Highlights from the GRAPES-3 Experiment

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Abstract

GRAPES-3 is an air shower array in the Nilgiri mountains in Southern India, consisting of a 560 m² muon telescope (G3MT) made up of proportional counters as well as a 25000 m² array of 400 plastic scintillator detectors. In operation since 2000, the muon telescope has proven itself to be a sensitive probe of cosmic, solar, heliospheric and atmospheric phenomena. Recently the observatory has validated its pointing by characterizing the shadow of the moon in data gathered over multiple years. Small angular scale anisotropies have been measured in the arrival directions of cosmic rays, confirming the existence of features reported previously by HAWC and Argo-YBJ. The multiplicity distribution of muons measured by G3MT allows the relative composition of proton primaries to be extracted from the all-particle air shower data. The extracted proton spectrum indicates a spectral hardening at ∼ 166 TeV, disfavoring a single power law description of the proton spectrum up to the knee. These emerging capabilities of GRAPES-3 as an astroparticle physics observatory are expected to be further enhanced by the completion of a new muon telescope, presently under construction. This will double the effective area over which the muon component of air showers can be measured.

Contents

1 Introduction

Ever since their discovery more than a century ago, the sites at and mechanisms by which high energy cosmic rays (CRs) are accelerated have been an enduring puzzle at the heart of fundamental physics. They are observed over a diverse energy range from 100 MeV to beyond 100 EeV, with a flux that has a power law dependence on energy, with a few established features such as the knee (at ∼ PeV energies) and the ankle (at ∼ EeV energies). Today direct detection experiments in space have taught us that CRs are predominantly made up of protons (∼ 90%) with helium nuclei making up 9% and heavier elements the remaining [1](#page-8-0)% [1]. Beyond \sim 100 TeV, the flux of CRs is too low to lead to significant event statistics in realistically sized direct detection experiments. Consequently, ground based indirect detection techniques leveraging the large area of the upper atmosphere have to be relied upon. Here, an array of particle detectors on the ground are used to sample the Extensive Air Shower (EAS) that the primary cosmic ray develops into after interacting with the upper atmosphere.

The **G**amma **R**ay **A**stronomy at **P**eV **E**nergie**S** - phase **3** (GRAPES-3) is such a detector array.

2 The GRAPES-3 Experiment

Located at an altitude of 2200 m above mean sea level in Ooty, Tamil Nadu, India (11.4◦N, 76.7◦E), GRAPES-3 consists of an array of 400 plastic scintillator detector modules spread over [2](#page-8-1)5000 m² - G[3](#page-8-2)SD [2] as well as a large area (560 m²) tracking muon telescope -G3MT [3].

G3SD is made up of an even mixture of 3 different detector types. In cone type detectors, 5 cm thick plastic scintillators of 1 m^2 total area, placed at the bottom of the pyramidal geometry

of the detector are exposed to a downward pointing Photomultiplier Tube (PMT) placed at the top vertex of the pyramid. The internal sides of the pyramid are coated with a reflective material. In fiber type detectors, the scintillators are optically coupled to the PMTs through wavelength shifting fibers embedded in grooves made along the scintillator plates. Due to the superior light collection efficiency of this method, scintillator plates of only 2 cm thickness are used in these detectors. A subset of the fiber type detectors have 2 PMTs, the second one coupled to fewer fibers and allowing for a higher dynamic range in the total number of particles detected.

The waveforms from the PMTs in the G3SD detector modules are digitized through Analog to Digital Converters (ADC) and Time to Digital Converters (TDCs). The ADC outputs, in conjunction with estimates of the gain of each detector obtained through calibration measurements enable the total number of particles in each detector (and thus the whole G3SD array) to be inferred, while the TDC outputs enable the inference of the times of flight of these particles. The former information is utilized to reconstruct the coordinates of the Core of the EAS as well as its age and size by fitting with the Nishimura-Kamata-Greisen function (NKG) [[4](#page-8-3)], while the latter information enables the arrival direction of the primary to be reconstructed.

With an 8 metre inter module separation, the G3SD is the densest (and consequently lowest energy threshold) array of its scale in the world, enabling it to trigger at ∼ 45 Hz and record \sim 3 million Extensive Air Showers (EAS) every year in the primary energy range of \sim 1 TeV - 10 PeV. This energy range gives it a unique overlap with both space based direct and ground based indirect CR detection experiments, bridging the gap between the two. Implementation of a shower age and size dependent curvature correction method [[5](#page-8-4)] to the shower front detected by the scintillator modules has enabled G3SD to reconstruct the arrival direction of a 300 TeV primary with a median resolution less than 0.3◦ . The astronomical pointing of G3SD has been validated to an accuracy of ∼ 0.032◦ in R.A. and ∼ 0.090◦ in Declination by characterizing the shadow of the moon as observed in CR arrival directions [[6](#page-8-5)].

