Causality and the Interpretation of Quantum Mechanics

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Abstract

From the ancient Einstein-Podolsky-Rosen paradox to the recent Sorkin-type impossible measurements problem, the contradictions between relativistic causality, quantum nonlocality, and quantum measurement have persisted. Based on quantum field theory, our work provides a framework that harmoniously integrates these three aspects. This framework consists of causality expressed by reduced density matrices and an interpretation of quantum mechanics that considers quantum mechanics to be complete. Specifically, we use reduced density matrices to represent the local information of the quantum state and show that the reduced density matrices cannot evolve superluminally. Unlike recent approaches that address causality by introducing new operators to represent detectors, our perspective is that everything—including detectors, the environment, and even humans—is made up of the same fundamental fields. This viewpoint leads us to question the validity of the Schrödinger's cat paradox and motivates us to propose an interpretation of quantum mechanics that requires no extra assumptions and remains fully compatible with relativity.

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1 Introduction

To discuss relativistic causality within the framework of quantum mechanics, we need the concept of locality. For the case of a single-particle state, we can impart quantum state locality by defining position operators [1]. However, it leads to superluminal propagation, as succinctly argued in [2]. Further variations and physical implications are discussed in [3–12]. In fact, particles are quantized waves rather than point-like entities. Due to pair production, we cannot precisely determine the position of a particle [2, 13, 14], which means that position operators are not suited for a genuine description of local phenomena in quantum field theory. Therefore, superluminal propagation induced by the use of position operators is not physically realizable. Furthermore, our later results indicate that a group of particles initially confined to a certain region will not propagate superluminally in the subsequent evolution.

Compared to superluminal propagation of particles, the "Fermi two-atom problem" can more accurately reflect the core of causality [15]. Fermi considered two atoms separated by a distance R, with one atom in the ground state and the other in the excited state. If causal influences propagate at the maximum speed c, then within the time interval $0 < t < R/c$, the excited atom will not have any impact on the ground-state atom through emitted photons.

The Fermi two-atom problem garnered renewed attention when revisited sixty years later by Hegerfeldt [16] (The introduction in [17] is a brief review of earlier work). Hegerfeldt fixed the local properties of states with the help of projection operators, and demonstrated that describing this experiment in terms of transition probabilities leads to results that violate causality. Subsequently, Buchholz and Yngvason used algebraic quantum field theory to analyze the problem and pointed out that Hegerfeldt's projection operators are not legitimate quantities, and expectation values should be used instead. [18]. In the modern development of the Fermi two-atom problem, the two atoms were replaced by two Unruh-DeWitt detectors to explore causality [19–26]. In this context, detectors are not composed of fields used to propagate influences over the distances; instead, one needs to enlarge the Hilbert space to introduce the quantum states of the detectors. Additionally, the initial states are bare states (direct product states), involving the projection operators required by Hegerfeldt [16], which, however, was precisely the point refuted by Buchholz and Yngvason [18].

In the framework of quantum field theory, all matter in the world is composed of the same fundamental fields, which means that we do not need to enlarge the Hilbert space to introduce detectors. More specifically, detectors are composed of elementary particles, and elementary particles are excitations of fundamental fields; therefore, detectors are essentially excitations of fundamental fields as well.Regardless of the number of detectors, the Hilbert space used to describe the entire system should remain the same. At the same time, this also implies that we cannot separate detectors from the systems being detected. Even if we consider quantum fields at different spatial points as different field operators, the interactions between fields at adjacent points lead to strong entanglement between different points in the quantum state. This also means that even basic and simple states like the vacuum state cannot be written as the direct product of "vacuum states" of different regions. The characteristic of quantum field theory has been noted by Hegerfeldt (see [16], last page) and further analyzed by Buchholz and Yngvason [18]. However, subsequent studies and discussions on causality have overlooked this characteristic and simply regarded the initial state as a bare state. Later, we will point out that the idea that "all matter

in the world is composed of the same fundamental fields" plays an important role in finding an interpretation of quantum mechanics compatible with relativity.

In addition to the two-detector system mentioned above, introducing a third detector will lead to "Sorkin-type impossible measurements problem" illustrating that the natural generalization of the non-relativistic measurement scheme to relativistic quantum theory leads to superluminal signaling [27–29]. Traditional quantum mechanics is governed by two laws: time evolution of quantum states and measurement theory. Relativity requires that these two laws together must ensure that information cannot propagate faster than light. However, non-relativistic measurement theories with traditional state update rule (Lüders' rule) cannot meet this requirement. To address this issue, various "update rules" have recently been proposed to describe the state update induced by measurements on the fields or on the detectors [28–34]. Similar to the Fermi two-atom problem, new operators are introduced to specifically describe the detectors, then the resulting "update rules" to some extent depend on the model construction. In fact, the Sorkin-type impossible measurements problem reveals a deeper issue, providing a window for exploring the interpretation of quantum mechanics. Since the physical law behind measurement theory are the interpretation of quantum mechanics [35], and traditional non-relativistic measurement theory has been shown to be inadequate, it suggests that traditional interpretations might also be flawed. A correct interpretation of quantum mechanics has the potential to provide a measurement theory that satisfies causality.

In addition to the method of introducing detectors as described above, one can also discuss causality by specifying rules for physical operations on quantum states. Quoting Witten's discussion of the Reeh-Schlieder theorem, he states [36], "If one asks about not mathematical operations in Hilbert space but physical operations that are possible in the real world, then the only physical way that one can modify a quantum state is by perturbing the Hamiltonian by which it evolves, thus bringing about a unitary transformation." This classic statement also serves as the starting point for earlier proofs regarding causality [37,38]. However, the assertion that "the only physical way that one can modify a quantum state is by 'perturbing the Hamiltonian' by which it evolves" has not been robustly demonstrated (for a more detailed elaboration on the concept of "perturbing the Hamiltonian," one can refer to the third hypothesis outlined in [37]). Especially when humans (or apparatus) interact with the system, it is generally challenging to regard the system as a pure state after the operations due to the entanglement caused by the interaction. Later, we will demonstrate that, the conclusion that physical operations can always be equivalent to a unitary operator need not rely on the notion of "perturbing the Hamilton" but can be derived from our results.

In this paper, we adopt a more realistic perspective to investigate causality, where everything (including detectors, environments, and humans) is composed of the same fundamental fields and collectively described by a pure state. This implies that there is no need to enlarge the Hilbert space to introduce detectors, and the Hilbert space of the entire system remains the same regardless of the number of detectors. Specifically, we employ reduced density matrices to characterize the local information of quantum states. Dividing the entire space into two regions, denoted as a and A, we trace out region A (a) to obtain the reduced density matrix which characterizes the information of the quantum state in region a (A). As shown in Fig. 1, region b is the t surface in the future domain of influence of region a. Consider two quantum states with unequal reduced

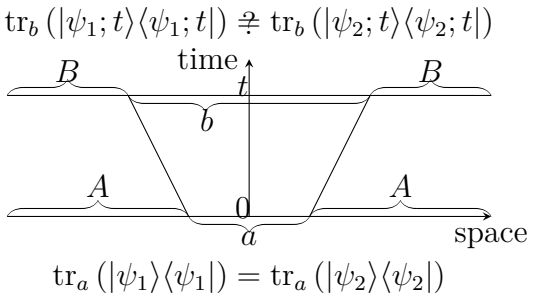

Figure 1 The relationships among regions A, a, B, and b

density matrices of region a and equal reduced density matrices of region A. We will demonstrate that, for local quantum field theory, the reduced density matrices of region B are equal for the two quantum states at time t. Furthermore, since detectors and the systems under measurement are both composed of the same fundamental fields, the "composite" system formed by a detector and a system under measurement cannot be written in the form of $\sum |\text{System}\rangle|\text{Detector}\rangle$. Instead, the entire system should be described by the quantum state of quantum field theory, while the system under measurement and the detector can be described by the corresponding reduced density matrices. However, when two quantum states with the same reduced density matrix in a certain region are superposed, due to spatial entanglement, the superposition state no longer retains the original reduced density matrix in that region. This characteristic of quantum field theory implies that there is no Schrödinger's cat paradox in quantum field theory and gives rise to an interpretation of quantum mechanics that does not require any additional assumptions and is compatible with relativity.

This paper is organized as follows. In Section 2, we demonstrate causality using the example of a free scalar field, providing a detailed exposition of our definition of causality. Subsequently, we extend the proof of causality to the general case of local quantum field theory. In Section 3, we analyze Witten's statement regarding physical operations when discussing the Reeh-Schlieder theorem. In Section 4, we point out a loophole in the derivation of Schrödinger's cat paradox and propose an interpretation of quantum mechanics that supports the completeness of quantum mechanics. We also discuss the differences of this interpretation from traditional hidden-variable theories and its compatibility with relativity. Appendix A provides the derivation of the field propagator used to compute the time evolution of the density matrix. Section 5 concludes the paper with a summary and some discussions, along with several directions for further investigations.

2 Localization and Causality

Ref. [8] argues that among various free quantum field theories, the simplest one—the free real scalar field theory (referred to as the "relativistic Schrödinger system" in Ref. [8])—actually violates causality. We do not agree with this conclusion. Therefore, in order to clearly demonstrate our definition of causality, we first use the free real scalar field as an example to illustrate and prove causality. Subsequently, we will prove causality for general local quantum field theories.

