

# QGSJET-III: predictions for extensive air shower characteristics and the corresponding uncertainties

Sergey Ostapchenko\*

Universität Hamburg, II Institut für Theoretische Physik, 22761 Hamburg, Germany

\* [sergey.ostapchenko@desy.de](mailto:sergey.ostapchenko@desy.de)



22nd International Symposium on Very High Energy Cosmic Ray Interactions (ISVHECRI 2024)  
Puerto Vallarta, Mexico, 8-12 July 2024  
doi:[10.21468/SciPostPhysProc.?](https://doi.org/10.21468/SciPostPhysProc.)

## Abstract

The physics content of the QGSJET-III Monte Carlo model of high energy hadronic interactions is briefly described. The predictions of the model for extensive air shower characteristics are presented in comparison to the corresponding results of other Monte Carlo generators of cosmic ray interactions. The results of a recent quantitative analysis of uncertainties for such predictions are discussed, notably, regarding possibilities to enhance the muon content of extensive air showers or to delay the air shower development.

Copyright attribution to authors.

This work is a submission to SciPost Phys. Proc.

License information to appear upon publication.

Publication information to appear upon publication.

Received Date

Accepted Date

Published Date

## Contents

<b>1 Introduction</b>	<b>2</b>
<b>2 QGSJET-III model</b>	<b>2</b>
<b>3 Uncertainties for the predicted EAS muon content</b>	<b>3</b>
<b>4 Uncertainties for the predicted EAS maximum depth</b>	<b>5</b>
<b>5 Conclusion</b>	<b>8</b>
<b>References</b>	<b>8</b>

## 1 Introduction

Experimental studies of ultra-high energy cosmic rays (UHECRs) are traditionally performed using indirect methods: based on measurements of the so-called extensive air showers (EAS) – nuclear-electromagnetic cascades initiated by interactions of UHECRs in the atmosphere [1]. Therefore, an analysis and interpretation of the corresponding experimental data requires an accurate description of EAS development, notably, regarding the cascade of nuclear interactions of both primary cosmic ray (CR) particles and of secondary hadrons produced. Here comes the importance of Monte Carlo (MC) generators of hadronic interactions, employed in EAS simulation procedures [2]. In turn, since such MC generators are largely phenomenological and involve a considerable extrapolation of the underlying physics into scarcely studied kinematic regimes, of considerable importance is to estimate the range of uncertainty regarding EAS predictions of such models [3, 4].

In the current contribution, the results of recent quantitative investigations of model uncertainties for EAS predictions [5, 6], in the framework of the QGSJET-III model [7, 8], are discussed. The corresponding studies were guided by three basic principles: i) the changes of the corresponding modeling were performed at a microscopic level; ii) the considered modifications were restricted by the requirement not to contradict basic physics principles; iii) the consequences of such changes, regarding a potential (dis)agreement with relevant accelerator data, were analyzed.

## 2 QGSJET-III model

The major development in QGSJET-III, compared to the previous model version, QGSJET-II-04 [9, 10], concerns the treatment of nonlinear corrections to perturbative hard scattering processes [7]. The corresponding standard approach in all present MC generators of hadronic collisions is based on the leading twist collinear factorization of perturbative quantum chromodynamics (pQCD) [11]. In that case, the inclusive parton jet production cross section is defined by a convolution of two parton momentum distribution functions (PDFs) of interacting hadrons (nuclei) with the Born parton scatter cross section, thus corresponding to a binary parton-parton scattering. However, since such a cross section explodes in the limit of small jet transverse momentum  $p_t$ , in MC models one is forced to introduce a low  $p_t$  cutoff for jet production and all the model predictions depend strongly on the choice of that cutoff.

In QGSJET-III, one considered a phenomenological implementation of a certain class of higher twist corrections to hard parton-parton scattering, namely, those which correspond to multiple coherent rescattering of final  $s$ -channel partons on correlated “soft” gluon pairs, characterized by very small light cone (LC) momentum fractions  $x$  [12, 13]. In such a case, the hardest scattering process is no longer a binary parton-parton scattering but generally involves an arbitrary number of soft gluons. As demonstrated in [14], such a development reduces considerably the dependence of model predictions on the low  $p_t$  cutoff.

An additional technical improvement in the QGSJET-III model concerned a more consistent treatment of the pion exchange process in hadronic collisions [8, 15], including a cross check of the approach, based on the data of the LHCf experiment on forward neutron production in proton-proton interactions [16, 17]. As demonstrated earlier in [10], assuming a dominance of the  $t$ -channel pion exchange in pion-nucleus collisions, over contributions of heavier Reggeon states (e.g., of  $\rho$ -mesons), gives rise to a substantial ( $\simeq 20\%$ ) enhancement of EAS muon content  $N_{\mu}$ , due to a significant increase of forward  $\rho$ -meson production. This is somewhat nontrivial, being a direct consequence of the isospin symmetry.<sup>1</sup> Enhancing central production

<sup>1</sup>While the isospin symmetry is not an exact one for strong interactions, it holds to a very good accuracy thanks

