Tau lepton averages by the HFLAV group

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Abstract

We document some features of the preliminary draft of the tau lepton averages for the incoming HFLAV report. We include the tau mass and lifetime averages, and a fit of measurements of tau branching fraction measurements. Elaborations of these results provide lepton universality tests and determination of $|V_{us}|$ with tau measurements.

Contents

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1 Introduction

The preliminary draft of the tau section of the HFLAV report includes the averages of the tau lifetime and of the tau mass, and a fit of measurements of the tau branching fractions. The tau lifetime average replicates one reported in the 2022 edition and 2023 update of the Review of Particle Physics [1] (PDG 2023). The average of the tau mass uses 10 τ mass measurements, including the recent Belle II measurement [2], which is not yet included in PDG 2023. The fit has $\chi^2/\text{d.o.f.} = 7.3/9$, corresponding to a confidence level CL = 61%, and reports $m_\tau = 1776.96 \pm 0.09\,\text{MeV}$. We report more details on the tau branching fractions fits and further elaborations to test the Standard Model lepton universality and to compute $|V_{us}|$ in the following sections.

2 Tau branching fractions fit

We perform an equality-constrained fit of tau branching fractions measurements following the procedures mentionned in the 2021 HFLAV report [3]. This edition of the fit differs from the 2021 one because we take into account the uncertainty on an external input, $\mathcal{B}(a_1^- \to \pi^-\gamma)$, which is used in the constraint equations that describe related measurements, according to our assumptions that

$$\begin{aligned}\mathcal{B}(\tau^- \to a_1^- \nu_\tau) \;&=\; \mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\nu_\tau\ (\text{ex.}K^0,\omega)) + \mathcal{B}(\tau^- \to \pi^- 2\pi^0 \nu_\tau\ (\text{ex.}K^0)) \\ &\quad + \mathcal{B}(\tau^- \to a_1^-(\pi^-\gamma)\nu_\tau)\,,\end{aligned}$$

neglecting the observed but negligible branching fractions to other modes, and that

$$\mathcal{B}(\tau^- \to a_1^-(\pi^-\gamma)\nu_\tau) \;=\; \mathcal{B}(\tau^- \to a_1^- \nu_\tau)\cdot\mathcal{B}(a_1^- \to \pi^-\gamma)\,.$$

In the other constraints, uncertainties on external inputs (mostly η, ω and ϕ branching fractions) are smaller in relative size, and negligible when compared with the typical uncertainties on the tau branching fractions. In the fit, an additional floating nuisance parameter describes $\mathcal{B}(a_1^- \to \pi^-\gamma)$, and an associated χ^2 term accounts for its ALEPH estimate of 0.0021 ± 0.0008 [4]. The current procedure is preferable to the past use the ALEPH estimate of $\mathcal{B}(\tau^- \to a_1^-(\pi^-\gamma)\nu_\tau)$, which depended on $\mathcal{B}(a_1^- \to \pi^-\gamma)$ and on the ALEPH measurements of $\mathcal{B}(\tau^- \to \pi^-\pi^+\pi^-\nu_\tau\ (\text{ex.}K^0,\omega))$ and $\mathcal{B}(\tau^- \to \pi^- 2\pi^0\nu_\tau\ (\text{ex.}K^0))$.

The fit uses 170 τ branching fractions and ratios measurements to optimize 137 fit parameters subject to 91 constraints. When considering just the τ branching fractions, omitting the nuisance fit parameters, the fit has $\chi^2/\text{d.o.f.} = 134/124$, corresponding to a confidence level $\text{CL} = 24.7\%$.

3 Tests of lepton universality

Using the tau branching fraction fit results and present world averages for the other quantities [1], we compute the universality tests

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}}\frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^{\mu e}}{\tau_\tau m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^{\tau e}}} = 1.0007 \pm 0.0015\,,$$

$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\mu e}}\frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^{\mu e}}{\tau_\tau m_\tau^5 f_{\tau \mu} R_\gamma^\tau R_W^{\tau \mu}}} = 1.0026 \pm 0.0015\,,$$

$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau e}}\frac{f_{\tau e} R^{\tau e}}{f_{\tau \mu} R^{\tau \mu}}} = 1.0019 \pm 0.0014\,,$$

where, for a higher mass lepton h and a lower mass lepton l,

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \quad f_{hl} = f\left(\frac{m_l^2}{m_h^2}\right),$$

$$R_\gamma^h = 1 + \frac{\alpha(m_h)}{2\pi}\left(\frac{25}{4} - \pi^2\right), \quad R_\gamma^\tau = 1 - 43.2\cdot 10^{-4}\ [5], \quad R_\gamma^\mu = 1 - 42.4\cdot 10^{-4}\ [5]\,,$$

$$R_W^{hl} = 1 + \frac{3}{5}\frac{m_h^2}{M_W^2} + \frac{9}{5}\frac{m_l^2}{M_W^2}\ \ [6\text{–}8]\,.$$

By assuming that $g_e = g_\mu$ we compute a synthetic tau electronic branching fraction $\mathcal{B}'_e$ using the fit results of the tau muonic and electronic branching fractions. The universality of the light leptons and tau couplings implies that:

$$\mathcal{B}'_e = \frac{\tau_\tau m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^{\tau e}}{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^{\mu e}} . \quad (1)$$

$\mathcal{B}'_e$ is a linear function of the tau lifetime, whose slope is known with a precision primarily limited by the uncertainty on the tau mass, which has been improved by the recent Belle II measurement [2]. Figure 1 illustrates the present status of this universality test.