2.1 Free Field Theory as An Example

Consider a free field theory described by the Hamiltonian $\hat{H} = \int \mathrm{d}^3x \Big[\frac{1}{2}\hat{\pi}^2(\boldsymbol{x}) + \frac{1}{2}\Big(\nabla\hat{\phi}(\boldsymbol{x})\Big)^2 + \frac{1}{2}m^2\hat{\phi}^2(\boldsymbol{x})\Big]$, and a quantum state $|\psi;t\rangle = \mathrm{e}^{-i\hat{H}t}|\psi\rangle$ evolving from an initial state $|\psi\rangle$ after a time t. In fact, the expression of H in terms of creation and annihilation operators is precisely the same as $\hat{H}_{rS}$ in equation (3.3a) of Ref. [8], which describes the relativistic Schrödinger system. To highlight the local information of the quantum state, we use a representation based on the eigenstates $|\phi\rangle$ of the field operator $\hat{\phi}$, rather than the traditional Fock representation. These states are normalized according to $\langle\phi|\phi'\rangle = \prod_{\boldsymbol{x}} \delta(\phi(\boldsymbol{x}) - \phi'(\boldsymbol{x}))$. Sometimes, we interchange the symbol ϕ with φ. Utilizing the field propagator derived in Appendix A, the density matrix $\hat{\rho}(t) = |\psi;t\rangle\langle\psi;t|$ can be expressed as

$$\begin{aligned}\rho(\phi,\phi';t) &= \langle\phi|\psi;t\rangle\langle\psi;t|\phi'\rangle \\ &= |N(t)|^2 \int \mathcal{D}\varphi\mathcal{D}\varphi'\ \rho(\varphi,\varphi')K(\phi,\phi',\varphi,\varphi';t)\ ,\end{aligned} \tag{1}$$

where

$$\rho(\varphi,\varphi') = \langle\varphi|\psi\rangle\langle\psi|\varphi'\rangle\ , \tag{2}$$

$$N(t) = \mathcal{N}\mathrm{e}^{-\frac{1}{2}\int \mathrm{d}^3x \int \mathrm{d}t\ G(\mathbf{0};t)}\ , \tag{3}$$

$$K(\phi,\phi',\varphi,\varphi';t) = \mathrm{e}^{iS(\phi,\varphi;t)-iS(\phi',\varphi';t)}\ , \tag{4}$$

$$\begin{aligned}S(\phi,\varphi;t) =& \frac{1}{2}\int \mathrm{d}^3x\mathrm{d}^3y\ G(\boldsymbol{x}-\boldsymbol{y};t)\Big[\phi(\boldsymbol{x})\phi(\boldsymbol{y}) + \varphi(\boldsymbol{x})\varphi(\boldsymbol{y})\Big] \\ &- \int \mathrm{d}^3x\mathrm{d}^3y\ g(\boldsymbol{x}-\boldsymbol{y};t)\phi(\boldsymbol{x})\varphi(\boldsymbol{y})\ ,\end{aligned} \tag{5}$$

and

$$\begin{aligned}G(\boldsymbol{x}-\boldsymbol{y};t) &= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^0\cos(p^0t)}{\sin(p^0t)}\mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\ , \\ g(\boldsymbol{x}-\boldsymbol{y};t) &= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^0}{\sin(p^0t)}\mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\ .\end{aligned} \tag{6}$$

Dividing the space into two parts, denoted as B and b, with the values of ϕ on the two regions denoted as ϕ_B and ϕ_b, the reduced density matrix of the region B in Fig. 1 is given by

$$\int \mathcal{D}\phi_b\ \rho(\phi,\phi';t)\Big|_{\phi_b=\phi'_b} = |N(t)|^2 \int \mathcal{D}\varphi\mathcal{D}\varphi'\rho(\varphi,\varphi')\int \mathcal{D}\phi_b\ K(\phi,\phi',\varphi,\varphi';t)\Big|_{\phi_b=\phi'_b}\ . \tag{7}$$

Further utilization of Eq. (4) and Eq. (5) yields the final integral of Eq. (7) as

$$
\begin{aligned}
&\int \mathcal{D}\phi_b \; K(\phi, \phi', \varphi, \varphi'; t)\Big|_{\phi_b = \phi'_b} \\
&= \exp\left\{ \frac{i}{2} \int_B \mathrm{d}^3x \int_B \mathrm{d}^3y \; G(\boldsymbol{x}-\boldsymbol{y};t) \Big[\phi_B(\boldsymbol{x})\phi_B(\boldsymbol{y}) - \phi'_B(\boldsymbol{x})\phi'_B(\boldsymbol{y}) \Big] \right. \\
&\qquad + \frac{i}{2} \int \mathrm{d}^3x \int \mathrm{d}^3y \; G(\boldsymbol{x}-\boldsymbol{y};t) \Big[\varphi(\boldsymbol{x})\varphi(\boldsymbol{y}) - \varphi'(\boldsymbol{x})\varphi'(\boldsymbol{y}) \Big] \\
&\qquad \left. - i \int_B \mathrm{d}^3x \int \mathrm{d}^3y \; g(\boldsymbol{x}-\boldsymbol{y};t) \Big[\phi_B(\boldsymbol{x})\varphi(\boldsymbol{y}) - \phi'_B(\boldsymbol{x})\varphi'(\boldsymbol{y}) \Big] \right\} \\
&\times \prod_{\boldsymbol{x}\in b} \delta\left[\int_B \mathrm{d}^3y \; G(\boldsymbol{x}-\boldsymbol{y};t) \left[\phi_B(\boldsymbol{y}) - \phi'_B(\boldsymbol{y}) \right] - \int \mathrm{d}^3y \; g(\boldsymbol{x}-\boldsymbol{y};t) \left[\varphi(\boldsymbol{y}) - \varphi'(\boldsymbol{y}) \right] \right].
\end{aligned}
\tag{8}
$$

According to Eq. (6), the inverse of the function $g(\boldsymbol{x}-\boldsymbol{y};t)$ can be determined as

$$
g_{-1}(\boldsymbol{x}-\boldsymbol{y};t) = \int \frac{d^3p}{(2\pi)^3} \frac{\sin(p^0 t)}{p^0} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})} \; . \tag{9}
$$

It is easy to verify that the functions $g(\boldsymbol{x}-\boldsymbol{y};t)$ and $g_{-1}(\boldsymbol{x}-\boldsymbol{y};t)$ satisfy

$$
\int \mathrm{d}^3y \; g_{-1}(\boldsymbol{x}-\boldsymbol{y};t) g(\boldsymbol{y}-\boldsymbol{z};t) = \delta(\boldsymbol{x}-\boldsymbol{z}) \; . \tag{10}
$$

Based on Eq. (6) and Eq. (9), we can also obtain the following useful formulas:

$$
\int \mathrm{d}^3y \; g_{-1}(\boldsymbol{x}-\boldsymbol{y};t) G(\boldsymbol{y}-\boldsymbol{z};t) = \frac{\partial}{\partial t} g_{-1}(\boldsymbol{x}-\boldsymbol{z};t) \; , \tag{11}
$$

$$
\int \mathrm{d}^3z \; \frac{\partial}{\partial t} g_{-1}(\boldsymbol{x}-\boldsymbol{z};t) G(\boldsymbol{z}-\boldsymbol{y};t) = g(\boldsymbol{x}-\boldsymbol{y};t) + \frac{\partial^2}{\partial t^2} g_{-1}(\boldsymbol{x}-\boldsymbol{y};t) \; . \tag{12}
$$

Based on Eq. (9), we also note an important equality: $g_{-1}(\boldsymbol{x}-\boldsymbol{y};t) = \Big[\hat{\phi}(\boldsymbol{x},t), \hat{\phi}(\boldsymbol{y},0) \Big]$, where $\hat{\phi}(\boldsymbol{x},t)$ is the field operator in the Heisenberg picture. Therefore, it is evident that

$$
g_{-1}(\boldsymbol{x};t) = 0 \; , \quad \boldsymbol{x}^2 - t^2 > 0 \; . \tag{13}
$$

As shown in Fig. 1, the region a is the $t=0$ surface in the past domain of dependence of region b. Combining Fig. 1 with Eq. (13), we can rewrite Eq. (10), Eq. (11), and Eq. (12) as follows:

$$
\int_b \mathrm{d}^3y \; g_{-1}(\boldsymbol{x}-\boldsymbol{y};t) g(\boldsymbol{y}-\boldsymbol{z};t) = \delta(\boldsymbol{x}-\boldsymbol{z}), \; \boldsymbol{x} \in a, \tag{14}
$$

$$
\int_b \mathrm{d}^3y \; g_{-1}(\boldsymbol{x}-\boldsymbol{y};t) G(\boldsymbol{y}-\boldsymbol{z};t) = \frac{\partial}{\partial t} g_{-1}(\boldsymbol{x}-\boldsymbol{z};t), \; \boldsymbol{x} \in a, \tag{15}
$$

$$
\int_b \mathrm{d}^3z \; \frac{\partial}{\partial t} g_{-1}(\boldsymbol{x}-\boldsymbol{z};t) G(\boldsymbol{z}-\boldsymbol{y};t) = g(\boldsymbol{x}-\boldsymbol{y};t) + \frac{\partial^2}{\partial t^2} g_{-1}(\boldsymbol{x}-\boldsymbol{y};t), \; \boldsymbol{x} \in a. \tag{16}
$$

Let's focus back on Eq. (8). The δ function in Eq. (8) indicates that

$$\int \mathrm{d}^3y\; g(\boldsymbol{x}-\boldsymbol{y};t)\left[\varphi(\boldsymbol{y})-\varphi'(\boldsymbol{y})\right]=\int_B \mathrm{d}^3y\; G(\boldsymbol{x}-\boldsymbol{y};t)\left[\phi_B(\boldsymbol{y})-\phi'_B(\boldsymbol{y})\right],\ \boldsymbol{x}\in b. \tag{17}$$

