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Adiabatic Ramsey interference in a pseudo-Hermitian two-level system

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Abstract

Ramsey interferometry involves two separate pulse fields with a controlled free evolution period between them, exploiting the coherent superposition of two distinct quantum states and their subsequent phase accumulation to manifest interference patterns. This technique has implications across quantum mechanics, quantum optics, and quantum information processing. The pseudo-Hermitian systems have garnered significant attention due to their unconventional properties, such as the existence of exceptional points (EPs) and parity-time ($\mathcal{PT}$) symmetry. Here, we investigate adiabatic Ramsey interference in a two-level pseudo-Hermitian system characterized by nonlinear self-interaction and nonreciprocal coupling between the levels, unveiling novel phenomena that transcends the traditional Hermitian frameworks. We implement a Ramsey interferometer that incorporates two temporally separated Rosen-Zener pulses. By harnessing the properties of the pseudo-Hermitian system, such as $\mathcal{PT}$ symmetric phase transition and associated EPs, we demonstrate that the interference pattern can be generated and controlled, corresponding to coherent control of the quantum states. In particular, the interference pattern observed vanishes as the system transitions from a $\mathcal{PT}$ symmetric phase to a broken-symmetry phase, which underscores the interplay between the symmetry properties of the system and the manifestation of quantum coherence. Our findings provide a theoretical perspective for the manipulation of quantum states in pseudo-Hermitian systems.

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Contents

1 Introduction

The utilization of discrete energy levels in systems to achieve quantum interference lies at the core of both fundamental and applied physics [1–6]. Ramsey interferometry [2,3,7,8], being one of the pivotal quantum interference schemes initially devised to probe molecular beam resonance, possesses extensive application values in various fields such as quantum information processing, quantum simulation [9,10], metrology [11–13], and quantum communication [9]. It forms the foundation of the basis of current primary time standards, such as cesium-beam or cesium-fountain clocks [14, 15]. The Ramsey's interferometric method has been applied in a number of aspects such as investigating the effect of interactions in a many-body system [8, 16–18], the detection of the Schrödinger-cat states of an electromagnetic field [11,19,20] and so on. Ramsey interference, typically involving two distinct energy levels or quantum states, exploits the coherent superposition of these states and their subsequent phase accumulation to manifest interference patterns. The quantum two-level system (TLS) model serves as a paradigm for exploring a range of quantum dynamical phenomena, including the Rosen-Zener (RZ) model [21], which investigates spin-flips in two-level atoms under the influence of a rotating magnetic field in Stern-Gerlach setups, and the Landau-Zener model [22,23], which describes the traversal of a system across an avoided level crossing at a specified sweeping velocity.

In recent years, non-Hermitian systems have garnered extensive attention both theoretically and experimentally, particularly in the domains of quantum physics and quantum optics, underscoring their significance and potential applications [24–34]. As a particular subset within the non-Hermitian systems, pseudo-Hermitian systems adhere to a more generalized Hermiticity condition [35–38], which stipulates the existence of an invertible Hermitian operator η such that the Hamiltonian H of the system fulfills the relation $H^\dagger = \eta H \eta^{-1}$. In this scenario, despite the Hamiltonian itself being non-Hermitian (i.e., $H \neq H^\dagger$), its eigenvalues can be either real or complex-conjugate pairs. The $\mathcal{PT}$-symmetric Hamiltonian [24,39], satisfying $[H, \mathcal{PT}] = 0$, constitutes a distinct subclass of pseudo-Hermitian Hamiltonians [40]. Adjustment of one parameter in a pseudo-Hermitian system triggers a quantum phase transition at the exceptional points (EPs), marking the transition from a $\mathcal{PT}$-symmetric phase characterized by real eigenvalues to a $\mathcal{PT}$-symmetry-broken phase featuring complex-conjugate eigenvalue pairs, accompanied by the simultaneous coalescence of eigenvalues and their respective eigenvectors [26, 31, 41–45]. Over the recent years, lots of fascinating phenomena have garnered substantial research attention in various pseudo-Hermitian systems, encompassing cavity optomechanical systems [46–49], waveguides [50], microcavities [51], cavity magnonics [52–54], and superconducting circuits [55–57], among others.

In the present work, we investigate RZ transition and adiabatic Ramsey interference in a pseudo-Hermitian TLS. As an important concept in quantum mechanics, adiabatic evolution [58,59] that has been widely applied in the preparation and control of quantum states [60–65]. Recently, adiabatic evolution within pseudo-Hermitian TLS has garnered attention in the context of Landau-Zener tunneling [32] and Landau-Zener-Stückelberg-Majorana interference [66], fostering a deeper understanding of their quantum dynamics. Differing from the Landau-Zener model the RZ model fixes the energy difference between two modes as a constant, while allowing the coupling strength to vary with time [67]. Herein, we focus on the unexplored and fascinating RZ transition, along with Ramsey interference, in pseudo-Hermitian quantum systems, unveiling novel phenomena for quantum state preparation and manipulation that transcend traditional Hermitian frameworks.

This paper constructs a Ramsey interferometer within the theoretical framework established in Refs. [68–70], employing two sequential, identical RZ transition processes interspersed with an adjustable holding interval. Using a simple two-level pseudo-Hermitian model, featuring nonlinear self-interactions and nonreciprocal coupling between the levels, we comprehensively delve into the physics governing the interference patterns. We commence our investigation by examining the RZ transition. In the absence of nonlinear interactions, we analytically derive expressions for the RZ transition probability, which manifest the influence of pseudo-Hermiticity and pulse periodicity, thereby providing a solid foundation for precise manipulation of quantum states in pseudo-Hermitian systems. Subsequently, we delve into the adiabatic Ramsey interference within such a pseudo-Hermitian TLS. Our analysis systematically explores the intricate interplay between nonlinear interactions, nonreciprocal coupling, and level asymmetry, and its consequential impact on the Ramsey interference patterns. In particular, we find that a strong nonlinear interaction, nonreciprocal coupling, as well as level asymmetry, all exert a pronounced inhibitory effect on the interference patterns. This comprehensive understanding not only enhances our theoretical grasp of quantum dynamics in non-Hermitian environments but also opens avenues for exploring novel quantum phenomena and prospective applications in quantum technologies.