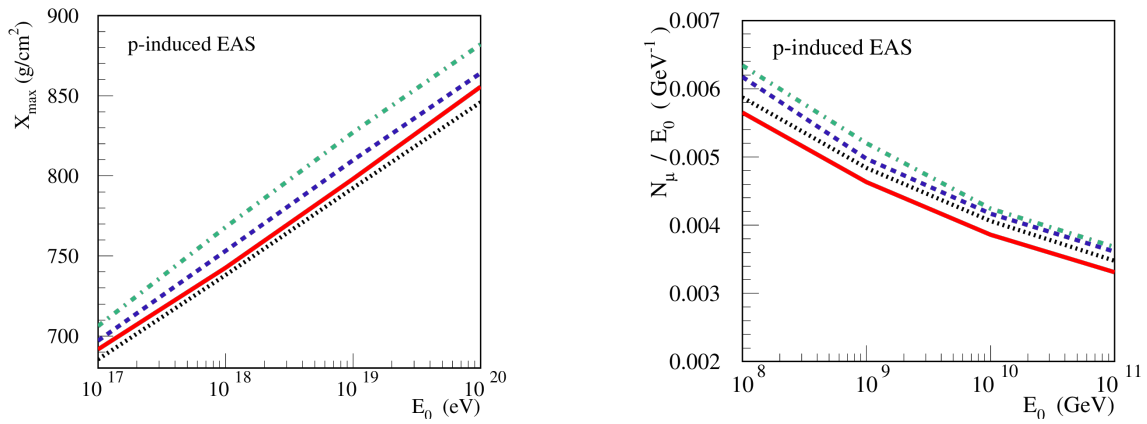


Figure 1: Dependence on primary energy of the shower maximum depth  $X_{\max}$  (left) and of the muon number  $N_{\mu}$  at sea level (right), for proton-initiated EAS, calculated using the QGSJET-III, QGSJET-II-04, EPOS-LHC, and SIBYLL-2.3 models – solid, dotted, dashed, and dash-dotted lines, respectively.

of  $\rho$ -mesons, at an expense of pions, would not have an appreciable impact on the predicted  $N_{\mu}$  (e.g., [5]): since  $\rho^{+}$ ,  $\rho^{-}$ , and  $\rho^{0}$  are created in proportion 1:1:1 and, upon decays of those mesons ( $\rho^{\pm} \rightarrow \pi^{\pm}\pi^{0}$ ,  $\rho^{0} \rightarrow \pi^{+}\pi^{-}$ ), the energy partition between the resulting charged and neutral pions is the same as for direct pion production ( $E_{\pi^{\pm}}:E_{\pi^{0}}=2:1$ ). On the contrary, for  $\rho$ -mesons resulting from the pion exchange process, this energy partition becomes 3:1 ( $\pi^{\pm} \xrightarrow{\pi^{0}} \rho^{\pm} \rightarrow \pi^{\pm}\pi^{0}$ ,  $\pi^{\pm} \xrightarrow{\pi^{\pm}} \rho^{0} \rightarrow \pi^{+}\pi^{-}$ ). Consequently, a higher pion exchange rate leads to a larger fraction of the primary particle energy, retained in the nuclear cascade at a given depth, instead of going into the electromagnetic “sink” via the  $\pi^{0} \rightarrow \gamma\gamma$  decay, thereby giving rise to a higher  $N_{\mu}$ .

Regarding the model predictions for basic EAS characteristics, those appeared to be rather similar to the ones of the previous model version, QGSJET-II-04: the difference for the predicted extensive air shower maximum depth  $X_{\max}$  being  $\leq 10$  g/cm<sup>2</sup>, while the one for EAS muon number  $N_{\mu}$  ( $E_{\mu} > 1$  GeV) amounting to 5% only. This is illustrated in Fig. 1, where the corresponding results of the two models are compared to each other and to predictions of two other CR interaction models, EPOS-LHC [18] and SIBYLL-2.3d [19]. Such a robustness of the calculated EAS characteristics may suggest that the relevant features of interaction models are sufficiently constrained by accelerator data.<sup>2</sup>

### 3 Uncertainties for the predicted EAS muon content

One of the traditional methods for high energy CR composition studies is based on measurements of ground lateral density of muons in extensive air showers [1, 21]. However, the use of this method for UHECRs is hampered presently by a persisting contradiction between the corresponding predictions of EAS simulations and the experimental data, the latter indicating a substantially higher EAS muon content [22, 23].

Generally, the predicted  $N_{\mu}$  is correlated with the multiplicity of hadron-air collisions (e.g., [3]), which can be understood using the simple Heitler’s qualitative picture for the cascade process [24, 25]. Yet the relation between the multiplicity and the shower muon size

to the small mass difference between the  $u$  and  $d$  quarks.

<sup>2</sup>Potential explanations for the somewhat different results of the EPOS-LHC and SIBYLL-2.3d models have been discussed in [20].