Based on Eq. (14), Eq. (17), Eq. (15), and Eq. (13), in region a we can obtain

$$\begin{aligned}
&\varphi_a(\boldsymbol{x})-\varphi'_a(\boldsymbol{x})\\
&=\int \mathrm{d}^3y\;\delta(\boldsymbol{x}-\boldsymbol{y})\left[\varphi_a(\boldsymbol{y})-\varphi'_a(\boldsymbol{y})\right]\\
&=\int \mathrm{d}^3y\int_b \mathrm{d}^3z\; g_{-1}(\boldsymbol{x}-\boldsymbol{z};t)g(\boldsymbol{z}-\boldsymbol{y};t)\left[\varphi_a(\boldsymbol{y})-\varphi'_a(\boldsymbol{y})\right]\\
&=\int_B \mathrm{d}^3y\int_b \mathrm{d}^3z\; g_{-1}(\boldsymbol{x}-\boldsymbol{z};t)G(\boldsymbol{z}-\boldsymbol{y};t)\left[\phi_B(\boldsymbol{y})-\phi'_B(\boldsymbol{y})\right]\\
&=\int_B \mathrm{d}^3y\int_b \mathrm{d}^3z\; g_{-1}(\boldsymbol{x}-\boldsymbol{z};t)G(\boldsymbol{z}-\boldsymbol{y};t)\left[\phi_B(\boldsymbol{y})-\phi'_B(\boldsymbol{y})\right]\\
&=0\ , \qquad\qquad \boldsymbol{x}\in a,
\end{aligned} \tag{18}$$

where, the second equality employs Eq. (14), the third equality employs Eq. (17), the fourth equality employs Eq. (15), and the final equality employs Eq. (13). Then, according to Eq. (18), the δ function in (8) can be expressed as

$$\begin{aligned}
&\propto\prod_{\boldsymbol{x}\in a}\delta\Big(\varphi_a(\boldsymbol{x})-\varphi'_a(\boldsymbol{x})\Big)\\
&\quad\times\prod_{\boldsymbol{x}\in b}\delta\Bigg(\int_B \mathrm{d}^3y\; G(\boldsymbol{x}-\boldsymbol{y};t)\left[\phi_B(\boldsymbol{y})-\phi'_B(\boldsymbol{y})\right]\\
&\qquad\qquad -\int_A \mathrm{d}^3y\; g(\boldsymbol{x}-\boldsymbol{y};t)\left[\varphi_A(\boldsymbol{y})-\varphi'_A(\boldsymbol{y})\right]\Bigg)\ .
\end{aligned} \tag{19}$$

The proportionality coefficient in Eq. (19) is independent of φ_a and φ'_a. Substituting Eq. (19) into Eq. (8), we obtain

$$
\begin{aligned}
&\int \mathcal{D}\phi_b \, K(\phi, \phi', \varphi, \varphi'; t)\Big|_{\phi_b = \phi'_b} \\
&\propto \exp\left\{ i \int_a \mathrm{d}^3x \, \varphi_a(\boldsymbol{x}) \left[\int_A \mathrm{d}^3y \, G(\boldsymbol{x}-\boldsymbol{y};t) \left[\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{y})\right] \right.\right. \\
&\qquad\qquad \left.\left. - \int_B \mathrm{d}^3y \, g(\boldsymbol{x}-\boldsymbol{y};t) \left[\phi_B(\boldsymbol{y}) - \phi'_B(\boldsymbol{y})\right] \right] \right\} \\
&\times \exp\left\{ \frac{i}{2} \int_B \mathrm{d}^3x \int_B \mathrm{d}^3y \, G(\boldsymbol{x}-\boldsymbol{y};t) \Big[\phi_B(\boldsymbol{x})\phi_B(\boldsymbol{y}) - \phi'_B(\boldsymbol{x})\phi'_B(\boldsymbol{y})\Big] \right. \\
&\qquad + \frac{i}{2} \int_A \mathrm{d}^3x \int_A \mathrm{d}^3y \, G(\boldsymbol{x}-\boldsymbol{y};t) \Big[\varphi_A(\boldsymbol{x})\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{x})\varphi'_A(\boldsymbol{y})\Big] \\
&\qquad \left. - i \int_A \mathrm{d}^3x \int_B \mathrm{d}^3y \, g(\boldsymbol{x}-\boldsymbol{y};t) \Big[\varphi_A(\boldsymbol{x})\phi_B(\boldsymbol{y}) - \varphi'_A(\boldsymbol{x})\phi'_B(\boldsymbol{y})\Big] \right\} \\
&\times \prod_{\boldsymbol{x}\in b} \delta\left(\int_B \mathrm{d}^3y \, G(\boldsymbol{x}-\boldsymbol{y};t) \left[\phi_B(\boldsymbol{y}) - \phi'_B(\boldsymbol{y})\right] - \int_A \mathrm{d}^3y \, g(\boldsymbol{x}-\boldsymbol{y};t) \left[\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{y})\right] \right) \\
&\times \prod_{\boldsymbol{x}\in a} \delta\Big(\varphi_a(\boldsymbol{x}) - \varphi'_a(\boldsymbol{x})\Big) \, .
\end{aligned}
\tag{20}
$$

Next, we proceed to prove that the first two lines (the first exponential term) of Eq. (20) equals 1. Note that the first δ function in Eq. (20) implies

$$
\int_B \mathrm{d}^3y \, G(\boldsymbol{x}-\boldsymbol{y};t) \left[\phi_B(\boldsymbol{y}) - \phi'_B(\boldsymbol{y})\right] = \int_A \mathrm{d}^3y \, g(\boldsymbol{x}-\boldsymbol{y};t) \left[\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{y})\right], \ \boldsymbol{x} \in b. \tag{21}
$$

According to Eq. (14), Eq. (15), Eq. (13), and Eq. (21), we can derive the following expression for $\boldsymbol{x} \in a$:

$$
\begin{aligned}
&\int_A \mathrm{d}^3y \, G(\boldsymbol{x}-\boldsymbol{y};t) \Big[\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{y})\Big] \\
&= \int_A \mathrm{d}^3y \int \mathrm{d}^3s \, G(\boldsymbol{x}-\boldsymbol{s};t) \delta(\boldsymbol{s}-\boldsymbol{y}) \Big[\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{y})\Big] \\
&= \int \mathrm{d}^3z \int \mathrm{d}^3s \, G(\boldsymbol{x}-\boldsymbol{s};t) g_{-1}(\boldsymbol{s}-\boldsymbol{z};t) \int_A \mathrm{d}^3y \, g(\boldsymbol{z}-\boldsymbol{y};t) \Big[\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{y})\Big] \\
&= \int \mathrm{d}^3z \, \frac{\partial}{\partial t} g_{-1}(\boldsymbol{x}-\boldsymbol{z};t) \int_A \mathrm{d}^3y \, g(\boldsymbol{z}-\boldsymbol{y};t) \Big[\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{y})\Big] \\
&= \int_b \mathrm{d}^3z \, \frac{\partial}{\partial t} g_{-1}(\boldsymbol{x}-\boldsymbol{z};t) \int_A \mathrm{d}^3y \, g(\boldsymbol{z}-\boldsymbol{y};t) \Big[\varphi_A(\boldsymbol{y}) - \varphi'_A(\boldsymbol{y})\Big] \\
&= \int_b \mathrm{d}^3z \, \frac{\partial}{\partial t} g_{-1}(\boldsymbol{x}-\boldsymbol{z};t) \int_B \mathrm{d}^3y \, G(\boldsymbol{x}-\boldsymbol{y};t) \left[\phi_B(\boldsymbol{y}) - \phi'_B(\boldsymbol{y})\right],
\end{aligned}
\tag{22}
$$

where, the second equality uses Eq. (14), the third equality uses Eq. (15), the fourth equality uses Eq. (13), and the final equality uses Eq. (21). Furthermore, utilizing Eq. (16), Eq. (22) can

be written as

$$\begin{aligned}
&\int_A \mathrm{d}^3y\; G(\boldsymbol{x}-\boldsymbol{y};t)\Big[\varphi_A(\boldsymbol{y})-\varphi'_A(\boldsymbol{y})\Big] - \int_B \mathrm{d}^3y\; g(\boldsymbol{x}-\boldsymbol{y};t)\Big[\phi_B(\boldsymbol{y})-\phi'_B(\boldsymbol{y})\Big] \\
&= \int_B \mathrm{d}^3y\; \frac{\partial^2}{\partial t^2} g_{-1}(\boldsymbol{x}-\boldsymbol{y};t)\Big[\phi_B(\boldsymbol{y})-\phi'_B(\boldsymbol{y})\Big] \\
&= 0\ ,
\end{aligned} \tag{23}$$

where $\boldsymbol{x}\in a$ and the last equality utilizes Eq. (13). From Eq. (23), it follows that the first two lines (the first exponential term) of Eq.(20) equals 1. Therefore, utilizing Eq. (23) and Eq. (20), Eq. (7) can be expressed in the following form:

$$\int \mathcal{D}\phi_b\ \rho(\phi,\phi';t)\Big|_{\phi_b=\phi'_b} = \int \mathcal{D}\varphi_A\mathcal{D}\varphi'_A F(\phi_B,\phi'_B,\varphi_A,\varphi'_A;t)\int \mathcal{D}\varphi_a\ \rho(\varphi,\varphi')\Big|_{\varphi_a=\varphi'_a}. \tag{24}$$

This indicates that the reduced density matrix of region B at time t in Fig. 1 is only determined by the reduced density matrix of region A at time $t=0$, and (13) is the key factor leading to this causality.

If the quantum state $|\psi\rangle$ describes a group of particles (which may not be an eigenstate of the particle number) confined in region a at $t=0$, the quantum state in region A is indistinguishable from the vacuum $|\Omega\rangle$, i.e., $\mathrm{tr}_a\left(|\psi\rangle\langle\psi|\right) = \mathrm{tr}_a\left(|\Omega\rangle\langle\Omega|\right)$. According to the established causality, for $|\psi;t\rangle = \mathrm{e}^{-i\hat{H}t}|\psi\rangle$, we have $\mathrm{tr}_b\left(|\psi;t\rangle\langle\psi;t|\right) = \mathrm{tr}_b\left(|\Omega\rangle\langle\Omega|\right)$, which means that at time t the quantum state in region B remains indistinguishable from the vacuum state, indicating that the propagation speed of particles does not exceed the speed of light.