The rest of the paper is organized as follows. In Sec. 2, we introduce the pseudo-Hermitian generalization of the Ramsey model. In Sec. 3, we study quantum tunneling in the absence of the nonlinear self-interaction and theoretically explain the phenomenon, emphasizing on the impact of EPs on quantum tunneling. Then, we respectively study the adiabatic Ramsey interference in the absence and presence of nonlinear interactions in Sec. 4. Section 5 is our discussions and summary.

2 Ramsey interferometry in a two-level pseudo-Hermitian system

We consider a two-level system with nonreciprocal coupling between the levels described by the Hamiltonian [32, 66].

$$H(\nu)=\begin{pmatrix} \frac{\gamma}{2}+c|a|^2 & \frac{\nu(t)}{2} \\ \frac{\nu(t)}{2}(1-\delta) & -\frac{\gamma}{2}+c|b|^2 \end{pmatrix}, \tag{1}$$

with a and b representing the probability amplitudes of the two components of the wave function in the two states, c is the nonlinear parameter describing the interatomic interaction among each component, where $c>0$ indicates a repulsive interaction in the whole article. Parameter γ represents the level bias characterizing the asymmetry of the system, $\nu(t)$ is the coupling strength between the levels, and $\delta>0$ denotes nonreciprocal coupling which leads to non-Hermiticity. Obviously, the Hamiltonian H of the system satisfying the pseudo Hermitian condition ($H^\dagger=\eta H\eta^{-1}$) with $\eta=\begin{pmatrix} 1 & 0 \\ 0 & 1-\delta \end{pmatrix}$.

We construct a Ramsey interferometer within the theoretical framework established in Refs. [68–70], wherein the variation of the coupling strength is assumed to be governed by two Rosen-Zener pulses separated by a period of free evolution lasting time τ. The interferometer can be mathematically formulated as follows:

$$\nu(t)=\begin{cases}0, & t<0,\\ \nu_0\sin^2\left(\frac{\pi t}{T}\right), & t\in[0,T],\\ 0, & t\in(T,T+\tau),\\ \nu_0\sin^2\left[\frac{\pi(t-T-\tau)}{T}\right], & t\in[T+\tau,2T+\tau],\\ 0, & t>2T+\tau.\end{cases} \tag{2}$$

where ν_0 denotes the maximum strength of the coupling, and T is the scanning period of RZ pulse. Since the Hamiltonian can be scaled by dividing by ν_0, for convenience, we can set $\nu_0=1$ as the energy unit hereafter. The first pulse constructs quantum superpositions, whereas the subsequent, delayed second pulse orchestrates the completion of the Ramsey sequence.

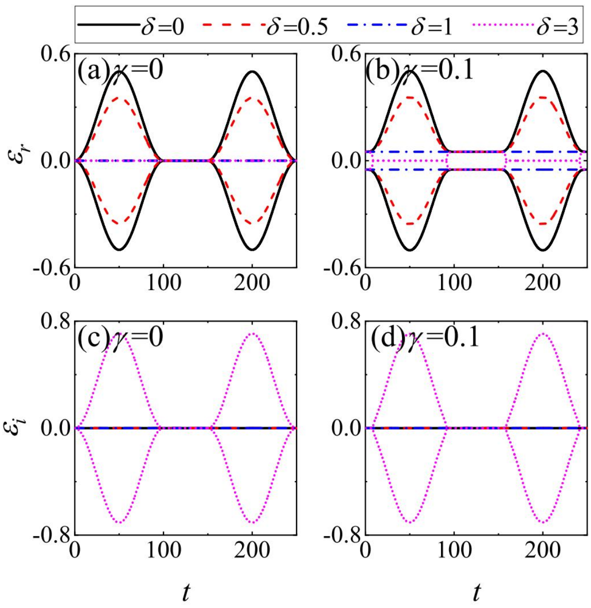

Figure 1: (Color online) The energy levels in the time domain of Ramsey interferometry within the two-level pseudo-Hermitian system. We implement a Ramsey interferometry that incorporates two temporally separated RZ pulses. The first pulse establishes quantum superpositions, while the subsequent, delayed second pulse completes the Ramsey sequence. Each RZ pulse has a period T, and the time interval between the two pulses, known as the holding period, is denoted by τ. (a) The real parts and (c) the imaginary parts of the energy levels for the level-symmetric case (i.e., $\gamma=0$). (b) The real parts and (d) the imaginary parts of the energy levels for the level-asymmetric case (i.e., $\gamma\neq 0$). For $\delta<1$, the imaginary parts of the energy levels are always zero, resulting in an overlap in plots (c) and (d) for this range. In contrast, when $\delta>1$ (e.g., $\delta=3$), the system undergoes transitions between $\mathcal{PT}$-symmetric and $\mathcal{PT}$-symmetry-broken phases at a series of EPs. Here, the specific values used are $T=100$ and $\tau=50$.

In Ramsey interferometry, the variations observed in the interference fringes serve as a sensitive indicator, typically mirroring the intricate dynamics of energy level transitions within the quantum system under investigation. In the absence of the nonlinear self-interaction (i.e., $c=0$), the eigenvalues of Hamiltonian (1) depend on the coupling strength as follows:

$$\varepsilon_\pm(t)=\pm\frac{1}{2}\sqrt{\gamma^2+(1-\delta)\nu(t)^2}. \tag{3}$$

When $\gamma^2+(1-\delta)\nu(t)^2<0$, the system allows imaginary energy levels to exist, i.e., $\varepsilon=\varepsilon_{\mathrm{r}}+i\varepsilon_{\mathrm{i}}$ with ε_{r} and ε_{i} are real and imaginary parts of energy, respectively.

