Quantum of action in Entangled Relativity

Olivier Minazzoli[1⋆]

1 Artemis, Université Côte d'Azur, CNRS, Observatoire Côte d'Azur, BP4229, 06304, Nice Cedex 4, France,
2 Bureau des Affaires Spatiales, 2 rue du Gabian, 98000 Monaco.

⋆ ominazzoli@gmail.com.

Abstract

In this article, we demonstrate that the novel general theory of relativity, named 'Entangled Relativity', is more economical than General Relativity in terms of universal dimensionful constants when both theories are considered through a path integral formulation. The sole parameter of Entangled Relativity is a quantum of energy squared. However, in order to recover standard Quantum Field Theory when gravity is neglected in the path integral, we show that this quantum of energy corresponds to the reduced Planck energy. But this result also implies that Planck's quantum of action $\hbar$ and Newton's constant G are not fixed constants in this framework but vary proportionally to a gravitational scalar degree of freedom, akin to typical scalar-tensor and $f(R)$ theories. In particular, it is derived that $\hbar$ is proportional to G in this framework. This establishes an explicit connection between the quantum and gravitational realms. Given the absence of a free parameter in the theory, we argue that this unique prediction can likely be probed observationally in the future. Furthermore, due to the deficit of dimensionful parameters in Entangled Relativity compared to standard physics, fundamental length or time scales cannot be defined within this framework. We argue that this aspect is expected to become significant in the non-perturbative quantum gravity regime of the theory.

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1 introduction

A major challenge in modern elementary physics is to understand quantum gravity. For decades, it has been asserted that General Relativity and Quantum Field Theory are incompatible, suggesting that merging the two frameworks necessarily leads to a meaningless theory [1]. However, as of today, there is absolutely no proof that this is indeed the case. Firstly, at the perturbative level, Quantum General Relativity is perfectly coherent as an Effective Field Theory [1–3], enabling the computation of unambiguous quantum corrections to classical phenomenology within this framework. More importantly, theoretical evidence from different lines of research now suggests that non-perturbative Quantum General Relativity might be renormalizable, despite being perturbatively non-renormalizable. This evidence notably comes from the Asymptotic Safety [4, 5] and the Causal Dynamical Triangulation [6] programs, which employ different theoretical techniques to explore the potential non-perturbative renormalizability of Quantum General Relativity. Remarkable outcomes from both programs include predictions of a particle physics landscape compatible with an asymptotically safe Quantum General Relativity within the Asymptotic Safety framework [7–9], notably the prediction of the Higgs mass before it was observed [10], and the emergence of a 4-dimensional quantum universe (with a positive renormalized cosmological constant) from first principles in the framework of Causal Dynamical Triangulation [11, 12]. Nevertheless, these approaches have their own open questions [6, 13].

In what follows, we do not argue that Quantum General Relativity has an issue per se, because, to date, no one actually knows [14]; instead, we propose another potential path toward quantum gravity, based on a novel general theory of relativity that is more economical than General Relativity, while it possesses both General Relativity and standard Quantum Field Theory as predictable limits of the theory. Moreover, as we will see, this theory precludes the definition of elementary units of time and space. Hence, given the central role of Planck time and length in all Quantum Gravity programs to date [15], we argue that this new direction offers a qualitative departure from all other approaches explored thus far.

Indeed, almost ten years ago, an alternative general theory of relativity was proposed, but it was considered a curiosity due to its unusual non-linear Lagrangian density [16]. It has recently been named 'Entangled Relativity' in [17], not because it is related to 'quantum entanglement' a priori, but because matter and gravity cannot be treated separately within this framework. Indeed, Entangled Relativity is a general theory of relativity that requires the existence of matter to even be defined, thereby realizing Einstein's original idea that a satisfying theory of relativity should not allow for the existence of vacuum solutions [18–23]. Indeed, vacuum solutions imply that inertia—which is defined from the metric tensor in a relativistic theory—could be defined in the total absence of matter, which would *de facto* violate the *principle of relativity of inertia* [18–23] that Einstein named *Mach's principle* in [19]. Despite its very unusual non-linear action—see Eq. (2) below—Entangled Relativity has been shown to possess General Relativity as a limit in fairly generic (classical) situations [16, 17, 24–26], indicating that, at least up to further scrutiny, the theory may be viable from an observational standpoint.