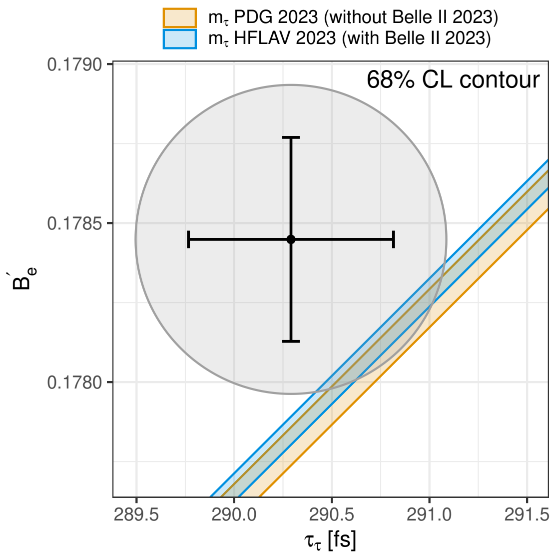

Figure 1: The plot shows the world average of the synthetic tau electronic branching fraction (see Section 3), which represents the electron and muon couplings, the average of the tau lifetime, which is related to the tau coupling, and the prediction of the Standard Model lepton universality, represented by a band whose width is primarily determined by the uncertainty on the tau mass average. The orange band corresponds to the PDG 2023 [1] tau mass average, while the thinner blue band corresponds to the preliminary HFLAV tau mass average, which includes the recent Belle II measurement [2].

3.1 Measurements of $|V_{us}|$

We proceed like in the previous public HFLAV report [3] to compute $|V_{us}|$ using the inclusive τ branching fraction to strange hadronic final states as [9, 10] and using the ratio of branching fractions $\mathcal{B}(\tau^- \to K^-\nu_\tau)$ and $\mathcal{B}(\tau^- \to \pi^-\nu_\tau)$, without giving details because of space limits.

We determine $|V_{us}|$ from the branching fraction $\mathcal{B}(\tau^- \to K^-\nu_\tau)$ using

$$\mathcal{B}(\tau^- \to K^-\nu_\tau) = \frac{G_F^2}{16\pi\hbar} f_{K\pm}^2 |V_{us}|^2 \tau_\tau m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{\text{EW}}^{\tau h} (1 + \delta_{\tau K}) . \quad (2)$$

We use $f_{K\pm} = 155.7 \pm 0.3\,\text{MeV}$ from the 2023 web update of the FLAG 2021 lattice QCD averages with $N_f = 2{+}1{+}1$ [11–16]. The universal short-distance electroweak correction for τ hadronic decays is $S_{\text{EW}}^{\tau h} = S_{\text{EW}}^{R\tau h} \cdot S_{\text{EW}}^{\text{sub,lep}} = 1.01910 \pm 0.00030$, where the radiative correction for

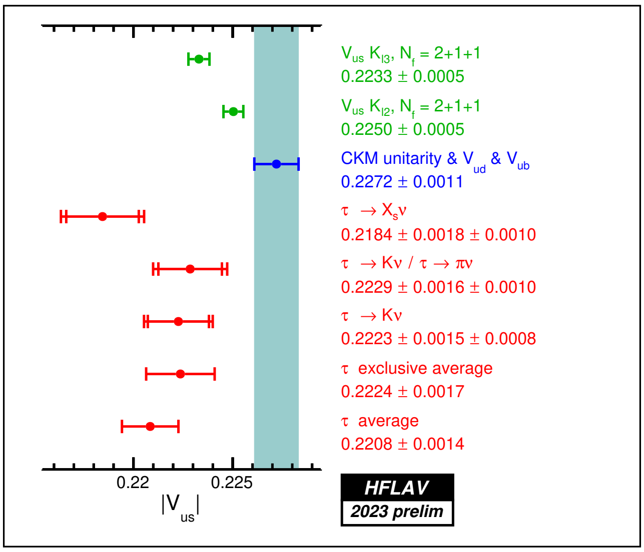

Figure 2: $|V_{us}|$ determinations. Ref. [21] has been used for $|V_{us}|_{K\ell 3}$, $|V_{us}|_{K\ell 2}$, $|V_{ud}|$ (from the average of the nuclear decay and neutron decay determinations), and for the $|V_{us}|$ value implied by CKM unitarity.

the tau spectral functions is $S_{\rm EW}^{R\tau h} = 1.02350 \pm 0.00030$ [6,17,18] and the sub-leading universal short-distance correction for the τ leptonic decays is $S_{EW}^{\rm sub,lep} = 0.9957$ [17]. The long-distance radiative correction for $\mathcal{B}(\tau^- \to K^- \nu_\tau)$ is $\delta_{\tau K} = (-0.15 \pm 0.57)\%$ [19]. The physical constants G_F and $\hbar$ are taken from CODATA 2018 [20]. We obtain $|V_{us}|_{\tau K} = 0.2224 \pm 0.0017$, which is 2.3σ below the CKM unitarity prediction.

Figure 2 reports our $|V_{us}|$ determinations using the τ branching fractions, compared with two determinations based on kaon data [21] and with the value obtained from $|V_{ud}|$ with CKM-matrix unitarity [21].

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