Figure 1 comprehensively depicts the temporal evolution of energy levels in the context of Ramsey interferometry within a two-level pseudo-Hermitian quantum system. Figs. 1(a) and (c) respectively illustrate the real and imaginary parts of the energy levels for the level-symmetric case, where $\gamma=0$. Conversely, Figs. 1(b) and (d) present the corresponding real and imaginary components for the level-asymmetric case, distinguished by $\gamma\neq 0$. A noteworthy observation is that for $\delta<1$, the imaginary parts of the energy levels vanish, leading to a congruent overlap in the presentations of Figs. 1(c) and (d) within this parameter range. However, as $\delta>1$ (e.g., $\delta=3$), the system experiences a dramatic transformation, transitioning between $\mathcal{PT}$-symmetric and $\mathcal{PT}$-symmetry-broken phases at a series of EPs. here are four EPs present in the time domain, each appearing at $t_1=\left[\frac{T}{\pi}\arcsin\left(\frac{\sqrt{\gamma}}{(1-\delta)^{1/4}\sqrt{\nu_0}}\right)\right]$, $t_2=T-t_1$, $t_3=T+\tau+t_1$, $t_4=2T+\tau-t_1$ respectively. Obviously, pseudo-Hermiticity relation leads to a special constraint on the eigenspectrum of a pseudo-Hermitian Hamiltonian: the spectrum consists of purely real eigenvalues and pairs of complex conjugated eigenvalues. These observations underscore profound deviations from traditional Hermitian framework, marking a significant transformation in the interference phenomena observed. The non-Hermitian nature of the quantum system under investigation leads to plenty of variations in the interference outcomes, thereby revealing novel and intricate behavior that challenges conventional quantum optical and quantum physical paradigms.

The dimensionless Schrödinger equation is $i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}a\\b\end{pmatrix}=H(\nu)\begin{pmatrix}a\\b\end{pmatrix}$, which can be solved numerically using standard Runge-Kutta fourth- and fifth-order algorithms. The Hamiltonian, as expressed in (1), explicitly demonstrates the presence of nonreciprocal coupling, which fundamentally underpins the potential for pronounced disparities in the probabilities of tunneling and interference phenomena originating from distinct energy levels. This nonreciprocal nature manifests in the form of asymmetric tunneling and interference patterns, collectively termed as ***nonreciprocal RZ transition*** and ***nonreciprocal Ramsey interference***, respectively.

In order to study the characteristics of nonreciprocal quantum interference, we assume that the system is initially found in the upper (or lower) energy level, i.e., $(a(t=0),\ b(t=0))^T=(0,\ 1)^T$ (or $(a(t=0),\ b(t=0))^T=(1,\ 0)^T$). Since Hamiltonian (1) is non-Hermitian, a notable characteristic emerges in the form of complex eigenvalues. Therefore, the time evolution is no longer unitary and the total population (i.e., $N(t)=|a(t)|^2+|b(t)|^2$) is not a conserved quantity. Thus, the occupancy probability of each energy level can be denoted by the following:

$$P_a(t)\equiv\frac{|a(t)|^2}{N(t)};\ P_b(t)\equiv\frac{|b(t)|^2}{N(t)}. \tag{4}$$

In our following study, we postulate that the quantum state is initially prepared on one energy level. Upon the application of an external field, a nonreciprocal RZ transition or nonreciprocal Ramsey interference arises between distinct energy levels. Our primary focus lies in elucidating the population dynamics that ensue under the influence of this external field. Specifically, we define the nonreciprocal RZ transition and nonreciprocal Ramsey interference as the probability by recording the occupancy probability P_a (or, P_b), after the coupling field have been turned off.

3 Nonreciprocal RZ transition in a pseudo-Hermitian two-level system

3.1 For the level-symmetric case (i.e., $\gamma = 0$)

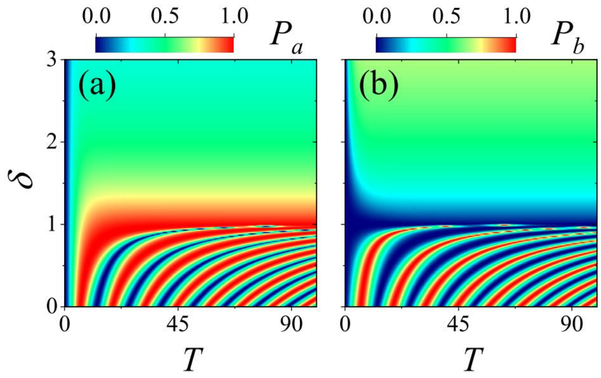

Figure 2: (Color online) Nonreciprocal RZ transition for level-symmetric (i.e., $\gamma = 0$) and linear (i.e., $c = 0$) case. Level occupation probabilities (a) P_a and (b) P_b for the system are initially found in the upper and lower energy levels, respectively. Here the RZ transition probabilities are obtained by recording the final occupancy probability P_a (or P_b) after the first pulse, i.e., $t = T$. In the sudden limit of the pulse duration ($T \ll \frac{2\pi}{\nu_0}$), the transition between the two energy levels becomes effectively frozen. Conversely, in the adiabatic limit ($T \gg \frac{2\pi}{\nu_0}$), a rich tapestry of transition phenomena is unveiled. When the system resides in the $\mathcal{PT}$-symmetric phase, characterized by $\delta < 1$, the RZ transition exhibits periodic oscillations with the pulse period T, with the oscillation period expanding as δ increases. An intriguing phenomenon arises when $\delta > 1$, signaling the breakdown of $\mathcal{PT}$-symmetry. In this regime, the RZ transition probability ceases to be dependent on the pulse period T and instead becomes solely a function of δ, specifically, $P_a = \frac{1}{\delta}$ and $P_b = \frac{\delta-1}{\delta}$. This non-reciprocal transition behavior provides a pivotal theoretical foundation for the precise manipulation of quantum states in non-Hermitian systems.