G3MT is made up of Proportional Counters (PRCs) - gas based detectors with ∼ *µ*s timing resolution. 6 m \times 10 cm \times 10 cm metal tubes filled with a 90% Argon, 10% Methane mixture and a Tungsten anode wire running through their center are placed in four layers. A two metre thick concrete absorber at the top with a grammage of 550 g cm⁻² ensures that only muons with an energy of at least 1 GeV penetrate down to the detection layers below. Each detection layer consists of 58 PRCs placed in such a way that the PRCs in even and odd layers are laid out perpendicularly to each other, enabling the arrival direction of these muons to be reconstructed with a resolution of ∼ 4 ◦ . Four modules as described above make up a station of G3MT, and four such stations together make up the 560 $m²$ detection area of G3MT over a field of view of 2.3 sr.

Data acquisition is triggered when a coincident detection is reported between at least one PRC in each of the four layers that make up a G3MT module. After reconstruction the muons are grouped into 225 (169 when excluding the outermost) directional bins, and the number of muons in each such bin is recorded grouped further into 10 s time bins. In addition, data acquisition is also triggered whenever G3SD reports the detection of an EAS, and the total number of muons in G3MT associated with the triggering EAS is recorded.

GRAPES-3 has been in continuous operation for more than two decades, during which time G3MT has been recording 4 billion muons a day. The simultaneous operation of both detector arrays gives GRAPES-3 the unique ability to measure the muon content in air showers triggered by G3SD using G3MT. This enables showers initiated by *γ*-rays to be distinguished from those originating in hadron primaries and also gives GRAPES-3 some sensitivity to the composition of the primary CR flux.

In addition to G3MT and G3SD, GRAPES-3 also operates several Boltek model EFM-100 Electric Field Monitors (EFMs) as well as monitors for weather related parameters, at a few locations around Ooty. These play a crucial role in studying and mitigating atmospheric effects on the data gathered by G3MT.

GRAPES-3 was constructed and is operated by a collaboration of researchers from seven Japanese and fourteen Indian institutions 1 1 . Of the air shower arrays in the Northern hemisphere, GRAPES-3 is closest to the Equator, giving it a unique advantage in studying the latitude dependence of the CR flux.

3 Physics highlights

Over the years GRAPES-3 has produced several pathbreaking results in the study of solar, heliospheric [[7](#page-8-6)] and atmospheric [[8](#page-8-7)] phenomena using muons recorded by G3MT, heralding an era in which the flux of CR secondaries will be extensively used as tools for monitoring the near Earth environment. In more recent times GRAPES-3 has emerged as an astroparticle physics observatory, using G3SD data with additional input from G3MT to study the properties of the CR flux at TeV-PeV energies.

In the following sections we summarize these results with a focus on the newer ones.

3.1 Solar, Heliospheric and Atmospheric Physics with G3MT

G3MT detects rapid variations in the muon flux over \sim 10 minute timescales during thunderstorms. Since only muons above 1 GeV make it to G3MT, this requires large electric potentials. Between April 2011 and December 2014, 184 such thunderstorm like events were detected both by G3MT and the Electric Field Monitors. Of this, an event on December 2014 [[8](#page-8-7)] was found to produce a peak time variation in the muon flux of almost -2% in specific directional bins. Over an 18 minute duration, the muon intensity variation was tracked across 45 out of the 169 non peripheral directional bins, moving at a velocity of 6.2◦min−¹ across the sky, spanning ∼ 75◦ along the North-South direction. The EFMs enabled the altitude and area of the thundercloud to be estimated at 11.4 km above mean sea level and \geq 380 km 2 respectively suggesting that it was moving at a velocity of \sim 60 km h $^{-1}.$

Detailed simulations of the acceleration of muons in atmospheric electric fields using the CORSIKA code allowed the muon intensity variations to be interpreted as an equivalent electric potential. This suggests that a peak potential of 1.8 GV (mean 1.3 GV) occurred during the 20 minute duration of the thunderstorm, implying the delivery of a power ≥ 2 GW by the thundercloud. These observations possibly provide the first direct evidence for the generation of GV potentials in thunderclouds, also explaining the production of the highest energy (100 MeV) *γ*-rays in terrestrial *γ*-ray flashes.