However, it was soon realized that the only parameter of the theory was a quantum parameter, as it does not appear in the field equations [27]. In the present paper, we formulate the theory through its path integral because this approach allows one to explicitly identify this parameter by requiring the theory to be consistent with standard Quantum Field Theory on 'flat spacetime'.[1]

2 Formulation and field equations

The path integral formulation of Entangled Relativity reads as follows

$$Z = \int [\mathcal{D}g] \prod_i [\mathcal{D}f_i] \exp(i\Theta), \tag{1}$$

where the quantum phase is given by

$$\Theta = -\frac{1}{2\epsilon^2} \int d_g^4x \frac{\mathcal{L}_m^2(f,g)}{R(g)}, \tag{2}$$

and where $\int[\mathcal{D}]$ relates to the sum over all possible (non-redundant) field configurations, R is the usual Ricci scalar that is constructed upon the metric tensor g, $d_g^4x := \sqrt{-|g|}d^4x$ is the spacetime volume element, with $|g|$ the metric g's determinant, and $\mathcal{L}_m$ is the Lagrangian density of matter fields f—such as gauge bosons, fermions and the Higgs—which could be the current *standard model of particle physics* Lagrangian density, but most likely a completion of it. It also depends on the metric tensor, a priori through to the usual *comma-goes-to-semicolon rule* [28] in order to recover General Relativity in some limit.[2] Let us note that, like General Relativity, Entangled Relativity does not specify what $\mathcal{L}_m$ should be. Given that the dimension of the term in the integral is an energy squared, the only parameter of the theory is a quantum of energy squared ϵ^2. This means, in particular, that Planck's quantum of action $\hbar$ is not a fundamental constant in this framework, nor is Newton's constant G, since they do not appear in the formulation of the theory.

In order to evaluate the limit at which gravity can be neglected, one first need to understand what gravity is in this framework. We do not have the pretension to evaluate the path integral Eq. (2) in this paper, but we can already take advantage of some lessons about classical gravity that we can learn from the study of the paths with stationary phases $\delta\Theta = 0$. As we will see, this alone enables the evaluation of the quantum of energy squared, ϵ^2. Those paths corresponds to the following field equations [16]

$$G_{\mu\nu} = \kappa T_{\mu\nu} + f_R^{-1}\left[\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right] f_R, \tag{3}$$

with

$$\kappa = -\frac{R}{\mathcal{L}_m}, \qquad f_R = \frac{1}{2\epsilon^2}\frac{\mathcal{L}_m^2}{R^2} = \frac{1}{2\epsilon^2\kappa^2}, \tag{4}$$

with the following stress-energy tensor

$$T_{\mu\nu} := -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}, \tag{5}$$

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[1] For the author, 'flat spacetime' is only a somewhat useful approximation for scales at which gravity can be neglected, but apart from that, it does not exist anywhere in the universe—as evidenced observationally with the acceleration of the expansion of the universe, and theoretically with the quantum vacuum.

[2] Strictly speaking, this condition is only necessary in some limit of the theory, but could perhaps be relaxed in general, as long as it then emerges in the required limit.

which is not classically conserved

$$\nabla_\sigma \left(\frac{\mathcal{L}_m}{R} T^{\alpha\sigma} \right) = \mathcal{L}_m \nabla^\alpha \left(\frac{\mathcal{L}_m}{R} \right). \tag{6}$$

The matter field equation, for any tensorial matter field χ, gets modified due to the non-linear coupling between matter and curvature as follows

$$\frac{\partial \mathcal{L}_m}{\partial \chi} - \frac{1}{\sqrt{-|g|}} \partial_\sigma \left(\frac{\partial \sqrt{-|g|} \mathcal{L}_m}{\partial \left(\partial_\sigma \chi \right)} \right) = \frac{\partial \mathcal{L}_m}{\partial \left(\partial_\sigma \chi \right)} \frac{R}{\mathcal{L}_m} \partial_\sigma \left(\frac{\mathcal{L}_m}{R} \right). \tag{7}$$

3 Decoupling

It has already been demonstrated that these equations lead to classical phenomenology very similar to, or even indistinguishable from, that of General Relativity in many cases [16, 17, 24–26, 29, 30]. This similarity primarily results from the *intrinsic decoupling* originally identified in scalar-tensor theories [31]. Specifically, as is common in $f(R)$ theories, the trace of the metric field equation produces the differential equation for the extra scalar degree of freedom, f_R, which is given by:

$$3 f_R^{-1} \Box f_R = \kappa \left(T - \mathcal{L}_m \right). \tag{8}$$

Therefore, whenever $\mathcal{L}_m = T$ on-shell, the extra degree of freedom ($f_R \propto \kappa^{-2}$) is not sourced and becomes constant in many cases, allowing one to recover General Relativity, minimally coupled to matter and without a cosmological constant, with very good accuracy [16, 17, 24–26]. It is worth noting that $\mathcal{L}_m = T$ is a valid assumption for a universe composed almost entirely of dust and electromagnetic radiation, which closely approximates the current content of our universe.