Let us now study the nonreciprocal RZ transition in the absence of nonlinear interaction. For the level-symmetric case, Figs. 2(a) and 2(b) depict nonreciprocal RZ transition probabilities P_a and P_b as a function of the nonreciprocity parameter δ and the scanning period of the RZ pulse T for the system that is initially found in the upper and lower energy levels, respectively. It is observed that the transition between two levels is frozen when the scanning period of the RZ pulse T is much smaller than the intrinsic period of the system $2\pi/\nu_0$, i.e., sudden limit case. For adiabatic limit case, i.e., $T \gg 2\pi/\nu_0$, RZ transition presents periodic oscillation in the weak nonreciprocal regime ($\delta < 1$), whereas the periodic oscillation is completely suppressed in the strong nonreciprocal regime ($\delta > 1$) due to the $\mathcal{PT}$-symmetry-broken. Meanwhile, RZ transition probabilities P_a and P_b observed in Figs. 2(a) and 2(b) exhibit obvious differences, indicating a clear nonreciprocal quantum transition. In the absence of a nonreciprocity (i.e., $\delta = 0$), the nonreciprocal RZ transition probabilities of P_a are the same as those of P_b, which corresponds to the Hermitian case. In the weak nonreciprocal regime, as the nonreciprocal parameter δ increases, the difference between P_a and P_b becomes increasingly apparent.

We now elucidate the previously mentioned numerical observations through analytical derivation for the linear and symmetric case. For the system initially found in the upper energy level, i.e., $(a(t=0),\ b(t=0))^T = (0,\ 1)^T$, we have

$$P_a = \frac{1}{\delta + (1-\delta)\csc^2\left(\frac{1}{4}T\sqrt{1-\delta}\right)} \quad (\delta < 1), \tag{5a}$$

$$P_a = \frac{1}{\delta + (\delta-1)\mathrm{csch}^2\left(\frac{1}{4}T\sqrt{\delta-1}\right)} \quad (\delta > 1). \tag{5b}$$

For the system initially found in the lower energy level, i.e., $(a(t=0),\ b(t=0))^T = (1,\ 0)^T$, we have

$$P_b = \frac{(\delta-1)\sinh^2\left(\frac{1}{4}\mathrm{T}\sqrt{\delta-1}\right)}{\cosh^2\left(\frac{1}{4}\mathrm{T}\sqrt{\delta-1}\right) + (\delta-1)\sinh^2\left(\frac{1}{4}\mathrm{T}\sqrt{\delta-1}\right)} \quad (\delta < 1), \tag{6a}$$

$$P_b = \frac{(1-\delta)\sin^2\left(\frac{1}{4}\mathrm{T}\sqrt{1-\delta}\right)}{\cos^2\left(\frac{1}{4}\mathrm{T}\sqrt{1-\delta}\right) + (1-\delta)\sin^2\left(\frac{1}{4}\mathrm{T}\sqrt{1-\delta}\right)} \quad (\delta > 1). \tag{6b}$$

In the context of Hermitian physics, wherein $\delta = 0$, both Eqs. (5) and (6) converge to an identical form, expressing the probabilities of occupation $P_a = P_b = \sin^2\left(\frac{T}{4}\right)$, in accordance with the findings documented in reference [67]. This concordance emphasizes the degeneracy that arises under Hermiticity, simplifying the dynamics to a shared functional form.

Further delving into the temporal aspects of the pulse duration, two distinct limits emerge with profound implications. In the sudden limit, i.e., $T \ll \frac{2\pi}{\nu_0}$, Eqs. (5) and (6) unequivocally reveal that $P_a = P_b = 0$. This striking result signifies that the transition between the two energy levels becomes effectively suppressed or 'frozen,' highlighting the inadequacy of the pulse to induce a discernible change in the quantum state.

Conversely, in the adiabatic limit ($T \gg \frac{2\pi}{\nu_0}$), P_a and P_b exhibit a periodic dependence on δ for $\delta < 1$ (as shown in Eqs. (5a) and (6a)), illustrating the delicate interplay between the pulse's temporal characteristics and the system's parameters.

However, a particularly intriguing phenomenon unfolds when $\delta > 1$. In the adiabatic limit ($T \gg \frac{2\pi}{\nu_0}$), the RZ transition probability transcends its conventional dependence on the pulse period T, evolving into a pure function of δ alone. Specifically, P_a becomes inversely proportional to δ, given by $P_a = \frac{1}{\delta}$ (as shown in Eq. (5b)), while P_b adopts a form, $P_b = \frac{\delta-1}{\delta}$ (as shown in Eq. (6b)). This transition in behavior emphasizes the profound impact that non-Hermiticity, introduced by $\delta > 1$, has on the quantum dynamics, enabling the manipulation of transition probabilities independently of the pulse's temporal envelope.

These theoretical predictions are corroborated by our numerical simulations, which display a remarkable agreement, validating the theoretical framework's capacity to capture the nuanced features of quantum transitions and state manipulation within non-Hermitian quantum physics and quantum optics.

3.2 For the level-asymmetric case (i.e., $\gamma \neq 0$)

Now, we delve into the nonreciprocal RZ transition within the adiabatic limit, $T \gg \frac{2\pi}{\nu_0}$. This condition ensures that the system's evolution adheres closely to the instantaneous eigenstates, facilitating a nuanced analysis of the transition dynamics. Figure 3 presents a comprehensive visualization of the adiabatic nonreciprocal RZ transition probabilities within the parametric space (δ, γ), specifically tailored for a pulse duration of $T = 100$. This representation meticulously dissects the intricate interplay between level asymmetry and nonreciprocity, as manifested in the transition probabilities P_a and P_b, respectively depicted in 3(a) and 3(b). Here, the system's initial preparation is deliberately chosen to be in either the upper or lower