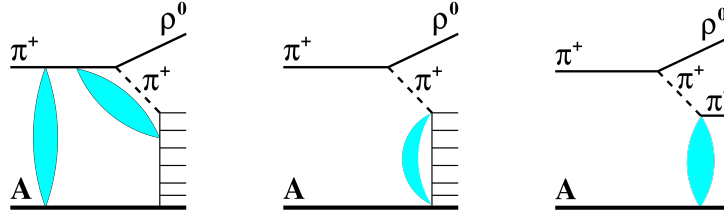


Figure 2: Left: schematic view of the pion exchange process in pion-nucleus interaction; shown by light-shaded ellipses are absorptive corrections due to additional rescatterings of the incident pion. Neglecting such corrections, one has to account both for the inelastic (middle) and elastic (right) interaction of the virtual pion.

is not straightforward since more energetic secondary hadrons capable of producing powerful enough subcascades give larger contributions to  $N_\mu$ , compared to much more copiously produced low energy hadrons. As demonstrated, e.g., in [5], the competition between an abundant production of low energy hadrons and larger muon yields from high energy secondaries leads to  $N_\mu$  being approximately proportional to the quantity  $\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi\text{-air}} \rangle$ , defined as

$$\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi\text{-air}}(E_0) \rangle = \int dx_E x_E^{\alpha_\mu} \frac{dn_{\text{stable}}^{\pi\text{-air}}(E_0, x_E)}{dx_E}, \quad (1)$$

where  $dn_{\text{stable}}^{\pi\text{-air}}/dx_E$  is the distribution, with respect to the energy fraction  $x_E$ , of “stable” secondary hadrons in pion-air collisions, i.e., those which have significant chances to interact in the atmosphere, instead of decaying. In turn,  $\alpha_\mu \simeq 0.9$  is the characteristic exponent for the dependence of  $N_\mu$  on the primary energy, for proton-induced EAS:  $N_\mu^p(E_0) \propto E_0^{\alpha_\mu}$ . Since  $\alpha_\mu$  is not too different from unity, the quantity  $\langle x_E^{\alpha_\mu} n_{\text{stable}}^{\pi\text{-air}} \rangle$  can be approximated by the average fraction of the parent pion energy  $\langle x_E n_{\text{stable}}^{\pi\text{-air}} \rangle$  (the case  $\alpha_\mu = 1$ ), taken by all stable secondary hadrons. Therefore, to predict a higher EAS muon content, a higher energy fraction taken by all stable secondaries in pion-air interactions is required.

Since experimental data on pion-proton and pion-nucleus collisions are available at fixed target energies only, one may try to enlarge the predicted  $N_\mu$  by enhancing the energy-rise of secondary hadron yields. The corresponding energy dependence is driven by (mini)jet production, which is, in turn, governed by gluon PDFs. Hence, the simplest way to obtain the desirable enhancement is to change the LC momentum partition between valence quarks and gluons (plus sea quarks) in the pion, in favor of the latter. It is worth remarking, however, that valence quark PDFs of the pion are seriously constrained by experimental studies of the Drell-Yan process in pion-proton scattering, while similar constraints on gluon PDFs come from measurements of direct photon and  $J/\psi$  production. Neglecting for the moment those constraints, one may try an extreme scenario: reducing the pion LC momentum fraction carried by valence quarks by factor two and enhancing correspondingly the gluon content of the pion. Yet such an extreme modification allows one to enlarge  $N_\mu$  by less than 1% [5]. This is because a noticeable enhancement of secondary hadron yields is obtained this way only in central rapidity region, i.e., for small  $x_E$ , and for sufficiently high pion energies corresponding to the top part of the nuclear cascade in the atmosphere.

Further, in view of the strong impact of the pion exchange process on the predicted  $N_\mu$ , one may try to change the energy dependence of that process. This dependence is governed in QGSJET-III by the so-called absorptive corrections, i.e., by the probability not to have additional inelastic rescatterings in pion-air interactions, see Fig. 2 (left). Indeed, such additional rescatterings would “suck out” energy from the pion, thereby preventing a production of  $\rho$ -mesons with large  $x_E$ . Because of the general energy-rise of multiple scattering, the absorptive corrections “push” the pion exchange process towards larger and larger impact parameters,

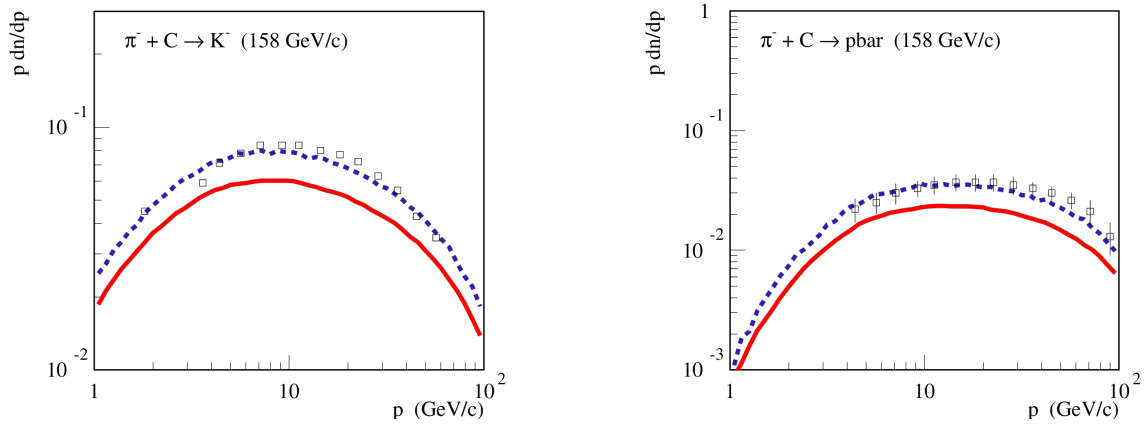


Figure 3: Momentum distributions in laboratory frame of  $K^-$  (left) and of  $\bar{p}$  (right) produced in  $\pi^-C$  collisions at 158 GeV/c, calculated using the default QGSJET-III model (solid lines) or considering 40% and 60% enhancement of kaon and (anti)nucleon yields, respectively, compared to NA61 data [27] (points).

