired
 634 by the imbalance in the transcription of genes due to obstruction, but the model is generic and

635 can be utilized to imitate any other non-equilibrium stochastic transport phenomena where
 636 dynamic defects are present.

637 To explore how dynamic defects influence the stationary-state characteristics of the system,
 638 we derive master equations in the thermodynamic limit under the framework of the continuum
 639 mean-field approximation. Moreover, we introduce a parameter termed the obstruction factor
 640 (\mathbf{z}), which amalgamates the impact of defect density on the lattice (ρ_d) and the affected
 641 hopping rate (p_d) on the system's stationary properties. In addition, we define an effective
 642 binding constant that incorporates the effect of obstruction on the binding constant. The
 643 system dynamics are controlled by the entry rate (α) and exit rate (β), and the other three
 644 important controlling parameters are \mathbf{z} , binding constant (K), and total detachment rate Ω_d .
 645 The explicit expression of analytical solutions for the density profile and phase boundaries
 646 are obtained for $K = 1/(1 - \mathbf{z})$ whereas, for the rest of the values of K , the stationary state
 647 solution has been implicitly expressed in the form of Lambert W function.

648 The theoretical solution has enabled us to delineate and extensively analyze the topology
 649 of the phase diagram. Since the proposed model doesn't obey the particle-hole symmetry, the
 650 analysis is performed for three distinct choices of binding constant, $K = 1, > 1$ and < 1 .
 651 In each case, the \mathbf{z} and Ω_d effects have been studied on the phase diagram. At the critical
 652 value of the obstruction factor (\mathbf{z}^c), the topology of the phase diagram changes significantly.
 653 At $\mathbf{z} = \mathbf{z}^c$, the phase diagram displays a richer structure consisting of either seven or five,
 654 or four stationary phases depending upon the value of Ω_d . Whereas for $\mathbf{z} \neq \mathbf{z}^c$, the phase
 655 diagram consists of either four or three stationary phases depending on Ω_d . In this case,
 656 with an increase in the magnitude of the obstruction factor, the LD or LD_M phases expand,
 657 whereas the S phase and the HD phase shrink. Furthermore, the impact of Ω_d on the phase
 658 diagrams is explored, revealing that an escalation in Ω_d diminishes the number of phases
 659 within the system. For $\mathbf{z} \neq \mathbf{z}^c$, there exists a unique critical value Ω_d^c about which the number
 660 of stationary phases changes from three to four. While for $\mathbf{z} = \mathbf{z}^c$, there exist two critical
 661 values of Ω_d^{c1} and Ω_d^{c2} such that about Ω_d^{c1} , the number of phases changes from seven to five
 662 whereas about Ω_d^{c2} , the number of phases changes from five to four. **The variation in the**
 663 **number of stationary phases with respect to the obstruction parameter \mathbf{z} can be understood as**
 664 **follows: with the increase in the obstruction factor, the dynamic defects increasingly hinder**
 665 **the particle flux in the bulk, effectively making the bulk dynamics more rate-limiting. Despite**
 666 **the bulk becoming rate-limiting, the boundary dynamics (α and β) still play a significant role**
 667 **in determining stationary phases. Consequently, the interplay between bulk obstructions and**
 668 **boundary conditions influences the number of distinct stationary phases that the system can**
 669 **possess. This behavior highlights the critical role of the obstruction parameter in dictating**
 670 **the system's overall phase structure.** Further, we examine the impact of the obstruction factor
 671 on the height (and position) of the de-localized shock, which is a monotonically decreasing
 672 (increasing) function of \mathbf{z} . Finally, we conclude that the proposed theoretical work aimed to
 673 simulate dynamic aspects of potential defective cellular and vehicular transport processes and
 674 to provide light on stationary qualities. The proposed study can be utilized to understand the
 675 role of the disorder in the form of defects on the stationary properties of the stochastic transport
 676 systems. Examples of such systems include the biological process of gene transcription, where
 677 DNA binding proteins and the low concentration of tRNA act as a disorder [25], transport
 678 processes along the microtubule where processive molecular motors switch between directed
 679 and diffusive motion [21] etc. The study can be expanded to include additional realistic aspects
 680 relevant to various physical and biological systems.

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684 A Monte Carlo simulations

685 For simulations, we utilize a Monte Carlo algorithm (Gillespie Algorithm) with a random se-
686 quential update rule [46]. A random site is selected and updated at each step according to
687 events like particle hopping attempts, attachment or detachment, and defect binding or un-
688 binding, chosen with rates outlined in Sec. 2. Time increments follow an exponentially dis-
689 tributed random pattern. The simulations are conducted for 10^8 time steps considering the
690 lattice size $L = 500$. The initial 5% of time steps are discarded to establish a steady state, and
691 the average particle density is calculated over an interval of $10L$. Phase boundaries are de-
692 termined with an estimated error of less than 2%, indicated by the marker sizes in the Monte
693 Carlo simulations.

694 B Numerical scheme

695 To derive analytical solutions for the second-order partial differential equation given by Eq.
696 (11a) can be challenging; hence, we present an alternative approach within the mean-field
697 theory. Retaining the time derivative within the system, we obtain density solutions at a steady
698 state in the limit as t tends to ∞ , where t denotes the number of time steps. Employing the
699 forward-in-time and central-in-space (FTCS) scheme, we derive the finite-difference equation
700 as:

$$\rho_j^{i+1} = \rho_j^i + \Delta t' \left((1 - \rho_{d,j}^i (1 - p_d)) \left(\frac{\epsilon}{2} \left(\frac{\rho_{j+1}^i - 2\rho_j^i + \rho_{j-1}^i}{\Delta x^2} \right) + \left(\frac{\rho_{j+1}^i - \rho_{j-1}^i}{2\Delta x} \right) (2\rho_j^i - 1) + \Omega_a (1 - \rho_j^i) \right) - \Omega_d \rho_j^i \right). \quad (\text{B.1})$$

$$\rho_{d,j}^{i+1} = \rho_{d,j}^i + \Delta t' (k^+ (1 - \rho_{d,j}^i) - k^- \rho_{d,j}^i). \quad (\text{B.2})$$

702 The symbols ρ_j^i and $\rho_{d,j}^i$ represent the numerical approximation of particle density and defect
703 density at the point (x_j, t_i) . Here, the spatial variable $\Delta x = 1/L$ and the temporal vari-
704 able $\Delta t'$ adhere to the stability criterion of the finite-difference scheme mentioned above,
705 $\Delta t' / \Delta x^2 \leq 1$. Similarly, Eq. (3) and Eq. (4) are employed to derive the finite-difference
706 equations at the left and right boundaries as:

$$\rho_1^{i+1} = \rho_1^i + \Delta t' \left((1 - \rho_{d,1}^i (1 - p_d)) (\alpha (1 - \rho_1^i) - \rho_1^i (1 - \rho_2^i)) \right), \quad (\text{B.3})$$

707 and

$$\rho_L^{i+1} = \rho_L^i + \Delta t' \left((1 - \rho_{d,L}^i (1 - p_d)) (\rho_{L-1}^i (1 - \rho_L^i) - \beta \rho_L^i) \right). \quad (\text{B.4})$$

708 respectively.

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