2.2 General Cases

Ref. [37] demonstrated that: For a quantum field theory that can be expressed in the form $\hat{H}=\int \mathrm{d}^3x\ \hat{h}(\boldsymbol{x},t)$, the reduced density matrix of spacelike-separated regions will not be affected by the action of a human, if "the action of a human on quantities defined at some point of coordinates x and t results only in changes $\Delta h(x,t)$ of the Hamiltonian density operator $h(x,t)$ defined at the same point" (refer to the third hypothesis in [37]). Ignoring the physical aspects and focusing solely on the mathematical expression, we can rephrase the mathematical conclusion in the aforementioned demonstration of Ref. [37] using the language of our paper as follows: If there exists a unitary operator $\hat{U}_a$ supported in region a such that $|\psi_2\rangle=\hat{U}_a|\psi_1\rangle$ and the relationship between regions a and b is as shown in Fig. 1, then we have $\mathrm{tr}_b\left(|\psi_1;t\rangle\langle\psi_1;t|\right) = \mathrm{tr}_b\left(|\psi_2;t\rangle\langle\psi_2;t|\right)$.

Next, we will prove that if two quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ satisfy $\mathrm{tr}_a\left(|\psi_1\rangle\langle\psi_1|\right) = \mathrm{tr}_a\left(|\psi_2\rangle\langle\psi_2|\right)$, then there must exist a unitary operator $\hat{U}_a$ supported in region a such that $|\psi_2\rangle=\hat{U}_a|\psi_1\rangle$.

Denoting the basis of the Hilbert space in regions A and a as $|A_n\rangle$ and $|a_m\rangle$ respectively, the quantum states $|\psi_2\rangle$ and $|\psi_1\rangle$ can be expressed as

$$|\psi_i\rangle = \sum_{m=1}^{M}\sum_{n=1}^{N} f_i(m,n)|a_m\rangle|A_n\rangle\ ,\quad i=1,2 \tag{25}$$

Let $\boldsymbol{f}_i$ denote an $M\times N$ matrix where $f_i(m,n)$ is the matrix element at the m-th row and n-th column, then the condition $\mathrm{tr}_a\left(|\psi_1\rangle\langle\psi_1|\right) = \mathrm{tr}_a\left(|\psi_2\rangle\langle\psi_2|\right)$ can be expressed as

$$\boldsymbol{f}_1^\dagger\boldsymbol{f}_1 = \boldsymbol{f}_2^\dagger\boldsymbol{f}_2\ . \tag{26}$$

Suppose the rank of the $\boldsymbol{f}_2$ is $k = M$. For the case where $N = M$, $\boldsymbol{f}_2$ has an inverse and then we have $\boldsymbol{f}_2 = (\boldsymbol{f}_2^\dagger)^{-1}\boldsymbol{f}_1^\dagger\boldsymbol{f}_1$ from Eq. (26). Consequently, a unitary matrix $\boldsymbol{U} \equiv (\boldsymbol{f}_2^\dagger)^{-1}\boldsymbol{f}_1^\dagger$ can be defined such that $\boldsymbol{f}_2 = \boldsymbol{U}\boldsymbol{f}_1$. For the case where $N > M$, it is possible to select M linearly independent column vectors from $\boldsymbol{f}_2$. Then we can form an $M \times M$ square matrix $\boldsymbol{F}_2$ using these M linearly independent vectors, and it is evident that $\boldsymbol{F}_2$ is an invertible matrix. Based on the positions of the M linearly independent vectors in $\boldsymbol{f}_2$, we can also correspondingly select column vectors from $\boldsymbol{f}_1$ to form another $M \times M$ square matrix $\boldsymbol{F}_1$. According to the definitions of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$, Eq. (26) immediately yields $\boldsymbol{F}_1^\dagger\boldsymbol{f}_1 = \boldsymbol{F}_2^\dagger\boldsymbol{f}_2$. Therefore, a unitary matrix $\boldsymbol{U} \equiv (\boldsymbol{F}_2^\dagger)^{-1}\boldsymbol{F}_1^\dagger$ can be defined such that $\boldsymbol{f}_2 = \boldsymbol{U}\boldsymbol{f}_1$.

Suppose the rank of the $\boldsymbol{f}_2$ is $k < M$. According to basic linear algebra knowledge, we know that $\mathrm{rank}\boldsymbol{f}_1 = \mathrm{rank}(\boldsymbol{f}_1^\dagger\boldsymbol{f}_1)$ and $\mathrm{rank}\boldsymbol{f}_2 = \mathrm{rank}(\boldsymbol{f}_2^\dagger\boldsymbol{f}_2)$. Thus, Eq. (26) indicates that the rank of $\boldsymbol{f}_1$ is $\mathrm{rank}\boldsymbol{f}_1 = \mathrm{rank}\boldsymbol{f}_2 = k$. It is possible to add $M - k$ normalized and mutually orthogonal column vectors to $\boldsymbol{f}_1$ ($\boldsymbol{f}_2$), such that these newly added vectors are orthogonal to the original ones in $\boldsymbol{f}_1$ ($\boldsymbol{f}_2$). After adding these vectors, $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ become extended matrices with rank M, denoted as $\boldsymbol{f}_1'$ and $\boldsymbol{f}_2'$. Due to the orthogonality and normalization of the new column vectors, as well as their orthogonality with the old column vectors, it is evident that (26) can be extended to $\boldsymbol{f}_1'^\dagger\boldsymbol{f}_1' = \boldsymbol{f}_2'^\dagger\boldsymbol{f}_2'$. Consequently, according to the proof in the preceding paragraph, there exists a unitary matrix $\boldsymbol{U}$ such that $\boldsymbol{f}_2' = \boldsymbol{U}\boldsymbol{f}_1'$. Focusing only on the transformation of the old column vectors, we immediately have $\boldsymbol{f}_2 = \boldsymbol{U}\boldsymbol{f}_1$.

In summary, we can always find a unitary matrix $\boldsymbol{U}$ such that $\boldsymbol{f}_2 = \boldsymbol{U}\boldsymbol{f}_1$. Let $U(m, m')$ be the matrix element of $\boldsymbol{U}$ at the m-th row and m'-th column. Using Eq. (25) and $\boldsymbol{f}_2 = \boldsymbol{U}\boldsymbol{f}_1$, we obtain

$$\begin{aligned}
|\psi_2\rangle &= \sum_{m=1}^{M}\sum_{n=1}^{N} f_2(m,n)|a_m\rangle|A_n\rangle \\
&= \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{m'=1}^{M} U(m,m')f_1(m',n)|a_m\rangle|A_n\rangle \\
&= \left(\sum_{m=1}^{M}\sum_{m'=1}^{M} U(m,m')|a_m\rangle\langle a_{m'}|\right)|\psi_1\rangle \ .
\end{aligned} \tag{27}$$

Eq. (27) indicates that there exist a unitary operator $\hat{U}_a$ supported in region a such that $|\psi_2\rangle = \hat{U}_a|\psi_1\rangle$. As mentioned at the beginning of this section, combining this with the results from Ref. [37], we can establish causality: In Fig. 1, if two quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ initially satisfy $\mathrm{tr}_a\left(|\psi_1\rangle\langle\psi_1|\right) = \mathrm{tr}_a\left(|\psi_2\rangle\langle\psi_2|\right)$, then after time t, the two quantum states satisfy $\mathrm{tr}_b\left(|\psi_1;t\rangle\langle\psi_1;t|\right) = \mathrm{tr}_b\left(|\psi_2;t\rangle\langle\psi_2;t|\right)$.

3 Physical Operations

Now, let's first analyze Witten's statement [36] quoted in Section 1: "If one asks about not mathematical operations in Hilbert space but physical operations that are possible in the real world, then the only physical way that one can modify a quantum state is by perturbing the Hamiltonian by which it evolves, thus bringing about a unitary transformation." Consider two

quantum states, where the state $|\psi_1\rangle$ only describes the system being operated on, while the other state $|\psi_2\rangle$ includes not only the same system but also an apparatus (or a human) in region a. At time $t = 0$, the apparatus in region a has not performed any operation on the system, so it is evident that the two quantum states are the same outside region a, specifically satisfying $\mathrm{tr}_a\left(|\psi_1\rangle\langle\psi_1|\right) = \mathrm{tr}_a\left(|\psi_2\rangle\langle\psi_2|\right)$. The apparatus complete the operation after Δt, and the two quantum states evolve into $|\psi_1;\Delta t\rangle$ and $|\psi_2;\Delta t\rangle$, respectively. Combining with Fig. 1 (where $t = \Delta t$), causality ensures that $\mathrm{tr}_b\left(|\psi_1;\Delta t\rangle\langle\psi_1;\Delta t|\right) = \mathrm{tr}_b\left(|\psi_2;\Delta t\rangle\langle\psi_2;\Delta t|\right)$. Based on the conclusion in Section 2.2, this indicate the existence of a unitary operator $\hat{U}_b$ supported in region b such that $|\psi_2;\Delta t\rangle = \hat{U}_b|\psi_1;\Delta t\rangle$. Assuming that $|\psi_1\rangle$ undergoes negligible changes during the short interval Δt (e.g., Witten chose $|\psi_1\rangle$ to be vacuum state in Ref. [36]), i.e., $|\psi_1;\Delta t\rangle \approx |\psi_1\rangle$, we then have $|\psi_2;\Delta t\rangle = \hat{U}_a|\psi_1\rangle$ in the ideal scenario ($\Delta t \to 0$, $a = b$), indicating that a physical operation can be equivalently represented by a unitary operator supported in the region where the apparatus is present. This way, we bypass Witten's "perturbing the Hamilton (or refer to the third hypothesis in [37])" and arrive at the final conclusion that physical operations are equivalent to unitary transformations.