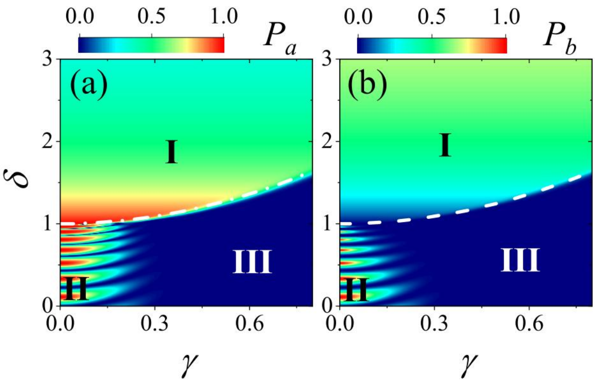

Figure 3: (Color online) The adiabatic nonreciprocal RZ transition probabilities in the parameter plane (δ, γ) for the duration of the RZ pulse in the adiabatic limit case (i.e., $T = 100$) for $c = 0$. Specifically, (a) P_a and (b) P_b are depicted, where the system is initially prepared in the upper and lower energy levels, respectively. These figures profoundly illustrate the combined effects of level asymmetry and nonreciprocity on the RZ transitions. Three distinct regions are evident: In Region I, nonreciprocity dominates the inter-level transitions, manifested as $P_a = \frac{1}{\delta}$ and $P_b = \frac{\delta-1}{\delta}$. In Region II, both level asymmetry and nonreciprocity contribute to the transitions. Specifically, the nonreciprocity parameter δ introduces periodic modulation, while an increase in the level asymmetry parameter γ leads to a decrease in the transition amplitude. Finally, in Region III, the sufficiently large energy level spacing completely suppresses the inter-level transitions. A single phase-change line, depicted as a dot-dash line, signifies the exceptional line, which comprises of EPs. This exceptional line is mathematically characterized by the equation $\delta = \gamma^2 + 1$.

energy level, allowing for a direct comparison of the transition dynamics from these distinct starting points.

The figures reveal three distinct regions, each characterized by a unique balance between the governing factors. In Region I, nonreciprocity emerges as the dominant force, shaping the inter-level transitions according to the inverse relationship $P_a = \frac{1}{\delta}$ and the scaled difference $P_b = \frac{\delta-1}{\delta}$. This region emphasizes the profound impact of nonreciprocal interactions on the transition probabilities, independent of the level asymmetry.

Transitioning into Region II, a harmonious interplay between level asymmetry and nonreciprocity becomes evident. The nonreciprocity parameter δ introduces a periodic modulation to the transition landscape, while the level asymmetry parameter γ exerts a countervailing influence, diminishing the transition amplitude as it increases. This region encapsulates the delicate balance between these two phenomena, highlighting their collective significance in determining the transition probabilities.

Finally, in Region III, the impact of the energy level spacing transcends both nonreciprocity and asymmetry. When this spacing reaches a sufficient magnitude, it effectively quarantines the system within its initial energy level, completely suppressing inter-level transitions. This phenomenon emphasizes the fundamental role of energy level structure in quantum dynamics, particularly in the context of non-Hermitian quantum systems where the conventional rules of energy conservation may be subtly altered.

Thus, Fig. 3 not only showcases the richness of the adiabatic nonreciprocal RZ transition dynamics but also stresses the critical importance of carefully navigating the parameter space to harness these phenomena for quantum control and manipulation.

4 Adiabatic Ramsey interference in the pseudo-Hermitian two-level system

We now delve into investigating adiabatic Ramsey interference within a pseudo-Hermitian two-level system, specifically focusing on the duration of the two RZ pulses in the adiabatic limit scenario, where $T \gg \frac{2\pi}{\nu_0}$. For our subsequent analysis, we set $T = 100$. To explore the distinct features of nonreciprocal Ramsey interference, we postulate that the system initially occupies either the lower or the upper energy level. Subsequently, we procure the nonreciprocal Ramsey interference patterns by monitoring the occupancy probabilities, P_a and P_b, respectively, once the coupling field has been turned off. These probabilities are formally defined as $P_a \equiv P_a(t = 2T + \tau)$ and $P_b \equiv P_b(t = 2T + \tau)$.

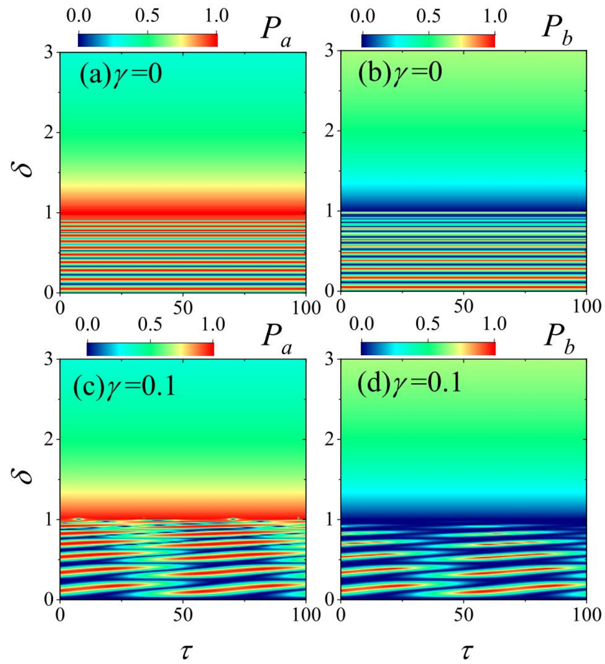

Figure 4: (Color online) The nonreciprocal adiabatic Ramsey interference patterns in the absence of nonlinear interactions (i.e., $c = 0$). Here, these patterns are observed by monitoring the occupancy probabilities, P_a and P_b, after the system is initially prepared in the upper and lower energy levels, respectively, and the coupling field is subsequently switched off at $t = 2T + \tau$, with $T = 100$. Specifically: For the level-symmetric case (i.e., $\gamma = 0$), (a) and (b) depict P_a and P_b, respectively. In this scenario, within the parameter plane (δ, τ), the Ramsey interference patterns do not depend on the holding time τ, but solely on the nonreciprocal parameter δ. This is because, in a symmetric system, phase accumulation is primarily governed by δ. For the level-asymmetric case (i.e., $\gamma = 0.1$), (c) and (d) display P_a and P_b, respectively. In this instance, the Ramsey interference patterns depend concurrently on both the holding time τ and the nonreciprocal parameter δ. This arises due to the additional phase accumulation introduced by the level asymmetry. Intriguingly, in both the level-symmetric and level-asymmetric scenarios, a notable phenomenon emerges in the strong nonreciprocal regime ($\delta > 1$): nonreciprocity dominates the inter-level interference, manifested as $P_a = \frac{1}{\delta}$ and $P_b = \frac{\delta-1}{\delta}$. This behavior stems from the breakdown of $\mathcal{PT}$-symmetry, highlighting the unique consequences of nonreciprocal interactions in quantum systems.