On 22nd June 2015, for a period of 2 hours beginning at 19.0 Hours (UT), the total muon intensity in all directions recorded by G3MT increased by more than 1% (\geq 50 standard deviations) [[7](#page-8-6)]. This was associated with a G4-class geomagnetic storm triggered by a Coronal Mass Ejection arriving at 18:40 UT and the related surge in the interplanetary magnetic field by almost 40 nT. Detailed simulations accounting for the different cutoff rigidities in nine different directions were able to closely match the observed muon rate variations within and across grouped directional bins, suggesting that a high energy (\sim 20 GeV) burst of Galactic cosmic rays caused by the lowering of cutoff rigidities due to a 17× compression of the interplanetary magnetic field to 680 nT followed by reconnection with the Geomagnetic Field. The simultaneous occurrence of the burst in all nine directions suggests its origin close to Earth, indicating a transient weakening of Earth's magnetic shield. These observations establish G3MT as an instrument that is sensitive to variations in the IMF at the level of a few nT. This line of inquiry

¹https://grapes-3.tifr.res.in/

holds clues for a better understanding and predictive power of solar superstorms that could cripple modern technological infrastructure on Earth, and endanger the lives of the astronauts in space.

An analysis of 22 years of G3MT data has provided possibly the first measurement by a ground based instrument of the parameters which affect the transport of Galactic CRs in the heliosphere and their dependence on solar activity levels at high rigidities (64-92 GV) [[9](#page-8-8)]. This was achieved by inferring the CR diffusion coefficient and the parallel mean free path along nine different directions (with different cutoff rigidities) from G3MT data within the diffusion-convection framework, which predicts an anticorrelation between the GCR intensity and the solar wind velocity.

The particle densities recorded by G3SD detector modules have been found to exhibit a dependence on the ambient temperature and atmospheric pressure, by studying a decade of G3SD data [[10](#page-8-9)]. These characterizations allow for improved correction algorithms, eventually improving the accuracy of shower parameter estimates obtained using G3SD.

3.2 Small scale anisotropy in the arrival directions of Cosmic Rays

Figure 1: Mollveide projection of the relative intensity skymap obtained after timescrambling with *∆*t = 24 hr and a smoothing radius of 10°. The large-scale deficit as well as the small-scale structures A and B are prominently seen. See Ref. [[11](#page-8-10)] from which this figure has been taken, for the associated significance skymap as well as more information.

Of the \sim 3.9 × 10⁹ EAS events observed by G3SD between 1st January 2013 and 31st December 2016, rejecting those with zenith angles greater than 60° leaves behind $\sim 3.7 \times 10^9$. Comparing the distribution of the NKG-size parameter of these events against that of CORSIKA simulated events, this sample is estimated to have a median energy of 16.2 TeV. Binning these events directionally into a HEALPix map in the celestial sphere in Equatorial coordinates and comparing against the average of twenty reference maps obtained from time-scrambling the same events, two anisotropic structures (see Figs [1](#page-4-1) and [2\)](#page-5-1) which deviate in a statistically significant manner from the expected isotropic distribution of CRs have been identified [[11](#page-8-10)]. Structure 'A' lies in the range 50° to 80° in R.A., -15° to 30° in DEC, while structure B lies between 110° and 140° in R.A, and extents beyond the declination range down to which GRAPES-3 is sensitive. Structures A and B have relative excesses at the level of $(6.5 \pm 1.3) \times 10^{-4}$ and

 $(4.9 \pm 1.4) \times 10^{-4}$, with the isotropic null hypothesis being rejected at 6.8 σ and 4.9 σ respectively as determined by the method of Li & Ma. Careful examination of the dependence of these results on choice of time scrambling window and smoothing angle, as well as comparison against similar analyses performed in anti-sidereal time were used to build confidence in the results and rule out spurious effects from sidereal effects as the origin of these anisotropies. The location and relative intensities of regions A and B as observed by GRAPES-3 are consistent with those reported by Milagro, ARGO-YBJ and HAWC, while these properties of region B as reported by IceCube in the declination range that overlaps with GRAPES-3 are also consistent.

Figure 2: Top: The distribution of events in bins of Right Ascension. The Background estimates are obtained using time scrambling. Bottom: The relative intensity, which is the ratio between the Signal + Background and Background curves in the top panel.