It is worth noting that $|\psi_2;\Delta t\rangle$ includes not only the system that has undergone the operation but also the apparatus (or the human) performing the operation. This may differ from previous understandings. Yet, this is natural because isolating the apparatus (or the human) from $|\psi_2;\Delta t\rangle$ generally results in the operated system no longer being in a pure state due to the interaction between the apparatus (or the human) and the system.

4 Schrödinger's cat paradox and A Possible Interpretation of Quantum Mechanics

In Section 3, complex macroscopic systems like apparatuses and humans can be described by quantum states and follow the Schrödinger equation, which implicitly assumes the completeness of quantum mechanics. Although the many-worlds interpretation [35, 39] aligns perfectly with the completeness of quantum mechanics, it requires people to accept the bizarre "many-worlds scenario" that arises from Schrödinger's cat paradox. This section will argue that in quantum field theory, we cannot derive Schrödinger's cat paradox as we do in non-relativistic quantum mechanics, which implies that we can get an interpretation satisfying the completeness of quantum mechanics without the need for the "many-worlds scenario". For convenience, we will figuratively refer to this new interpretation as the "one-world interpretation". We also discuss the differences of this interpretation from traditional hidden-variable theories and its compatibility with relativity.

4.1 There is No Schrödinger's Cat Paradox in Quantum Field Theory

In quantum mechanics, there is a fundamental assumption often cited as additional "axiom (0)" [40, 41]: "The states of composite quantum systems are represented by a vector in the tensor product of the Hilbert spaces of its components." This leads to Schrödinger's cat paradox [42]. We can refer to the mathematical symbols used in Ref. [35] to demonstrate the derivation of the paradox. let D be a detector acting on the system S. Let the states $|s_i\rangle$ be eigenstates of some

observable and let the detector D measure that observable. Let $|d_i\rangle$ be the state of the detector D indicating that the system S is in the state $|s_i\rangle$; in other words, after measuring S when it is in the state $|s_i\rangle$, the detector D will be in the state $|d_i\rangle$. Let the initial state of the detector be $|d_0\rangle$. Then, according to "axiom (0)", the composite system of the system S and the detector D can be written as $|s_i\rangle|d_0\rangle$, and the measurement process can be expressed as

$$|s_i\rangle|d_0\rangle \to |s_i\rangle|d_i\rangle \ . \tag{28}$$

In fact, the final state does not need to be a product state, meaning we can represent $|s_i\rangle|d_i\rangle$ as $|s_i, d_i\rangle$. However, this does not affect the subsequent derivation concerning Schrödinger's cat paradox.

In general, the initial state of the system S is a superposition $|s\rangle = \sum_i c_i|s_i\rangle$, then unitary time evolution during the measurement takes this initial state to

$$|s\rangle|d_0\rangle = \sum_i c_i|s_i\rangle|d_0\rangle \to \sum_i c_i|s_i\rangle|d_i\rangle \ . \tag{29}$$

The final state is a superposition of different measurement outcomes, unable to yield a unique classical measurement result. In the Schrödinger's cat experiment, such a state is akin to being both dead and alive simultaneously. One might argue that the above derivation does not consider environmental factors; however, if we let the detector D include the state of the environment, making the entire composite system isolated, the same paradox will still arise. Various interpretations of quantum mechanics are presented in Ref. [35] to resolve this paradox, including the well-known interpretation proposed by Wigner, where collapse occurs whenever a conscious human being observes a detector in a superposed state. Here we specifically focus on the many-worlds interpretation, which holds that each measurement outcome $|s_i\rangle|d_i\rangle$ displayed in Eq. (29) is real, with different measurement outcomes corresponding to different worlds.

The above derivation of the cat paradox is based on non-relativistic quantum mechanics. However, in quantum field theory, the system S and the detector D are composed of the same fundamental fields. Therefore strictly speaking, we cannot fully distinguish between the system S and the detector D. Even if we consider quantum fields at different spatial points as different field operators, the interactions between fields at adjacent points lead to strong entanglement between different points in the quantum state. Besides, the ubiquitous and constant interactions among fields cause field mixing, making it impossible to disentangle different types of fields [43]. Even the vacuum state, as fundamental and simple as it is, cannot be expressed as the direct product of "vacuum states" of different regions, nor can it be expressed as the direct product of "vacuum states" of different types of fields (bare vacuum). Especially when the system S and the detector D are in the same spatial region during interactions, it becomes more challenging to distinguish them, rendering the use of axiom (0) inappropriate. Therefore, in the framework of quantum field theory, the initial state of the "composite" system formed by the detector D and the system being measured S cannot be written in the form of $|s\rangle|d_0\rangle$.

Although we cannot strictly distinguish the detector D from the system being measured S at the level of quantum states, we can distinguish them in space using reduced density matrices. The reduced density matrices characterize the information in a particular region. When the system S

and the detector D do not overlap spatially, we can describe their respective states using reduced density matrices. Let the quantum state $|s_1, d_0\rangle$ represent the system S in state s_1 and the detector D in state d_0, and let $|s_2, d_0\rangle$ represent the system S in state s_2 and the detector D in state d_0. Suppose the detector D is located in region A while the system S is in region a. Since the detectors of both quantum states $|s_1, d_0\rangle$ and $|s_2, d_0\rangle$ are in state d_0, we have

$$\mathrm{tr}_a\left(|s_1, d_0\rangle\langle s_1, d_0|\right) = \mathrm{tr}_a\left(|s_2, d_0\rangle\langle s_2, d_0|\right) \ . \tag{30}$$

According to the derivation in Section 2.2, it is known that there exists a unitary operator $\hat{U}_a$ supported in region a, such that $|s_2, d_0\rangle = \hat{U}_a|s_1, d_0\rangle$. Next, we examine the properties of the quantum state $|\psi_3\rangle \equiv c_1|s_1, d_0\rangle + c_2|s_2, d_0\rangle$ in region A where the detector is located. Based on the relationship between $|s_1, d_0\rangle$ and $|s_2, d_0\rangle$, it is easy to obtain the relationship between $|\psi_3\rangle$ and $|s_1, d_0\rangle$ as $|\psi_3\rangle = \left(c_1 + c_2\hat{U}_a\right)|s_1, d_0\rangle$. Although the operator $c_1 + c_2\hat{U}_a$ is supported in region a, it is generally not unitary. Therefore, $|\psi_3\rangle$ may exhibit different properties in region A compared to $|s_1, d_0\rangle$. This implies that the detector D of $c_1|s_1, d_0\rangle + c_2|s_2, d_0\rangle$ may not necessarily be in the state d_0 if we still believe that the detector remains in region A. Unless we adopt axiom (0) to describe quantum states, in which case the quantum state $c_1|s_1\rangle|d_0\rangle + c_2|s_2\rangle|d_0\rangle = (c_1|s_1\rangle + c_2|s_2\rangle)|d_0\rangle$ explicitly indicates that the detector D is necessarily in the state d_0.

Regarding the above discussion, Ref. [44] provides a specific example. For a real scalar field theory, the quantum state corresponding to the classical field $\phi_{\text{class}}(\boldsymbol{x})$ and $\pi_{\text{class}}(\boldsymbol{x})$ is

$$\begin{aligned}\langle\phi|\varphi\rangle = \mathcal{N}' \exp\left\{-\frac{1}{2}\int d^3x d^3y\ \mathcal{E}(\boldsymbol{x},\boldsymbol{y})\left[\phi(\boldsymbol{x}) - \phi_{\text{class}}(\boldsymbol{x})\right]\left[\phi(\boldsymbol{y}) - \phi_{\text{class}}(\boldsymbol{y})\right]\right\} \\ \times \exp\left\{\frac{i}{\hbar}\int d^3x\ \pi_{\text{class}}(\boldsymbol{x})\phi(\boldsymbol{x})\right\}\ ,\end{aligned} \tag{31}$$

where $\mathcal{N}'$ is the normalization coefficient, and $\mathcal{E}(\boldsymbol{x},\boldsymbol{y})$ is defined by

$$\mathcal{E}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{\hbar^5}\int \frac{\mathrm{d}^3p}{(2\pi)^3} e^{\frac{i}{\hbar}\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})} E_{\boldsymbol{p}}\ . \tag{32}$$

Dividing the entire space into two regions, denoted as A and a, if the classical field in region A is in the classical vacuum state (i.e., $\phi_{\text{class}}(\boldsymbol{x}_A) = \pi_{\text{class}}(\boldsymbol{x}_A) = 0$ for $\boldsymbol{x}_A \in A$), then the quantum state $|\varphi\rangle$ corresponding to this classical field is equivalent to the vacuum state $|\Omega\rangle$ in region A (i.e., $\mathrm{tr}_a\left(|\varphi\rangle\langle\varphi|\right) = \mathrm{tr}_a\left(|\Omega\rangle\langle\Omega|\right)$). Let $\phi_{\text{class}}(\boldsymbol{x}_a) = \phi_a(\boldsymbol{x}_a), \pi_{\text{class}}(\boldsymbol{x}_a) = \pi_a(\boldsymbol{x}_a)$ for $\boldsymbol{x}_a \in a$, then we can figuratively denote $|\varphi\rangle$ as $|(\phi_a, \pi_a), \Omega_A\rangle$, representing a quantum state that describes the classical field (ϕ_a, π_a) in region a and is equivalent to the vacuum in region A. Similarly, according to Eq. (31), we can easily write another quantum state $|(\phi'_a, \pi'_a), \Omega_A\rangle$, which is also equivalent to the vacuum in region A. Contrary to intuition, even if $|(\phi_a, \pi_a), \Omega_A\rangle$ and $|(\phi'_a, \pi'_a), \Omega_A\rangle$ are equivalent to the vacuum state $|\Omega\rangle$ in region A, their superposition state $\frac{1}{\sqrt{2}}\Big(|(\phi_a, \pi_a), \Omega_A\rangle + |(\phi'_a, \pi'_a), \Omega_A\rangle\Big)$ is no longer equivalent to the vacuum state $|\Omega\rangle$ in region A. Therefore, when two quantum states with the same reduced density matrix in a certain region are superposed, due to spatial entanglement, the superposition state no longer retains the original reduced density matrix in that region. This also indicates that while the spatial entanglement of quantum states does not undermine the consistency of the correspondence between quantum state and classical fields, it does lead to other counterintuitive conclusions.