4.1 In the absence of nonlinear interaction (i.e., $c = 0$)

The nonreciprocal adiabatic Ramsey interference patterns in the absence of nonlinear interactions, as vividly illustrated in Fig. 4. These patterns explore how the occupancy probabilities, denoted as P_a and P_b, evolve following the preparation of the system in either the upper or lower energy level, respectively, and subsequent deactivation of the coupling field at a precise temporal juncture of $t = 2T + \tau$, where $T = 100$.

For the level-symmetric scenario, where γ vanishes, the Ramsey interference patterns exhibit an intriguing dependence within the parameter plane $(\delta,\ \tau)$. Notably, these patterns decouple from the holding time τ, solely entrusting their characteristics to the nonreciprocal parameter δ. This singular behavior stems from the fundamental physics underpinning the system: in a perfectly symmetric configuration, the phase accumulation is predominantly governed by δ, reflecting the dominance of nonreciprocal effects in shaping the quantum dynamics. Figures 4(a) and 4(b) vividly illustrate this phenomenon, where P_a and P_b respectively portray the occupation dynamics under varying δ, unperturbed by temporal delays introduced by τ.

Transitioning to the level-asymmetric case, where $\gamma = 0.1$ introduces a fundamental asymmetry, Figs. 4(c) and 4(d) paint a different narrative. Here, the Ramsey interference patterns become intertwined with both the holding time τ and the nonreciprocal parameter δ. This dual dependence underscores the profound impact of level-asymmetry, which introduces additional phase accumulation, thereby enriching the interference dynamics. The interplay between these two parameters orchestrates a complex tapestry of quantum state evolutions.

A particularly intriguing aspect emerges in both level-symmetric and level-asymmetric cases within the strong nonreciprocal regime ($\delta > 1$). Here, nonreciprocity ascends to prominence, dictating the inter-level interference. This dominance manifests vividly through the analytical expressions $P_a = \frac{1}{\delta}$ and $P_b = \frac{\delta-1}{\delta}$, which emphasize the breakdown of $\mathcal{PT}$-symmetry. This breakdown, a hallmark of non-Hermitian systems, fundamentally alters the dynamics, leading to novel phenomena such as the suppression of certain transitions and the enhancement of others, all governed by the intricate balance between reciprocity and nonreciprocity encapsulated within δ.

To further investigate the profound impact of the level asymmetry γ on interference, in the absence of nonlinear interactions, we show the nonreciprocal adiabatic Ramsey interference patterns in the parameter plane $(\gamma,\ \delta)$ in Fig. 5. Figures 5(a) and 5(b) meticulously portray the occupancy probabilities, P_a and P_b, respectively, corresponding to the system's initial preparation in the upper and lower energy levels. These probability distributions illuminate the underlying physics and quantum state manipulation capabilities in three distinct regions:

1. **Region I**: Here, the Ramsey interference patterns, disrupted by the inherent nonreciprocity, exhibit a remarkable monotonic dependence solely on the nonreciprocal parameter δ. This singular behavior is mathematically encapsulated in the analytical expressions $P_a = \frac{1}{\delta}$ and $P_b = \frac{\delta-1}{\delta}$, highlighting the dominant role of nonreciprocal effects in shaping the quantum dynamics. This region stresses the precision with which quantum states can be tuned by manipulating the nonreciprocal parameter, offering a unique avenue for quantum control.

2. **Region II**: Transitioning to Region II, we observe a more complex scenario where both level asymmetry and nonreciprocity contribute synergistically to the quantum transitions. The nonreciprocal parameter δ and the level asymmetry parameter γ jointly introduce periodic modulations in the occupancy probabilities. Notably, an increase in the level asymmetry parameter γ leads to a decrease in the interference amplitude, revealing a delicate balance between these two parameters that governs the quantum dynamics.

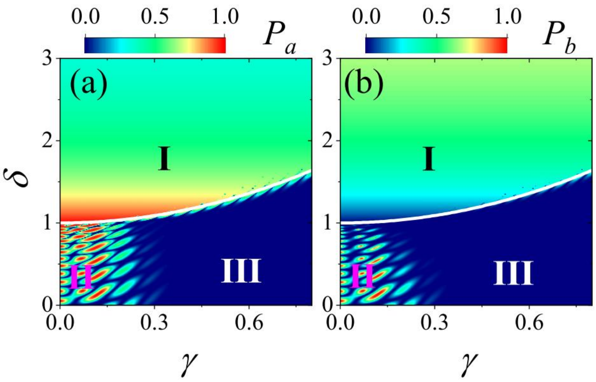

Figure 5: (Color online) In the absence of nonlinear interactions (i.e., $c = 0$), the nonreciprocal adiabatic Ramsey interference patterns in the parameter plane (γ, δ), with $T = 100$ and $\tau = 50$ as reference values. (a) and (b) illustrate the occupancy probabilities P_a and P_b, respectively, for the system initially prepared in the upper and lower energy levels. These plots reveal three distinct regions: In Region I, the Ramsey interference patterns, disrupted by nonreciprocity, exhibit a monotonic dependence solely on the nonreciprocal parameter δ. This is manifested by the analytical expressions $P_a = \frac{1}{\delta}$ and $P_b = \frac{\delta - 1}{\delta}$, highlighting the dominance of nonreciprocity in this region. Region II is characterized by the combined influence of level asymmetry and nonreciprocity on the interference. Both the nonreciprocal parameter δ and the level asymmetry parameter γ introduce periodic modulations, with an increase in γ resulting in a decrease in the transition amplitude. This region underscores the intricate interplay between these two parameters. Finally, in Region III, the sufficiently large energy level spacing results in a complete suppression of inter-level interference. A notable feature is the exceptional line, depicted as a dot-dash line. This line is mathematically defined by the equation $\delta = \gamma^2 + 1$, marking a boundary where unique physical phenomena occur.