with increasing energy, giving rise to a slow decrease of the pion exchange rate [20]. Yet here one has some space for a model dependence, e.g., if additional inelastic rescatterings take only small energy fractions, like in the SIBYLL model (see the corresponding discussion in [26]). Therefore, one may try an extreme modification: neglecting such absorptive corrections completely and having thus an energy-independent probability for the pion exchange process. Paradoxically, such a change would result in a decrease of the predicted  $N_\mu$  (by up to 10% at  $E_0 = 10^{19}$  eV) [5]. In the absence of absorptive corrections, in addition to the inelastic interaction of the virtual pion with the target nucleus, shown in Fig. 2 (middle), one has to take into account the contribution of pion elastic scattering of Fig. 2 (right). It is the scarce hadron production in the latter case which causes the decrease of  $N_\mu$  [5].

Thus, the only viable option for enlarging significantly the predicted EAS muon content is to enhance relative yields of secondary kaons and (anti)nucleons in pion-air collisions, at the expense of pions – since this would decrease the energy leak into neutral pions. Comparing in Fig. 3 the corresponding results of the QGSJET-III model with the data of the NA61 experiment, we see that a significant enhancement of the predicted yields is required to match the data:  $\simeq 40\%$  for kaons and  $\simeq 60\%$  for (anti)nucleons.<sup>3</sup> Applying such changes allows one to enhance the predicted  $N_\mu$  by up to 10%, see Fig. 4 [5].

## 4 Uncertainties for the predicted EAS maximum depth

Let us now turn to the EAS maximum depth  $X_{\max}$  which is the main air shower characteristic used for UHECR composition studies [21]. Unlike the EAS muon content which depends on the whole history of the nuclear cascade in the atmosphere,  $X_{\max}$  is largely governed by interactions of primary CR particles. Consequently, the corresponding model results are seriously constrained by experimental data of the Large Hadron Collider (LHC). Nonetheless, there exist some tension between the predictions of EAS simulations for  $X_{\max}$  and the data of the Pierre Auger Observatory: as demonstrated in [28], to reach a consistency with the measurements, a significantly slower air shower development is required.

What are the possibilities to have a larger  $X_{\max}$  predicted? First of all, a smaller  $pp$  inelastic

<sup>3</sup>However, as discussed in [5], such modifications would lead to a serious tension with results of other experiments on kaon and (anti)proton production in  $pp$  and  $\pi p$  collisions.

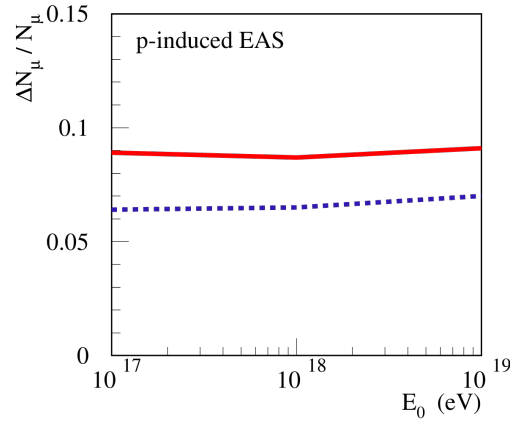


Figure 4: Dependence on primary energy of the relative change of the muon number  $N_\mu$  at sea level ( $E_\mu > 1$  GeV), for proton-initiated air shower, for 60% enhancement of (anti)nucleon production (solid line) and for 40% enhancement of kaon production (dashed line).

cross section,  $\sigma_{pp}^{\text{inel}}$ , would correspond to a smaller cross section for proton collisions with air,  $\sigma_{p\text{-air}}^{\text{inel}}$ , in the Glauber-Gribov formalism [29,30]. In turn, this would enlarge the proton mean free path in air,  $\lambda_p \propto 1/\sigma_{p\text{-air}}^{\text{inel}}$ , thereby shifting the whole air shower profile towards larger depths. A similar effect can be obtained by increasing the rate of diffractive interactions,  $\sigma_{p\text{-air}}^{\text{diffr}}/\sigma_{p\text{-air}}^{\text{inel}}$ , with  $\sigma_{p\text{-air}}^{\text{diffr}}$  being the cross section for inelastic diffraction on air. Indeed, since in diffractive collisions the proton loses typically a small portion of its energy, the effect of diffraction on EAS development is more or less equivalent to a redefinition of the proton mean free path:  $\lambda_p \rightarrow \lambda_p(1 + \sigma_{p\text{-air}}^{\text{diffr}}/\sigma_{p\text{-air}}^{\text{inel}})$ .