Consequently, in quantum field theory, we have $|s, d_0\rangle \neq \sum_i c_i |s_i, d_0\rangle$. As a result, even if we can write (28) as $|s_i, d_0\rangle \to |s_i, d_i\rangle$, we cannot derive a formula similar to (29) (i.e., $|s, d_0\rangle = \sum_i c_i |s_i, d_0\rangle \to \sum_i c_i |s_i, d_i\rangle$). Instead, we have

$$|s, d_0\rangle \nrightarrow \sum_i c_i |s_i, d_i\rangle \ . \tag{33}$$

This indicates that Schrödinger's cat paradox cannot be derived in quantum field theory.

4.2 One-world Interpretation

To clarify the discussion, we need to provide a more detailed explanation of "macroscopic state". A macroscopic state refers to a state described using classical concepts, such as a dead cat, a live cat, classical fields, a detector displaying measurement outcomes, and so on. Since classical descriptions are generally more ambiguous compared to quantum descriptions, a macroscopic state often corresponds to many quantum states. For example, many quantum states can describe "a dead cat". Even detectors displaying measurement outcomes correspond to multiple quantum states. This is because quantum states encompass not only specific displayed measurement readings but also factors like the average temperature of the detector, ubiquitous phonons in the detector, and so forth. More precisely, quantum states include the states of each atomic and molecular constituent composing the macroscopic detector. Of course, there are also instances where each "macroscopic state" corresponds to a single quantum state. For example, in free scalar field theory, the classical vacuum ($\phi = \pi = 0$) corresponds uniquely to the vacuum state $|\Omega\rangle$ in quantum field theory.

As demonstrated in the previous subsection, there is no Schrödinger's cat paradox in quantum field theory. Therefore, a crucial possibility arises: After the measurement process, the entire composite system evolves into a definite macroscopic state, rather than a superposition of macroscopic states. In other words, in the Schrödinger's cat experiment, the cat is either alive or dead, and not in a superposition of being both alive and dead. We don't need (consciousness) to observe a detector to collapse it into a specific macroscopic state; instead, the detector automatically evolves to a definite macroscopic state, mirroring the spirit of the measurement in classical mechanics. This naturally leads to a new interpretation of quantum mechanics: quantum mechanics is complete, and the time evolution of quantum states can describe measurement processes, yielding a unique measurement outcome for each measurement. This new interpretation and the many-worlds interpretation both consider quantum mechanics to be complete [35]. However, the difference lies in the fact that this new interpretation argue that there aren't "many worlds" but rather there is only "one world". Therefore, we may figuratively refer to this new interpretation as the "one-world interpretation". It is worth noting that this interpretation does not oppose the existence of superposition states of macroscopic states that are vastly different, such as a quantum superposed cat that is dead and alive at the same time. However, this interpretation suggests that it is not possible to prepare a quantum superposed cat that is dead and alive by following the steps of the Schrödinger's cat experiment.

A natural follow-up question arises: where does the probability in quantum mechanics come from? Looking back, the reason why probability was used to describe experimental results in

the early days of quantum mechanics was that experiments with the same initial setup yielded different results. Only by statistically analyzing the results of multiple experiments could patterns emerge. In fact, however, the initial state of each experiment is different. The initial setup includes the quantum state of the system being measured and the state of the detector (including the measuring device and the environment), but the state of the detector is a "macroscopic state", and a "macroscopic state" can correspond to multiple quantum states. Because we cannot control that every atom and molecule in the macroscopic detector remains unchanged, the initial quantum state of the entire system is not the same in each repeated experiment, which leads to the randomness of measurement results. Just like tossing a coin, although the entire process follows classical mechanics, the initial state varies each time, resulting in different outcomes for the coin's orientation. Ref. [45] formulates a mechanism for how the first droplet in a cloud chamber track arises, making no reference to quantum measurement axioms. This provides a specific example illustrating that randomness comes from initial conditions. The physics of a Geiger counter and a cloud chamber have a lot in common, then similar analysis can be applied to a Geiger counter as well [46].

If we toss a coin multiple times and statistically analyze the results, we can obtain a stable probability distribution, from which we can extract some information about the coin. For example, if the probability of landing heads up is not 1/2, it indicates that the coin may be asymmetric. In quantum mechanics, measurements follow a similar principle. Through multiple repetitions of experiments and statistical analysis of the results, the randomness of the initial state can be averaged out, revealing information that remains constant across each experiment. Since the state of the system being measured remains the same in each experiment, the randomness inherent in the detector itself is averaged out, thereby revealing information about the system being measured.

Regarding the measurement process, a more specific but qualitative description is as follows. When the system being measured interacts with the detector, the composite system is in a highly unstable state. Subsequently, the entire system rapidly evolves into a stable macroscopic state, namely the state where the detector displays a specific reading. If the system being measured is in an eigenstate of the observable before the measurement, then the stable macroscopic state of the composite system formed by the system and the detector is unique, and the composite system naturally evolves to a definite outcome. If the system being measured is in a superposition of eigenstates of the observable before the measurement, then there are multiple stable state of the composite system. Because the composite system is in a highly unstable state during the measurement process, even tiny changes in the initial state lead to the composite system evolving to different stable states, leading to various possible measurement outcomes. Therefore, the determination of which outcome the composite system evolves to depends on the precise initial quantum state of the composite system.

The mechanism for the formation of the first droplet in the Cloud Chamber, as presented in Ref. [45], serves as an example illustrating the measurement process described above. Specifically, when the decay product appear in the Cloud Chamber, the composite system is in a highly unstable state. At some time and in some location, a cluster of fortuitous size appears (as a result of thermal fuctuations) with an enormous ionization cross section, then the decay product wavefunction quickly becomes collimated at the location of the cluster, and the system rapidly evolves into a stable state—the state where the first droplet forms.

Finally, it is worth mentioning that in the one-world interpretation, there is no wave function collapse. Wave function collapse arises from treating the system under measurement as an independent entity from the detector at the quantum level, and describing the system being measured with a separate quantum state (wave function). For instance, in (29), if we describe the system being measured as $|s\rangle$ and it evolves into $|s_i\rangle$ after the measurement process, we would say that the system has collapsed from $|s\rangle$ to $|s_i\rangle$. However, in quantum field theory, since all matter in the world is composed of the same fundamental fields, and there are interactions between the detector and the system being measured, the fields that make up the system being measured must also be part of the detector's composition according to the field mixing effect [43]. Therefore, the system being measured cannot be described by a separate quantum state as in the non-relativistic case, especially when the measured system and the detector merge into one entity during their interaction (overlapping in space). Consequently, it is evident that in the one-world interpretation, there is no concept of wave function collapse.

4.3 Violation of Statistical Independence and Compatibility with Relativity

As demonstrated in the previous subsection, it can be seen that the one-world interpretation is actually a hidden-variable theory, where the hidden variables are the initial quantum states of the entire systems. This naturally raises the question: Will this interpretation be ruled out by experiments [47] related to Bell inequalities? Recalling the key formula used by Bell in deriving Bell's inequalities [48]:

$$P(\boldsymbol{a},\boldsymbol{b}) = \int \mathrm{d}\lambda\ \rho(\lambda) A(\boldsymbol{a},\lambda) B(\boldsymbol{b},\lambda)\ , \tag{34}$$

where the probability distribution $\rho(\lambda)$ of the hidden variable λ is independent of the macroscopic states of the detectors, i.e. $\boldsymbol{a}$ and $\boldsymbol{b}$ (specifically $\boldsymbol{a}$ and $\boldsymbol{b}$ are polarizer settings). In the one-world interpretation, the hidden variables λ might superficially seem to correspond to the initial states of the detectors, however due to the failure of axiom (0), we cannot consider the two detectors separately, nor can we separate the detectors from the system being measured. Therefore, the hidden variables are actually the initial quantum state of the entire system $|\psi\rangle$. However, the quantum state $|\psi\rangle$ needs to satisfy certain constraints. In addition to ensuring that the initial states of the system being measured are consistent (i.e., their reduced density matrices of the system being measured are the same), it is also required that the corresponding macroscopic states of the detectors be $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively. This constraint indicates that the range of values of the hidden variable λ is not the same for different macroscopic states $\boldsymbol{a}$ and $\boldsymbol{b}$ of the detectors. Consequently, the distribution function $\rho(\lambda)$ is actually dependent on the macroscopic states $\boldsymbol{a}$ and $\boldsymbol{b}$, and should be denoted as $\rho(\lambda|\boldsymbol{a},\boldsymbol{b})$, which differs from Bell's assumption. In fact, the assumption that the hidden variables do not in any way depend on measurement settings, i.e. $\rho(\lambda|\boldsymbol{a},\boldsymbol{b}) = \rho(\lambda)$, is commonly known as "Statistical Independence" [49, 50]. Therefore, although the one-world interpretation is a hidden variable theory, it violate Statistical Independence and does not lead to Bell's inequalities as in Ref. [48], and as a result, it will not be ruled out by experiments related to Bell's inequality.