Let us stress that the whole field equations are well-behaved at the limits $R \to 0$ and $\mathcal{L}_m \to 0$, even though it may not be apparent at first glance. Indeed, the behavior of the ratio between R and $\mathcal{L}_m$ is dictated by the entire field equations, and in particular by Eq. (8), just as the ratio between R and T is constrained by the trace of Einstein's equation in General Relativity. This is exemplified in the spherically charged black-hole solution found in [25], which is such that $(\mathcal{L}_m, R) \propto Q^2$, where Q is the charge of the black hole. As a consequence, the ratio between R and $\mathcal{L}_m$, or κ, turns out to tend to a constant in the $\mathcal{L}_m \to 0$ limit, which also corresponds to the $R \to 0$ limit. Let us emphasize that when the ratio between R and $\mathcal{L}_m$ becomes constant, we exactly recover General Relativity, minimally coupled to matter fields. Thus, General Relativity emerges as a limit of Entangled Relativity in the regime of weak matter field density. This is also exemplified by the solutions for a spherically neutral black hole immersed in a uniform electric or magnetic background, as found in [29]. These solutions reduce to the Schwarzschild black hole of General Relativity when the background electric or magnetic field vanishes.

Interrestingly, the whole set of Eqs. (3-8) can be recovered by this alternative Einstein-dilaton phase instead [16]

$$\Theta_{Ed} = \frac{1}{\epsilon^2} \int d^4_g x \frac{1}{\kappa} \left(\frac{R(g)}{2\kappa} + \mathcal{L}_m(f, g) \right), \tag{9}$$

provided that $\mathcal{L}_m \neq \emptyset$, and where κ is a dimensionful scalar-field, whose on-shell value matches the definition in Eq. (4). This is similar to the usual equivalence between $f(R)$ and Scalar-Tensor theories [32]. Eq. (9) corresponds to a special case of the theories studied in [31, 33], which are such that κ is indeed a weakly sourced gravitational field due to the *intrinsic decoupling* mentioned above. As a consequence, κ varies even less than the spacetime metric $g_{\mu\nu}$. In the Solar System, for instance, the metric's perturbation is of order $\mathcal{O}(c^{-2})$, whereas κ's perturbation is of order $\mathcal{O}(c^{-4})$, as shown in [31]. The scalar field's perturbation remains smaller than the metric's perturbation, even for neutron stars [17], which are the densest objects in the universe that are not hidden behind an event horizon.

4 Standard particle physics

As a consequence, for any quantum phenomenon where gravity can be neglected, the path integral in Eqs. (1-2) can be approximated by

$$Z \approx \int \prod_i [\mathcal{D} f_i] \exp\left[\frac{i}{\kappa \epsilon^2} \int d^4x \mathcal{L}_m(f)\right]. \tag{10}$$

Therefore, to recover the standard Quantum Field Theory in scenarios where gravity is negligible, one must ensure that in the limit corresponding to Eq. (10), one has

$$\kappa \epsilon^2 = \hbar c. \tag{11}$$

This allows one to identify the only free parameter of the theory in Eq. (2), ϵ^2, as the squared reduced Planck energy. This is akin to determining the value of the coupling constant κ in General Relativity, where κ in General Relativity must be chosen so that General Relativity reproduces Newtonian physics in the Newtonian limit.

5 Discussion

In Entangled Relativity, the value of κ is determined by its cosmic evolution and by its specific value when it began to stabilize at the onset of the matter era. For instance, assuming a Friedmann-Lemaître-Robertson-Walker metric with a universe filled with dust and electromagnetic radiation, Eq. (8) simplifies to $\ddot{f}_R + 3H\dot{f}_R = 0$, with $\kappa^2 \propto f_R^{-1}$ from Eq. (4), and where H is the Hubble parameter, leading to f_R (hence κ) quickly stabilizing ($\dot{f}_R \propto \exp[-3\int H dt]$) close to the value it held during a previous cosmic era.