This region showcases the intricate interplay between nonreciprocity and level asymmetry, offering unprecedented control over quantum state transitions and interference effects.

3. **Region III**: Finally, within this area, the quantum landscape undergoes a profound transformation. The sufficiently large energy level spacing acts as a barrier, completely suppressing inter-level interference. This suppression not only simplifies the quantum dynamics but also opens up new possibilities for quantum state manipulation.

Notably, a distinct phase-change line, depicted as a dot-dash line, emerges as an exceptional line comprising EPs. Mathematically defined by the equation $\delta = \gamma^2 + 1$, this exceptional line marks the boundary between qualitatively different quantum behaviors and serves as a pivot for extraordinary quantum phenomena, such as enhanced sensitivity to perturbations and unconventional quantum state evolution.

The study of nonreciprocal adiabatic Ramsey interference patterns, particularly in the absence of nonlinear interactions, provides deep insights into the fundamental mechanisms governing quantum dynamics and state manipulation. The observed dependencies on nonreciprocal parameters and system asymmetry, along with the emergence of $\mathcal{PT}$-symmetry breaking in the strong nonreciprocal regime, stressing the immense potential of these systems for advancing quantum technologies and probing the frontiers of quantum physics.

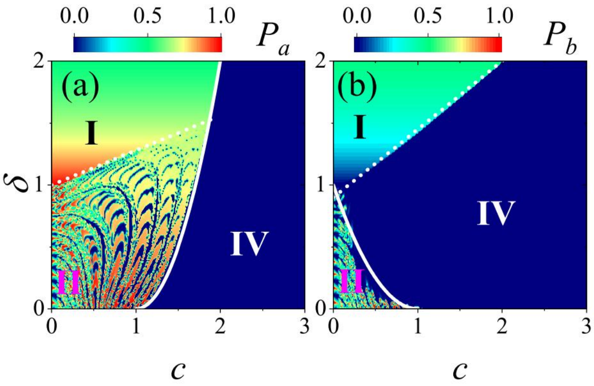

Figure 6: (Color online) In the presence of nonlinear interaction (where $c \neq 0$) and under level-symmetric conditions (i.e., $\gamma = 0$), the nonreciprocal adiabatic Ramsey interference patterns in the parameter plane (δ, c), taking $T = 100$ and $\tau = 50$ as reference values. (a) and (b) depict the occupancy probabilities P_a and P_b, respectively, for the system initially prepared in the upper and lower energy levels. In addition to the Regions I and II presented in Fig. 5, a novel Region IV emerges. In this region, the self-trapping effect induced by strong nonlinear interactions suppresses the quantum transitions between energy levels, leading to the breakdown of interference.

4.2 In the presence of nonlinear interaction

In the framework of non-Hermitian quantum mechanics, the intricate interplay between nonlinear interactions and non-Hermitian properties reveals profound consequences on the dynamics. Now, we delve into the intricate mechanisms governing quantum transitions and state manipulation in the presence of nonlinear interactions. For the level-symmetric conditions (i.e., $\gamma = 0$), we show the nonreciprocal adiabatic Ramsey interference patterns in the parameter plane (δ, c), with $T = 100$ and $\tau = 50$ as reference values. Figures 6(a) and 6(b) meticulously illustrate the occupancy probabilities P_a and P_b, respectively, corresponding to the system's initial preparation in the upper and lower energy levels. These figures underscore the delicate quantum state manipulation capabilities within the explored parameter space.

Beyond the well-established Regions I and II depicted in Fig. 5, our analysis unveils a novel **Region IV**, where the physics undergoes a qualitative transformation. In this intriguing region, the self-trapping effect, a direct consequence of the predominant strong nonlinear interactions, exerts a profound influence, effectively suppressing quantum transitions between energy levels [67,71–76]. This suppression, in turn, leads to a breakdown of the characteristic interference patterns, revealing a new paradigm for quantum state control and manipulation.

Furthermore, Figs. 6(a) and 6(b) highlight the presence of distinct phase-transition lines that delineate various regimes within Region IV. Specifically, one phase-transition line (solid line) is estimated to follow the relation $\delta = 2(c-1)^2$, while another (dashed line) exhibits different slopes, approximated as $\delta = 0.28c + 1$ in Fig. 6(a) and $\delta = 0.55c + 0.9$ in Fig. 6(b). These phase-change lines mark the boundaries between regions characterized by qualitatively different quantum behaviors, underscoring the intricate physics and rich phase structure that emerges in the presence of strong nonlinear interactions and non-Hermitian properties. By precisely tuning the system parameters along these lines, one can achieve precise control over quantum state evolution and transitions, opening up new avenues for quantum technologies and applications.