In addition, the one-world interpretation does not belong to the category of "nonlocal hidden-variable theory" as defined by Leggett in Ref. [51], because the nonlocal hidden-variable theory defined by Leggett satisfies Statistical Independence. Therefore, the one-world interpretation is not excluded by experiments [52] violating the inequality proposed by Leggett [51].

In Section 3, it was proved that a physical operation must be equivalent to a local unitary operator. In the Sorkin-type impossible measurements problem, such a unitary operation is also referred to as a "kick". Studying the effect of a kick on measurements that are spacelike separated is central to the Fermi two-atom problem and the Sorkin-type impossible measurements problem. In the Fermi two-atom problem, there is only one measurement, and there are no other measurements between the kick and this measurement. According to the derivation in Section 2, it is evident that the kick cannot affect the measurement result if the measurement is space-like separated from the kick.

In the Sorkin-type impossible measurements problem, there are additional measurement processes between the kick and the spacelike separated measurement. As mentioned in Section 1, traditional quantum mechanics is governed by two laws: the time evolution of quantum states and measurement theory. According to the traditionally accepted measurement theory, if there are additional measurement processes between the kick and the spacelike separated measurement, the kick has the ability to influence the results of the spacelike separated measurement, which violates relativistic causality. However, within the framework of the one-world interpretation and in combination with the causality demonstrated in Section 2, this troublesome Sorkin-type impossible measurements problem can be resolved. The one-world interpretation holds that quantum mechanics is complete, and thus, the underlying physical laws behind the measurement theory corresponding to this interpretation are still the time evolution of quantum states. Therefore, regardless of whether there are additional measurements between the kick and the spacelike separated measurement, the entire system can always be described using the evolving quantum states over time, where "state update" also falls under the time evolution of quantum states. Consequently, according to the causality demonstrated in Section 2, the kick cannot influence measurements spacelike separated from it. Furthermore in the one-world interpretation, each measurement process has a definite final outcome, and the quantum states encompass all the results of measurements, including those of intermediate measurements. Each measurement result can be read from the reduced density matrix of the region where the measurement occurs. In this sense, the one-world interpretation is a deterministic theory, but surprisingly it neither violates causality nor undermines the completeness of quantum mechanics.

5 Conclusion and Outlook

We use the method of reduced density matrices to represent the local information of quantum states, thereby providing a general definition of causality in quantum field theory. We illustrate the time evolution of reduced density matrices in detail using the example of a free real scalar field theory, providing a very specific demonstration of causality. As for the proof of causality in general local quantum field theory, it is derived based on the mathematical conclusions of Ref. [37] and further elaborated upon. It should be emphasized that the third hypothesis in Ref. [37] is

precisely the statement about physical operations quoted from Witten in Section 1: "the only physical way that one can modify a quantum state is by perturbing the Hamiltonian by which it evolves." However, the derivation in Section 2.2 did not adopt this physical viewpoint, but only used the mathematical conclusions it leads to in Ref. [37]. It is only after we fully derive causality that we analyze Witten's statement about physical operations. Our analysis indicates that we can bypass "perturbing the Hamiltonian" and arrive at the final conclusion that physical operations are equivalent to unitary transformations.

On the other hand, the idea that all matter is composed of the same fundamental fields also suggests that the initial state of the measurement process cannot be described by a product state. Furthermore, when two quantum states with the same reduced density matrix in a certain region are superposed, due to spatial entanglement, the superposition state no longer retains the original reduced density matrix in that region. Therefore, there is no Schrödinger's cat paradox in quantum field theory, giving rise to a new interpretation. This interpretation considers quantum mechanics to be complete (like the many-worlds interpretation) but does not need to introduce multiple worlds to deal with the cat paradox. Instead, the new interpretation proposes that there is only one world, which we figuratively refer to as the "one-world interpretation".

In the one-world interpretation, measurement outcomes are deterministic, and they are determined by the initial state of the entire system. However, we cannot maintain the exact same state of every atom and molecule in the detector (including the environment) in each repetition of the experiment, leading to the uncertainty of experimental results. Clearly, this interpretation is a kind of hidden variable theory. However, as explained in Section 4, because it violate Statistical Independence (for the meaning of "Statistical Independence", see Ref. [49, 50]), it does not fall under Bell's local hidden variable theory [48] or Leggett's definition of "nonlocal hidden-variable theory" [51]. Therefore, it will not be ruled out by experiments related to Bell's inequality and Leggett's inequality. In fact, the one-world interpretation closely resembles a superdeterministic theory [50]; however, unlike traditional superdeterministic approaches, it maintains that quantum mechanics is a complete theory. Combining the causality defined in this paper, it can be concluded that the one-world interpretation is compatible with special relativity, and situations involving superluminal transmission of information do not arise, nor does the Sorkin-type impossible measurements problem.

While our work provides a framework to harmoniously integrate relativistic causality, quantum non-locality, and quantum measurement, we still lack a quantitative description of the one-world interpretation. Most crucially, we are uncertain about the correspondence between quantum states and macroscopic states, where "macroscopic states" refer to states described using classical concepts, as introduced in Section 4.2. For some simple cases, we can find correspondences between quantum states and macroscopic states [44]. However, it is often challenging to find the corresponding quantum state for general complex macroscopic states. For instance, it is difficult to determine the quantum state describing a cat.

In addition to understanding the correspondence between macroscopic states and quantum states, there are many fundamental questions about the measurement process that require further investigation. For example, how to write down the entire initial quantum state of a composite system, or how to quantitatively demonstrate that microscopic changes in the initial state can be amplified into macroscopic changes during the measurement process. Another question is how to

quantitatively demonstrate that the final quantum state can evolve into a definite macroscopic state rather than a superposition state. Perhaps a method worth considering is to use a bound state containing a large number of atoms to simulate the detector in quantum field theory. However, while we aspire for the detector model to be entirely quantum mechanical, semi-classical detector models such as those in Ref. [45] and Ref. [46] can also address some of these questions and provide insights for further research. Moreover, since we have already demonstrated that there is no Schrödinger's cat paradox in quantum field theory, these semi-classical detector models may suffice to adequately describe the measurement process. In addition to the aforementioned issues regarding interpretations of quantum mechanics, our work also contributes to a fundamental understanding of the Sorkin-type impossible measurements problem. This may help us develop a universally applicable measurement theory that is compatible with relativity.

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A The derivation of the free field propagator

The propagator for a quantum harmonic oscillator, denoted as $\langle q|\mathrm{e}^{-i\hat{H}_o t}|q_1\rangle$, takes the form $\mathrm{e}^{iS(q,q_1;t)}$, where $S(q,q_1;t)$ is the extreme of the action under fixed path-boundaries (i.e., for fixed $q(0)=q$ and $q(t)=q_1$). The free field theory described by the Hamiltonian $\hat{H}=\int \mathrm{d}^3x\,\left[\frac{1}{2}\hat{\pi}^2(\boldsymbol{x})+\frac{1}{2}\Big(\nabla\hat{\phi}(\boldsymbol{x})\Big)^2+\frac{1}{2}m^2\hat{\phi}^2(\boldsymbol{x})\right]$ can be regarded as a collection of many harmonic oscillators, therefore we can similarly guess that the field propagator$\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle$ also takes the same form:

$$\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle = N(t)\mathrm{e}^{iS(\phi,\phi_1;t)}\ , \tag{35}$$

where $S(\phi,\phi_1;t)$ is the extremum of the action evaluated for fixed initial state ϕ_1 and final state ϕ, and $N(t)$ is independent of ϕ and ϕ_1. In fact, substituting the solutions satisfying the Euler-Lagrange equations into $S=\int \mathrm{d}^4x\ \mathcal{L}(\phi(x),\dot{\phi}(x))$ yields

$$\begin{aligned} S(\phi,\phi_1;t) = &\frac{1}{2}\int \mathrm{d}^3x\mathrm{d}^3y\ G(\boldsymbol{x}-\boldsymbol{y};t)\Big[\phi(\boldsymbol{x})\phi(\boldsymbol{y})+\phi_1(\boldsymbol{x})\phi_1(\boldsymbol{y})\Big] \\ &-\int \mathrm{d}^3x\mathrm{d}^3y\ g(\boldsymbol{x}-\boldsymbol{y};t)\phi(\boldsymbol{x})\phi_1(\boldsymbol{y})\ , \end{aligned} \tag{36}$$

where

$$\begin{aligned} G(\boldsymbol{x}-\boldsymbol{y};t) &= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^0\cos(p^0t)}{\sin(p^0t)}\mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\ , \\ g(\boldsymbol{x}-\boldsymbol{y};t) &= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^0}{\sin(p^0t)}\mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}\ . \end{aligned} \tag{37}$$

Eq. (35) is just our conjecture regarding the field propagator $\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle$. We need to verify that it reduces to $\prod_x \delta(\phi(x)-\phi_1(x))$ at $t=0$, and satisfies the Schrödinger equation for any t:

$$i\frac{\partial}{\partial t}\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle = \hat{H}\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \ . \tag{38}$$

Note that in Eq. (38), the first $\hat{H}$ on the right-hand side actually represents the Hamiltonian in the representation defined by the eigenstates of the field operator $\hat{\phi}(x)$. Since the entire derivation involves only one representation, writing it this way does not introduce ambiguity. In the representation defined by the eigenstates of the field operator $\hat{\phi}(x)$, we have $\hat{\phi}(x)=\phi(x)$ and $\hat{\pi}(x)=-i\frac{\delta}{\delta\phi(x)}$.