In the QGSJET-III model, one can study the combined effect of both, a smaller  $\sigma_{p\text{-air}}^{\text{inel}}$  and a larger  $\sigma_{p\text{-air}}^{\text{diffr}}$ , by increasing the cross section for low mass diffraction in  $pp$  collisions [6]. Enhancing the low mass ( $\leq 3.4$  GeV) in  $pp$  by  $\simeq 30\%$  and being still compatible with the corresponding results of the TOTEM experiment [31], one obtains  $\simeq 15\%$  higher rate of diffractive-like proton-air interactions characterized by a small ( $< 10\%$ ) energy loss of leading nucleons. On the other hand, since a higher diffraction is bound to a stronger inelastic screening effect [30], this leads to smaller total, inelastic, and elastic proton-proton cross sections, as illustrated in Fig. 5 (left), all becoming compatible with the data of the ATLAS experiment, shown by the open stars in the Figure. Yet the corresponding reduction of  $\sigma_{p\text{-air}}^{\text{inel}}$  is  $\leq 1\%$ , see Fig. 5 (right), since a proton-nucleus cross section is largely dominated by the nuclear size. Therefore, the effect of the considered changes on  $X_{\text{max}}$  is largely caused by the enhanced diffraction rate in proton-air interactions. The obtained shift of the average EAS maximum depth is limited by  $\simeq 8$  g/cm<sup>2</sup> [6], in a good agreement with earlier studies [35].

Another possibility to obtain a larger  $X_{\text{max}}$  predicted is to slow down the energy rise of the inelasticity  $K_{p\text{-air}}^{\text{inel}}$  of proton-air interactions: since this would enlarge somewhat the average number of hadron “generations” in the nuclear cascade, thereby elongating the air shower profile. The energy rise of the inelasticity is a generic feature: since the rate of multiple scattering in hadronic collisions increases with energy. However, the speed of predicted energy rise of  $K_{p\text{-air}}^{\text{inel}}$  may vary from model to model, depending, e.g., on how much energy is taken by a single inelastic rescattering process [26]. Since the energy dependence of multiple scattering rate is driven by a fast increase of (mini)jet production, one may try to tame that rise by considering stronger higher twist effects in the QGSJET-III model, which would further suppress the emission of minijets of relatively small transverse momenta [6]. On the other hand, the increase of the inelasticity would be reduced if additional inelastic scatterings took only

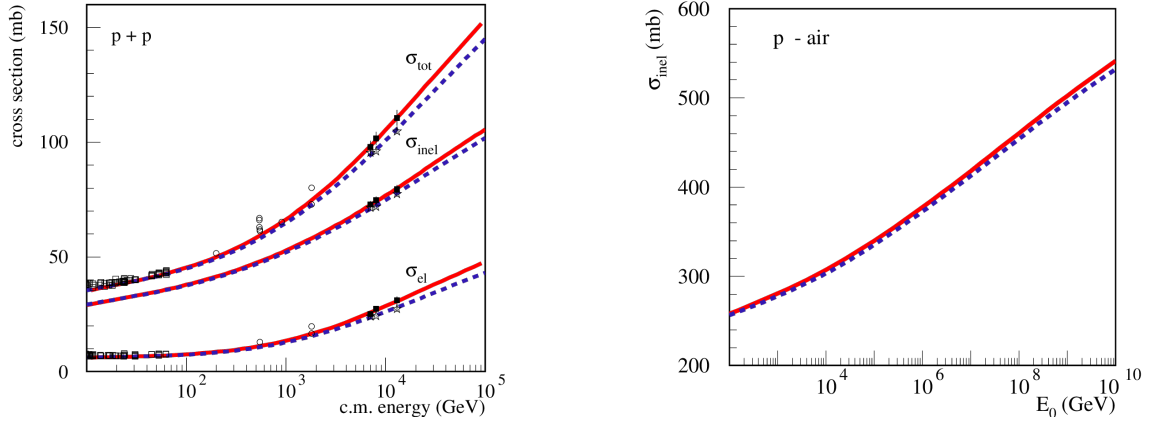


Figure 5: Center of mass (c.m.) energy dependence of the total, inelastic, and elastic  $pp$  cross sections, compared to experimental data [32–34] (left) and laboratory energy dependence of  $\sigma_{p\text{-air}}^{\text{inel}}$  (right), calculated with the default QGSJET-III model (solid lines) and considering a 30% enhancement of low mass diffraction (dashed lines).