To gain further insight into the effects of nonreciprocity parameter δ, asymmetry parameter γ, and nonlinear interaction c on quantum interference, in Fig. 7, the intricate adiabatic Ramsey interference patterns, exhibiting pronounced nonreciprocity within the (c, γ) param-

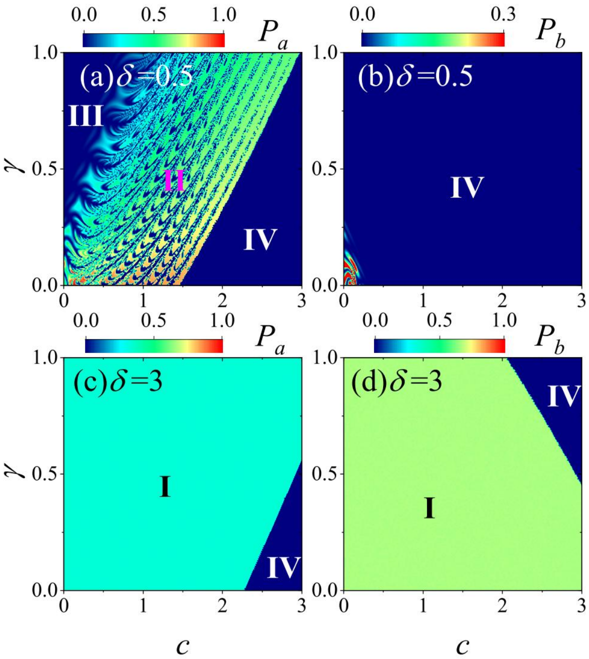

Figure 7: (Color online) The adiabatic Ramsey interference patterns exhibiting nonreciprocity in the (c, γ) parameter plane are illustrated for $T = 100$ and $\tau = 50$. The upper panel (a, b) corresponds to a weak nonreciprocal parameter of $\delta = 0.5$, while the lower panel (c, d) represents the case with a strong nonreciprocal parameter of $\delta = 3$, respectively. These patterns reveal distinct nonreciprocal interference signatures. In the regime of weak nonreciprocity, a substantial level-asymmetric parameter combined with sufficient nonlinear interaction effectively suppresses inter-level quantum transitions, thereby disrupting quantum interference phenomena, particularly in Regions III and IV. Conversely, in Region II, the intricate interplay between level asymmetry and nonlinear interaction fosters a rich tapestry of interference patterns. In the strong nonreciprocal regime, the emergence of Region I can be attributed to the breaking of $\mathcal{PT}$-symmetry. Furthermore, the presence of a sufficiently strong nonlinear interaction fully quenches quantum transitions, giving rise to the manifestation of Region IV. This intricate interplay underscores the profound impact of nonreciprocity and nonlinearity on quantum dynamics.

eter space, are meticulously depicted for specific temporal durations of $T = 100$ and $\tau = 50$. The upper panel encompassing (a, b) for a weak nonreciprocal parameter $\delta = 0.5$, and the lower panel comprising (c, d) for a strong nonreciprocal parameter $\delta = 3$—emphasizes the distinctiveness of the interference signatures imparted by varying degrees of nonreciprocity.

In the weak nonreciprocity, the intricate interplay between a significant level-asymmetric parameter and a robust nonlinear interaction serves as a potent suppressor of inter-level quantum transitions. This suppression mechanism effectively disrupts the onset of quantum interference phenomena, most notably in Regions III and IV, where the disruption is most pronounced. Conversely, in Region II, a delicate balance between level asymmetry and nonlinear interaction fosters a vibrant tapestry of interference patterns, each illustrating the complex dynamics at play.

In the strong nonreciprocity, the emergence of Region I stands as a testament to the profound consequences of $\mathcal{PT}$-symmetry breaking. This fundamental shift in the system's underlying symmetries profoundly alters the quantum landscape, enabling the manifestation of novel phenomena. Additionally, the presence of a sufficiently potent nonlinear interaction acts as an implacable barrier to quantum transitions, leading to the complete quenching of such processes and, consequently, the emergence of Region IV. This intricate interplay between nonreciprocity, nonlinearity, and symmetry breaking underlines the intricate and profound impact these phenomena have on the quantum dynamics and state manipulation capabilities within our system.

By delicately tuning the nonreciprocity parameter δ, asymmetry parameter γ, and nonlinear interaction c, we gain unprecedented control over the quantum states and their evolution, enabling the exploration of exotic quantum phenomena and facilitating the development of advanced quantum technologies that harness these unique properties. The manipulation of quantum states through the modulation of nonreciprocity and nonlinearity represents a frontier in quantum physics and quantum optics, with prospective applications ranging from quantum computing to precision sensing and metrology.

5 Conclusions

The research presented in this work delves into the phenomena of adiabatic Ramsey interference within a pseudo-Hermitian two-level system, revealing novel insights into the control and manipulation of quantum states beyond the traditional Hermitian frameworks. The study harnesses the distinctive attributes of pseudo-Hermitian systems, such as the presence of EPs and $\mathcal{PT}$ symmetry, to explore how can quantum interference patterns be generated and controlled through various system parameters.

The implementation of a Ramsey interferometer involving two temporally separated Rosen-Zener pulses allowed us to construct and analyze quantum superpositions and the subsequent interference patterns. The research uncovered that in the absence of nonlinear self-interaction, the eigenvalues of the Hamiltonian are influenced by the coupling strength, with the system demonstrating unique behaviors when transitioning between $\mathcal{PT}$-symmetric and $\mathcal{PT}$-symmetry-broken phases.

The investigation comprehensively examined the effects of nonlinear interactions, nonreciprocal coupling, and level asymmetry on the Ramsey interference patterns. Notably, it was discovered that strong nonlinear interactions, along with nonreciprocal coupling and level asymmetry, significantly suppress the interference patterns. This understanding enhances the theoretical knowledge of dynamics in non-Hermitian quantum physics.

Furthermore, the study provides a foundation for the precise manipulation of quantum states in pseudo-Hermitian systems. The findings illustrate that by carefully tuning the nonreciprocity parameter (δ), level asymmetry parameter (γ), and nonlinear interaction strength (c), the evolution of quantum states can be controlled, which is essential for the development of advanced quantum technologies.

In conclusion, this work deepens the fundamental understanding of non-Hermitian quantum physics, particularly in the context of adiabatic Ramsey interference. It opens new avenues for exploring novel quantum phenomena and their prospective applications in quantum technologies, such as quantum computing, precision sensing, and metrology. The ability to control quantum state transitions through the modulation of nonreciprocity and nonlinearity is a significant advancement in the field of quantum optics and quantum physics.

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