Eq. (11) suggests that the same applies to the value of $\hbar$. Given that $\hbar$ does not appear in Eq. (2), it should have been apparent from the outset that $\hbar$ could not be a fundamental constant in Entangled Relativity. Eq. (11) indicates that $\hbar$ is an emergent constant, whose constancy is only relevant in the limit where gravity can be entirely neglected. It is important to emphasize that this is not in contradiction with standard physics, as standard Quantum Field Theory, particularly the Standard Model of particle physics, entirely omits gravity from its framework. In fact, in Entangled Relativity, the concept of a *quantum of action* is only pertinent in the semi-classical limit of the theory, where gravity can be treated as a classical background field. At the non-perturbative quantum gravity level, the notion of a *quantum of action* does not exist in Entangled Relativity.[3]

This brings us to another significant aspect of Entangled Relativity: the theory lacks sufficient dimensionful universal constants to define elementary units of time and space. Indeed, the only two dimensionful constants present are the energy squared, ϵ^2, and the causal structure constant, c. Considering the pivotal role of the Planck time and length in all existing approaches to quantum gravity [15], this suggests that Quantum Entangled Relativity

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[3] See Appendix A for a discussion on massive matter fields.

could exhibit qualitatively distinct behavior from all other approaches in the non-perturbative regime.

Let us indeed note that in Eq. (4), one finds $\epsilon^2\kappa^2 = \kappa\hbar c = \ell_P^2$, where ℓ_P represents the reduced Planck length. It is quite intriguing that the new gravitational scalar degree of freedom in Entangled Relativity, which arises from the non-linearity of the Lagrangian density, is proportional to the inverse of the squared Planck length, $f_R \propto \ell_P^{-2}$. This elucidates the fact that in Entangled Relativity, the Planck length (ℓ_P) and time (ℓ_P/c) are not constants. The only constant is the reduced Planck energy squared, ϵ^2.

Another important lesson from Eq. (11) is that in Entangled Relativity, the weak gravity limit, $\kappa \rightarrow 0$, effectively corresponds to the classical limit, $\hbar \rightarrow 0$. This demonstrates an explicit connection between the quantum and gravitational realms within Entangled Relativity, offering a coherent and simplified perspective on elementary physics. Indeed, Eq. (2) is simply a non-linear, more economical reformulation of General Relativity.

However, Eq. (11) reveals something more profound about quantum mechanics and Quantum Field Theory: the procedure of *canonical quantization* should be valid only when gravity can be neglected. Indeed, *canonical quantization* depends on the existence of a constant quantum of action to elevate classical variables (c-numbers) to operators (q-numbers) through Dirac's procedure [15, 34–37]. Consequently, since a quantum of action is not a fundamental constant in Entangled Relativity, there's no basis to expect that *canonical quantization* will yield accurate results within this framework when gravity cannot be ignored. Actually, Heisenberg's uncertainty principle is also a priori only valid at the limit of the theory where κ is constant. But the fact that canonical quantization does not necessarily depict the mathematics underlying nature at a fundamental level is not inconsistent a priori. Indeed, it is possible that the path integral approach in Eq. (1) is the only viable method when dealing with gravity, and that the two approaches are equivalent when gravity is neglected only. Besides, this observation does not challenge established physics, as Quantum Field Theory has been verified experimentally only in conditions where κ's variation is negligible. Nevertheless, exploring how *canonical quantization* could be adapted to account for a variable quantum of action, $\hbar$, presents an intriguing avenue for future research.

6 Numerical evaluation

Eq. (11) enables the derivation of the expected numerical amplitude for variations of $\hbar$ in various contexts. Employing the post-Newtonian analysis from [31], it can be determined that within the Solar System, for example, the anticipated relative numerical variation of $\hbar$ between the surface of the Sun and Earth is

$$\frac{\Delta\hbar}{\hbar} = \frac{GM_\odot^P}{c^2}\left(\frac{1}{r_\odot} - \frac{1}{r_\oplus}\right) \approx \frac{GM_\odot^P}{r_\odot c^2} \sim 2.4 \times 10^{-12}, \tag{12}$$

where $r_\odot$ and $r_\oplus$ are the position of the surface of the Sun and of the Earth respectively, in heliocentric coordinates, and where a new type of mass term for a given body A, produced solely by pressure, has been defined as follows:

$$M_A^P := \int_A \frac{P(r)}{c^2} d^3r. \tag{13}$$

The numerical evaluations that led to Eq. (12) can be found at https://github.com/ominazzoli/hbar-in-SS, and rely on the model S [38] for the Sun's pressure. Let us emphasize that Eq. (12) is independent of any free theoretical parameters, making it a potential tool for empirically testing Entangled Relativity, despite the extremely small range of the variations involved.