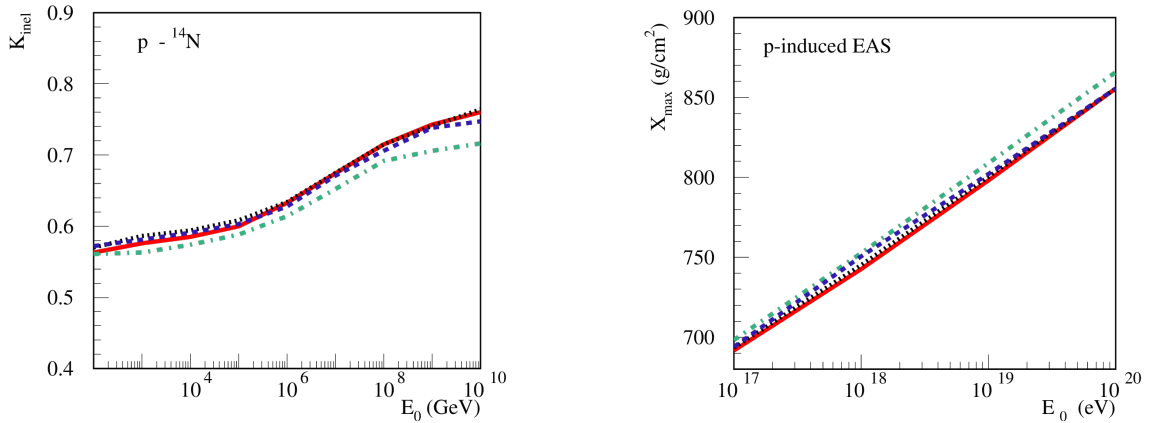


Figure 6: Laboratory energy dependence of  $K_{pN}^{\text{inel}}$  (left) and primary energy dependence of  $X_{\text{max}}$  for  $p$ -induced EAS (right), for the default QGSJET-III model (solid line) and for the model modifications discussed in the text; dotted, dashed, and dash-dotted lines correspond to  $\alpha_{\text{sea}} = 0.65, 0.8, \text{ and } 0.9$ , respectively.

small portions of energy of the incident proton. This can be achieved by choosing a softer distribution,  $\propto x^{-\alpha_{\text{sea}}}$ , for the LC momentum fraction  $x$  of constituent sea (anti)quarks involved in such rescattering processes [4, 26], i.e., using a larger value for  $\alpha_{\text{sea}}$ .

Considering twice stronger higher twist effects, using  $\alpha_{\text{sea}} = 0.8$  and  $\alpha_{\text{sea}} = 0.9$ , in addition to the default value  $\alpha_{\text{sea}} = 0.65$ , and adjusting the parameters of the hadronization procedure of the QGSJET-III model in order to keep an agreement with accelerator data, one arrives to the energy dependence of the inelasticity of proton-nitrogen interactions,  $K_{pN}^{\text{inel}}$ , shown in Fig. 6 (left), while the corresponding results for EAS maximum depth are plotted in Fig. 6 (right). As one can see in Fig. 6, noticeable changes both for  $K_{pN}^{\text{inel}}$  and  $X_{\text{max}}$  are caused only by modifications of LC momentum distributions of constituent partons. In particular, using  $\alpha_{\text{sea}} = 0.9$ , one obtains up to  $\simeq 6\%$  reduction of  $K_{pN}^{\text{inel}}$  and up to  $\simeq 12 \text{ g/cm}^2$  larger  $X_{\text{max}}$  at the highest energies [6].

Generally, one could have expected a stronger dependence of the inelasticity and of the predicted EAS maximum depth on the momentum distributions of constituent partons [36]. How-

ever, at very high energies, this naive picture is substantially modified due to the dominance of (semi)hard scattering processes over purely nonperturbative soft interactions. Indeed, in any partial semihard rescattering, the hardest parton-parton scattering is usually preceded by multiple emission of “softer” partons (so-called initial state radiation): with each “parent” parton in the cascade having a higher momentum fraction than its “daughter”. Hence, the total momentum fraction taken by the first  $s$ -channel partons produced in such perturbative cascades constitutes a lower bound on the inelasticity.

In principle, one may consider a more exotic scenario, assuming that the standard hadron production pattern is significantly modified at very high energies by collective effects. While this could allow one to increase the predicted  $X_{\max}$  more significantly, by up to  $\simeq 30$  g/cm<sup>2</sup>, such modifications are seriously disfavored both by the data of the LHCf experiment [16, 17], regarding forward neutron production in  $pp$  collisions at LHC, and by measurements of the muon production depth at the Pierre Auger Observatory [37], as demonstrated in [6].

## 5 Conclusion

In this contribution, I briefly discussed the physics content of the QGSJET-III model and presented its predictions for basic extensive air showers characteristics. Overall, the EAS results of QGSJET-III are rather close to the ones of the QGSJET-II-04 model: with the predicted  $X_{\max}$  being up to 10 g/cm<sup>2</sup> larger and with  $N_{\mu}$  being reduced by  $\simeq 5\%$ . Such a robustness of the model predictions for EAS development is not occasional but rather reflects the fact that the relevant features of high energy interaction treatment are sufficiently constrained by accelerator measurements, notably, from the Large Hadron Collider. Indeed, performing a quantitative analysis of the corresponding model uncertainties, within the standard physics picture and within the limits allowed by available accelerator data, one was able to further increase the predicted  $X_{\max}$  by up to  $\simeq 10$  g/cm<sup>2</sup> only, while the predicted EAS muon content could be enhanced by  $\simeq 10\%$  [5, 6].

**Funding information** This work was supported by Deutsche Forschungsgemeinschaft (project number 465275045).

## References

- [1] M. Nagano and A. A. Watson, *Observations and implications of the ultrahigh-energy cosmic rays*, Rev. Mod. Phys. **72**, 689 (2000), doi:[10.1103/RevModPhys.72.689](https://doi.org/10.1103/RevModPhys.72.689).
- [2] R. Engel, D. Heck, and T. Pierog, *Extensive air showers and hadronic interactions at high energy*, Ann. Rev. Nucl. Part. Sci. **61**, 467 (2011), doi:[10.1146/annurev.nucl.012809.104544](https://doi.org/10.1146/annurev.nucl.012809.104544).
- [3] R. Ulrich, R. Engel, and M. Unger, *Hadronic multiparticle production at ultrahigh energies and extensive air showers*, Phys. Rev. D **83**, 054026 (2011), doi:[10.1103/PhysRevD.83.054026](https://doi.org/10.1103/PhysRevD.83.054026).
- [4] R. D. Parsons, C. Bleve, S. S. Ostapchenko, and J. Knapp, *Systematic uncertainties in air shower measurements from high-energy hadronic interaction models*, Astropart. Phys. **34**, 832 (2011), doi:[10.1016/j.astropartphys.2011.02.007](https://doi.org/10.1016/j.astropartphys.2011.02.007).