To handle the Schrödinger equation (38), we need to compute $\int \mathrm{d}^3x\ \hat{\pi}^2(x)\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle$. Next, let's proceed to compute it. Firstly, according to Eq. (37), it is straightforward to calculate the specific expressions for $\frac{\partial}{\partial t}G(\boldsymbol{x}-\boldsymbol{y};t)$ and $\frac{\partial}{\partial t}g(\boldsymbol{x}-\boldsymbol{y};t)$:

$$\begin{aligned}
\frac{\partial}{\partial t}G(\boldsymbol{x}-\boldsymbol{y};t) &= -\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{(p^0)^2}{\sin^2(p^0t)}\mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})} \ , \\
\frac{\partial}{\partial t}g(\boldsymbol{x}-\boldsymbol{y};t) &= -\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{(p^0)^2\cos(p^0t)}{\sin^2(p^0t)}\mathrm{e}^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})} \ .
\end{aligned} \tag{39}$$

Secondly, utilizing Eq. (39), we can obtain the following formulas:

$$\begin{aligned}
\int \mathrm{d}^3x\ G(\boldsymbol{x}-\boldsymbol{y};t)G(\boldsymbol{x}-\boldsymbol{z};t) &= -\frac{\partial}{\partial t}G(\boldsymbol{z}-\boldsymbol{y};t)+\nabla_z^2\delta^3(\boldsymbol{z}-\boldsymbol{y})-m^2\delta^3(\boldsymbol{z}-\boldsymbol{y}), \\
\int \mathrm{d}^3x\ G(\boldsymbol{x}-\boldsymbol{y};t)g(\boldsymbol{x}-\boldsymbol{z};t) &= -\frac{\partial}{\partial t}g(\boldsymbol{z}-\boldsymbol{y};t) \ , \\
\int \mathrm{d}^3x\ g(\boldsymbol{x}-\boldsymbol{y};t)g(\boldsymbol{x}-\boldsymbol{z};t) &= -\frac{\partial}{\partial t}G(\boldsymbol{z}-\boldsymbol{y};t) \ .
\end{aligned} \tag{40}$$

Finally, utilizing (40), the expression for $\int \mathrm{d}^3x\ \hat{\pi}^2(x)\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle$ can be obtained as

$$
\begin{aligned}
&\int \mathrm{d}^3x\ \hat{\pi}^2(x)\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\
&= -i\int \mathrm{d}^3x\ G(\mathbf{0};t)\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\
&\quad + \Bigg[\int \mathrm{d}^3y\mathrm{d}^3z\ \phi(\boldsymbol{y})\phi(\boldsymbol{z})\int \mathrm{d}^3x\ G(\boldsymbol{x}-\boldsymbol{y};t)G(\boldsymbol{x}-\boldsymbol{z};t) \\
&\qquad + \int \mathrm{d}^3y\mathrm{d}^3z\ \phi_1(\boldsymbol{y})\phi_1(\boldsymbol{z})\int \mathrm{d}^3x\ g(\boldsymbol{x}-\boldsymbol{y};t)g(\boldsymbol{x}-\boldsymbol{z};t) \\
&\qquad - \int \mathrm{d}^3y\mathrm{d}^3z\ \phi_1(\boldsymbol{y})\phi(\boldsymbol{z})\int \mathrm{d}^3x\ g(\boldsymbol{x}-\boldsymbol{y};t)G(\boldsymbol{x}-\boldsymbol{z};t) \\
&\qquad - \int \mathrm{d}^3y\mathrm{d}^3z\ \phi(\boldsymbol{y})\phi_1(\boldsymbol{z})\int \mathrm{d}^3x\ G(\boldsymbol{x}-\boldsymbol{y};t)g(\boldsymbol{x}-\boldsymbol{z};t)\Bigg]\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\
&= -i\int \mathrm{d}^3x\ G(\mathbf{0};t)\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\
&\quad - \left[\int \mathrm{d}^3x\ \left(\nabla\phi(x)\right)^2 + m^2\int \mathrm{d}^3x\ \phi^2(x)\right]\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\
&\quad - \frac{\partial}{\partial t}\Bigg[-2\int \mathrm{d}^3x\mathrm{d}^3y\ g(\boldsymbol{x}-\boldsymbol{y};t)\phi(\boldsymbol{x})\phi_1(\boldsymbol{y}) \\
&\qquad + \int \mathrm{d}^3x\mathrm{d}^3y\ G(\boldsymbol{x}-\boldsymbol{y};t)\Big[\phi(\boldsymbol{x})\phi(\boldsymbol{y})+\phi_1(\boldsymbol{x})\phi_1(\boldsymbol{y})\Big]\Bigg]\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle\ .
\end{aligned}
\tag{41}
$$

With Eq. (41), we immediately know that the expression on the right-hand side of the Schrödinger equation (38) is given by

$$
\begin{aligned}
&\hat{H}\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\
&= -i\frac{1}{2}\int \mathrm{d}^3x\ G(\mathbf{0};t)\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\
&\quad - \frac{1}{2}\frac{\partial}{\partial t}\Bigg[-2\int \mathrm{d}^3x\mathrm{d}^3y\ g(\boldsymbol{x}-\boldsymbol{y};t)\phi(\boldsymbol{x})\phi_1(\boldsymbol{y}) \\
&\qquad + \int \mathrm{d}^3x\mathrm{d}^3y\ G(\boldsymbol{x}-\boldsymbol{y};t)\Big[\phi(\boldsymbol{x})\phi(\boldsymbol{y})+\phi_1(\boldsymbol{x})\phi_1(\boldsymbol{y})\Big]\Bigg]\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle\ .
\end{aligned}
\tag{42}
$$

The left-hand side of the Schrödinger equation (38) can be directly computed as

$$\begin{aligned} & i\frac{\partial}{\partial t}\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\ =& i\frac{1}{N(t)}\frac{dN(t)}{dt}\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\ & -\frac{1}{2}\frac{\partial}{\partial t}\Bigg[-2\int \mathrm{d}^3x\mathrm{d}^3y\; g(\boldsymbol{x}-\boldsymbol{y};t)\phi(\boldsymbol{x})\phi_1(\boldsymbol{y}) \\ & \qquad +\int \mathrm{d}^3x\mathrm{d}^3y\; G(\boldsymbol{x}-\boldsymbol{y};t)\Big[\phi(\boldsymbol{x})\phi(\boldsymbol{y})+\phi_1(\boldsymbol{x})\phi_1(\boldsymbol{y})\Big]\Bigg]\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle\ . \end{aligned} \tag{43}$$

Substituting the specific expressions from Eq. (42) and Eq. (43) into the Schrödinger equation (38), we obtain an equation involving $N(t)$:

$$\frac{d}{dt}\ln N(t) = -\frac{1}{2}\int \mathrm{d}^3x\; G(\mathbf{0};t)\ . \tag{44}$$

This is consistent with the earlier assumption that $N(t)$ is independent of ϕ and ϕ_1, indicating that our previous conjecture Eq. (35) indeed satisfies the Schrödinger equation, and the specific expression for $N(t)$ is

$$N(t) = \mathcal{N}\mathrm{e}^{-\frac{1}{2}\int \mathrm{d}^3x\int dt\; G(\mathbf{0};t)}\ . \tag{45}$$

However, merely demonstrating that (35) satisfies the Schrödinger equation as a solution is not sufficient. We also need to examine the behavior of $\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle$ as $t\to 0$. Utilizing (45) and (36), we can express (35) in a more specific form:

$$\begin{aligned} & \langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\ & = \mathcal{N}\mathrm{e}^{-\frac{1}{2}\int \mathrm{d}^3x\int dt\; G(\mathbf{0};t)}\exp\Bigg\{-i\int \mathrm{d}^3x\mathrm{d}^3y\; g(\boldsymbol{x}-\boldsymbol{y};t)\phi(\boldsymbol{x})\phi_1(\boldsymbol{y}) \\ & \qquad +\frac{i}{2}\int \mathrm{d}^3x\mathrm{d}^3y\; G(\boldsymbol{x}-\boldsymbol{y};t)\Big[\phi(\boldsymbol{x})\phi(\boldsymbol{y})+\phi_1(\boldsymbol{x})\phi_1(\boldsymbol{y})\Big]\Bigg\}\ . \end{aligned} \tag{46}$$

As $t\to 0$, it is easy to obtain from (37) that $G(\boldsymbol{x}-\boldsymbol{y};t)\to\frac{1}{t}\delta^3(\boldsymbol{x}-\boldsymbol{y})$ and $g(\boldsymbol{x}-\boldsymbol{y};t)\to\frac{1}{t}\delta^3(\boldsymbol{x}-\boldsymbol{y})$. Consequently, the behavior of (46) as $t\to 0$ can be obtained:

$$\begin{aligned} & \lim_{t\to 0}\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle \\ & = \mathcal{N}\lim_{t\to 0}\mathrm{e}^{-\frac{1}{2}\int \mathrm{d}^3x\delta^3(\mathbf{0})\ln(t)}\mathrm{e}^{\frac{i}{2t}\int \mathrm{d}^3x[\phi(\boldsymbol{x})-\phi_1(\boldsymbol{x})]^2}\ . \end{aligned} \tag{47}$$

Note that after regularization, the term $\delta^3(\mathbf{0})$ effectively becomes $\frac{1}{\mathrm{d}x^3}$. Expanding the integral in the exponential of Eq. (47) into a product of exponentials, we obtain:

$$\lim_{t\to 0}\langle\phi|\mathrm{e}^{-i\hat{H}t}|\phi_1\rangle = \mathcal{N}\lim_{t\to 0}\prod_x \frac{1}{\sqrt{t}}\mathrm{e}^{\frac{i(\mathrm{d}x)^3}{2t}[\phi(\boldsymbol{x})-\phi_1(\boldsymbol{x})]^2} = \prod_x \delta(\phi(x)-\phi_1(x))\ , \tag{48}$$

where $\mathcal{N} \equiv \prod_x \left(\frac{(\mathrm{d}x)^3}{2\pi i} \right)$. This implies that (46) not only serves as a solution to the Schrödinger equation, but also represents the inner product between field operator eigenstates as $t \to 0$, indicating that (46) and (35) are the correct expressions for the field propagator.

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