The largest variation of $\hbar$ in the observable universe is expected between the surface of a neutron star and a distant observer. Using Eq. (11), numerical simulations from [17,24] estimate this variation to be at the level of a few percent for the densest neutron stars conceivable. Although these simulations did not consider the impact of the variation in $\hbar$ on the neutron star's equation of state, the relatively minor extent of this variation suggests that this approximation was indeed a reasonable starting point, unlikely to significantly affect the estimations in [17,24].

7 Conclusion

Entangled Relativity predicts that the quantum of action $\hbar$ is not a fundamental constant of nature but emerges as a constant only in the limit where gravity can be entirely neglected. The potential variation of $\hbar$ is relevant not only to the community interested in gravity but also to a broader range of physicists, as it may impact other aspects of quantum physics, such as quantum entanglement between remote particles in different gravitational fields, or possibly even decoherence. Nevertheless, given the minuscule level of variation of $\hbar$ in the solar system evaluated in Sec. 6, the predicted variation of $\hbar$ does not impact much how quantum mechanics and quantum field theory describe quantum phenomena at the experimental level on Earth. However, it has also been argued in Sec. 6 that the variation of $\hbar$ could reach the percent level for the most compact objects in the universe, thereby also providing a potential way to check this prediction. Should the variation of $\hbar$ be quantitatively confirmed at the observational or experimental level, it would likely imply that Entangled Relativity is better than General Relativity in order to describe the relativistic laws of physics in general. This would stem not only from a theory that is more economical than General Relativity in terms of fundamental constants but also from a theory that better aligns with the whole set of principles Einstein initially proposed to construct General Relativity—see Sec. 1—while reducing to General Relativity in many instances to an extremely good level of accuracy.

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A Fields with finite range

It might be argued that $\hbar$ explicitly appears in the matter Lagrangian $\mathcal{L}_m$ of massive fields in the standard model of particle physics. However, fundamentally, what one calls 'massive fields' are just 'fields with finite range', specified by their (reduced) Compton wavelength λ_C. This is because any spacetime derivative in the kinetic term of massive fields in the matter Lagrangian has to be compensated by a constant with the dimension of length^{-1} in the potential term—each $\partial/\partial x^\alpha$ in the kinetic term has to be compensated by λ_C^{-1} in the potential term. The reason why $\hbar$ appears in the Lagrangian of standard physics is precisely because it is assumed from the outset that $\hbar$ is constant, allowing one to convert the fundamental Compton wavelength into a mass scale as $\lambda_C^{-1} = mc/\hbar$. But if $\hbar$ is not a fundamental constant, then one is no longer allowed to do so, and everything has to be kept consistent in terms of dimensions. As a consequence, only the Compton wavelength λ_C appears in the definition of fields with finite range when $\hbar$ is not assumed to be constant. For instance, the quantum phase of a Dirac field with finite range simply reads

$$\Theta_{Dirac} = \int d^4x \bar{\psi}(i\not{D} - \lambda_C^{-1})\psi, \tag{A.1}$$

in both standard physics and Entangled Relativity when gravity can entirely be neglected—see Sec. 4. Obviously, $\hbar$ plays no role in the definition of a Dirac field with finite range. Similarly, any field with finite range—such as the Higgs field—must involve in its formulation the Compton wavelength that characterizes its finite range. This is imposed by purely dimensional considerations.

However, the Langrangian of matter fields appearing in Eq. (2) must have the dimension of an energy density. Given that c, ϵ and λ_C are the only available dimensionful constants for a Dirac field with finite range in Entangled Relativity, its Lagrangian must be:

$$\mathcal{L}_{Dirac} = \epsilon\lambda_C \; \bar{\Psi}(i\not{D} - \lambda_C^{-1})\Psi. \tag{A.2}$$

Using Eq. (9), the resulting quantum phase reads

$$\Theta_{Dirac} = \int d^4x \frac{\lambda_C}{\epsilon\kappa} \bar{\Psi}(i\not{D} - \lambda_C^{-1})\Psi, \tag{A.3}$$

which, when gravity can entirely be neglected, can be identified with Eq. (A.1) with the following field redefinition $\psi = \sqrt{\lambda_c/(\epsilon\kappa)}\Psi$.

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