Confining the analysis only to EAS events which produce at least 2 coincident muon tracks within G3MT, 98% of the *γ*-ray events within the sample can be rejected (as estimated from simulation), leaving behind \sim 1.9 × 10 9 events in total. However the peak relative excesses of structures A and B in this sample were found to be within 1 σ of the relative excesses obtained with the full sample, strongly hinting that these anisotropic structures are of hadronic origin.

The differences in morphologies of these structures as reported by the different observatories point to a need to further explore latitude dependent systematics by jointly analyzing the data from the different observatories.

The origin of these anisotropic structures is presently unclear. Models attributing them to the proximity of supernova explosions, the turbulent magnetic field within the CR scattering length, CR scattering by Alfven waves created by turbulent cascades in the local field direction as well as magnetic reconnection in the heliosphere have all been proposed. See Ref. [[11](#page-8-10)] for a deeper discussion.

3.3 Evidence of a hardening in the CR proton spectrum at ∼ 166 **TeV**

The number of muons produced within an EAS is strongly dependent on the atomic mass number of the primary initiating the shower. GRAPES-3 has recently been able to exploit this to measure the CR proton spectrum [[12](#page-8-11)]. Starting with a sample of $\sim 1.75 \times 10^9$ EAS events triggered by G3SD between 1st January 2014 and 26th October 2015, stringent quality cuts were imposed to isolate a sample of \sim 7.8 × 10^6 events over an effective livetime of 460 days. A crescent shaped fiducial area (see Fig. [3\)](#page-6-1) defined by requiring the reconstructed core of the EAS to be within 50 metres of the centre of G3SD but at least 60 metres away from the centre of G3MT was imposed on the sample. This suppresses the level of events misreconstructed

Figure 3: A schematic of GRAPES-3: G3SD detector modules (shaded triangle), the 16 modules of G3MT (green squares). The pink shaded area represents the fiducial area for the proton spectrum analysis.

because they're near the edge of the detector to \leq 1% while ensuring that the number of associated muon tracks reported by G3MT is not significantly contaminated by hadron punch through. Inclined events were rejected by requiring that the reconstructed zenith be less than 17.8°. Only events with an NKG size $\geq 10^4$ where the detector trigger efficiency is $\geq 90\%$ were selected. After imposing further quality cuts to reject events affected by issues with the readout electronics and data acquisition channels, the muon multiplicities of the events as well as their NKG sizes were compared using Gold's unfolding procedure against Monte Carlo events utilizing the QGSJET-II-04 hadronic interaction model to extract the relative composition of proton primaries. The proton energy spectrum thus extracted (see Fig [4\)](#page-7-1) favours a broken power law over a single power law at 3.2*σ* significance, with the break found to be at 164 $±$ 55 TeV. Systematic uncertainties from various sources such as uncertainties in the detector acceptance, the priors used in the unfolding algorithm and limited Monte Carlo statistics were factored into the fit when comparing the single power law against the broken power law.

The proton+Helium spectrum reported by the direct experiment DAMPE also suggests a hardening at ∼ 150 TeV [[13](#page-8-12)]. These observations suggest that the idealized description of a single unbroken power law for the CR spectrum all the way to the knee is no longer valid, and calls for more realistic modeling of the injection and propagation of CRs within the Galaxy. Models capable of explaining these features require multiple classes of sources with different rigidity cutoffs. See Ref. [[12](#page-8-11)] for a deeper discussion.

4 The future

The efforts of the GRAPES-3 collaboration are now focused towards the completion of a new muon telescope, the construction of which began in 2016 but had been interrupted by the COVID 19 pandemic. Construction resumed in 2023 and preliminary data taking is expected to commence by late 2025. The commissioning of the new muon telescope will double the area over which the muons associated with G3SD EAS triggers can be counted. A proportionate

Figure 4: CR proton energy spectrum measured by GRAPES-3 (red circles) compared with results from direct and indirect observations. See Ref. [[12](#page-8-11)] from which this figure has been taken, for more details.

expansion of G3SD to significantly improve the sensitivity of GRAPES-3 as a *γ*-ray instrument is currently under consideration.

On the analysis side the collaboration is now working on expanding the CR composition and spectrum measurements to all nuclear species, and in improving *γ*-hadron separation strategies for enhancing the sensitivity of *γ*-ray searches.

5 Conclusions and Discussion

In this manuscript we have presented a summary of the key highlight results from GRAPES-3. Over the past two decades GRAPES-3 has carved a niche out for itself in the measurements of CR secondary fluxes as probes of near Earth phenomena. In recent times it has emerged as an astroparticle physics observatory.

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