- [5] S. Ostapchenko and G. Sigl, *On the model uncertainties for the predicted muon content of extensive air showers*, *Astropart. Phys.* **163**, 103004 (2024), doi:[10.1016/j.astropartphys.2024.103004](https://doi.org/10.1016/j.astropartphys.2024.103004).
- [6] S. Ostapchenko and G. Sigl, *Model uncertainties for the predicted maximum depth of extensive air showers*, *Phys. Rev. D* **110**, 063041 (2024), doi:[10.1103/PhysRevD.110.063041](https://doi.org/10.1103/PhysRevD.110.063041).
- [7] S. Ostapchenko, *QGSJET-III model of high energy hadronic interactions: The formalism*, *Phys. Rev. D* **109**, 034002 (2024), doi:[10.1103/PhysRevD.109.034002](https://doi.org/10.1103/PhysRevD.109.034002).
- [8] S. Ostapchenko, *QGSJET-III model of high energy hadronic interactions: II. Particle production and extensive air shower characteristics*, *Phys. Rev. D* **109**, 094019 (2024), doi:[10.1103/PhysRevD.109.094019](https://doi.org/10.1103/PhysRevD.109.094019).
- [9] S. Ostapchenko, *Monte Carlo treatment of hadronic interactions in enhanced Pomeron scheme: QGSJET-II model*, *Phys. Rev. D* **83**, 014018 (2011), doi:[10.1103/PhysRevD.83.014018](https://doi.org/10.1103/PhysRevD.83.014018).
- [10] S. Ostapchenko, *QGSJET-II: physics, recent improvements, and results for air showers*, *EPJ Web Conf.* **52**, 02001 (2013), doi:[10.1051/epjconf/20135202001](https://doi.org/10.1051/epjconf/20135202001).
- [11] J. C. Collins, D. E. Soper, and G. F. Sterman, *Factorization of hard processes in QCD*, *Adv. Ser. Direct. High Energy Phys.* **5**, 1 (1989), doi:[10.1142/9789814503266\\_0001](https://doi.org/10.1142/9789814503266_0001).
- [12] Jianwei Qiu and Ivan Vitev, *Resummed QCD power corrections to nuclear shadowing*, *Phys. Rev. Lett.* **93**, 262301 (2004), doi:[10.1103/PhysRevLett.93.262301](https://doi.org/10.1103/PhysRevLett.93.262301).
- [13] Jian-Wei Qiu and Ivan Vitev, *Coherent QCD multiple scattering in proton-nucleus collisions*, *Phys. Lett. B* **632**, 507 (2006), doi:[10.1016/j.physletb.2005.10.073](https://doi.org/10.1016/j.physletb.2005.10.073).
- [14] S. Ostapchenko and M. Bleicher, *Taming the energy rise of the total proton-proton cross-section*, *Universe* **5**, 106 (2019), doi:[10.3390/universe5050106](https://doi.org/10.3390/universe5050106).
- [15] S. Ostapchenko, *QGSJET-III model: novel features*, *Phys. At. Nucl.* **44**, 1017 (2021), doi:[10.1134/S1063778821130238](https://doi.org/10.1134/S1063778821130238).
- [16] O. Adriani *et al.* (LHCf Collaboration), *Measurement of very forward neutron energy spectra for 7 TeV proton-proton collisions at the Large Hadron Collider*, *Phys. Lett. B* **750**, 360 (2015), doi:[10.1016/j.physletb.2015.09.041](https://doi.org/10.1016/j.physletb.2015.09.041).
- [17] O. Adriani *et al.* (LHCf Collaboration), *Measurement of inclusive forward neutron production cross section in proton-proton collisions at  $\sqrt{s} = 13$  TeV with the LHCf Arm2 detector*, *JHEP* **11**, 073 (2018), doi:[10.1007/JHEP11\(2018\)073](https://doi.org/10.1007/JHEP11(2018)073).
- [18] T. Pierog, Iu. Karpenko, J. M. Katzy, E. Yatsenko, and K. Werner, *EPOS LHC: Test of collective hadronization with data measured at the CERN Large Hadron Collider*, *Phys. Rev. C* **92**, 034906 (2015), doi:[10.1103/PhysRevC.92.034906](https://doi.org/10.1103/PhysRevC.92.034906).
- [19] F. Riehn, R. Engel, A. Fedynitch, T. K. Gaisser, and T. Stanev, *Hadronic interaction model Sibyll 2.3d and extensive air showers*, *Phys. Rev. D* **102**, 063002 (2020), doi:[10.1103/PhysRevD.102.063002](https://doi.org/10.1103/PhysRevD.102.063002).
- [20] S. Ostapchenko, *Cosmic ray interactions in the atmosphere: QGSJET-III and other models*, *SciPost Phys. Proc.* **13**, 004 (2023), doi:[10.21468/SciPostPhysProc.13.004](https://doi.org/10.21468/SciPostPhysProc.13.004).
- [21] K.-H. Kampert and M. Unger, *Measurements of the cosmic ray composition with air shower experiments*, *Astropart. Phys.* **35**, 660 (2012), doi:[10.1016/j.astropartphys.2012.02.004](https://doi.org/10.1016/j.astropartphys.2012.02.004).

- [22] A. Aab *et al.* (Pierre Auger Collaboration), *Muons in air showers at the Pierre Auger Observatory: Mean number in highly inclined events*, Phys. Rev. D **91** (2015) 032003, doi:[10.1103/PhysRevD.91.032003](https://doi.org/10.1103/PhysRevD.91.032003).
- [23] A. Aab *et al.* (Pierre Auger Collaboration), *Testing hadronic interactions at ultrahigh energies with air showers measured by the Pierre Auger Observatory*, Phys. Rev. Lett. **117**, 192001 (2016), doi:[10.1103/PhysRevLett.117.192001](https://doi.org/10.1103/PhysRevLett.117.192001).
- [24] W. Heitler, *The Quantum Theory of Radiation*, Oxford University Press (1954).
- [25] J. Matthews, *A Heitler model of extensive air showers*, Astropart. Phys. **22**, 387 (2005), doi:[10.1016/j.astropartphys.2004.09.003](https://doi.org/10.1016/j.astropartphys.2004.09.003).
- [26] S. Ostapchenko, M. Bleicher, T. Pierog, and K. Werner, *Constraining high energy interaction mechanisms by studying forward hadron production at the LHC*, Phys. Rev. D **94**, 114026 (2016), doi:[10.1103/PhysRevD.94.114026](https://doi.org/10.1103/PhysRevD.94.114026).
- [27] H. Adhikary *et al.* (NA61/SHINE Collaboration), *Measurement of hadron production in  $\pi^-$ -C interactions at 158 and 350 GeV/c with NA61/SHINE at the CERN SPS*, Phys. Rev. D **107**, 062004 (2023), doi:[10.1103/PhysRevD.107.062004](https://doi.org/10.1103/PhysRevD.107.062004).
- [28] A. Abdul Halim *et al.* (Pierre Auger Collaboration), *Testing Hadronic-Model Predictions of Depth of Maximum of Air-Shower Profiles and Ground-Particle Signals using Hybrid Data of the Pierre Auger Observatory*, Phys. Rev. D **109**, 102001 (2024), doi:[10.1103/PhysRevD.109.102001](https://doi.org/10.1103/PhysRevD.109.102001).
- [29] R. J. Glauber, *High-energy collision theory*, in: Lectures in theoretical physics, Ed. by W. E. Brittin and L. G. Dunham, Interscience Publishers, New York, 1959, vol. 1, p. 315.
- [30] V. N. Gribov, *Glauber corrections and the interaction between high-energy hadrons and nuclei*, Sov. Phys. JETP **29**, 483 (1969).
- [31] G. Antchev *et al.* (TOTEM Collaboration), *Measurement of proton-proton inelastic scattering cross-section at  $\sqrt{s} = 7$  TeV*, Europhys. Lett. **101**, 21003 (2013), doi:[10.1209/0295-5075/101/21003](https://doi.org/10.1209/0295-5075/101/21003).
- [32] R. L. Workman *et al.* (Particle Data Group), *Review of Particle Physics*, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022), doi:[10.1093/ptep/ptac097](https://doi.org/10.1093/ptep/ptac097).
- [33] G. Antchev *et al.* (TOTEM Collaboration), *First measurement of elastic, inelastic and total cross-section at  $\sqrt{s} = 13$  TeV by TOTEM and overview of cross-section data at LHC energies*, Eur. Phys. J. C **79**, 103 (2019), doi:[10.1140/epjc/s10052-019-6567-0](https://doi.org/10.1140/epjc/s10052-019-6567-0).
- [34] G. Aad *et al.* (ATLAS Collaboration), *Measurement of the total cross section and  $\rho$ -parameter from elastic scattering in pp collisions at  $\sqrt{s} = 13$  TeV with the ATLAS detector*, Eur. Phys. J. C **83**, 441 (2023), doi:[10.1140/epjc/s10052-023-11436-8](https://doi.org/10.1140/epjc/s10052-023-11436-8).
- [35] S. Ostapchenko, *LHC data on inelastic diffraction and uncertainties in the predictions for longitudinal extensive air shower development*, Phys. Rev. D **89**, 074009 (2014), doi:[10.1103/PhysRevD.89.074009](https://doi.org/10.1103/PhysRevD.89.074009).
- [36] S. S. Ostapchenko, *Contemporary models of high-energy interactions: Present status and perspectives*, J. Phys. G **29**, 831 (2003), doi:[10.1088/0954-3899/29/5/305](https://doi.org/10.1088/0954-3899/29/5/305).
- [37] A. Aab *et al.* (Pierre Auger Collaboration), *Muons in Air Showers at the Pierre Auger Observatory: Measurement of Atmospheric Production Depth*, Phys. Rev. D **90**, 012012 (2014), doi:[10.1103/PhysRevD.90.012012](https://doi.org/10.1103/PhysRevD.90.012012).