Gauge-Dominated Boson–Fermion Universality in Large-N 4D Yang–Mills Theories.

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(Dated: today)

We show that four-dimensional $SU(N)$ Yang–Mills theories coupled to massive fundamental matter , with or without scalar self-interactions—exhibit a common large-N behavior in their gauge-invariant current correlators. The large-N dynamics is governed entirely by the universal Yang–Mills sector, leading to identical current correlators for fermionic and scalar theories at leading order in $1/N$, independent of the 't Hooft coupling. This establishes a new form of boson–fermion universality in four dimensions, distinct from the familiar $(2+1)$-dimensional Chern–Simons–matter dualities, and shows that the mechanism extends beyond topological gauge dynamics.

INTRODUCTION

Dualities between bosonic and fermionic theories have played a central role in quantum field theory across dimensions. In two dimensions, bosonization establishes an exact correspondence between the massless Thirring and sine-Gordon models [1]. In three dimensions, a broad family of boson–fermion dualities has been uncovered in Chern–Simons gauge theories coupled to scalar or fermionic matter [2] - [11] In these $(2+1)$-dimensional systems, gauge fixing to $A_0 = 0$ removes non-linear self-interactions, allowing matter loops to dominate and generate non-trivial λ-dependent current correlators. Such dualities form the modern *duality web* of three-dimensional gauge–matter systems and have found applications ranging from the quantum Hall effect to strongly correlated condensed matter physics [12].

In contrast, the structure of four-dimensional Yang–Mills theories is far less understood. Unlike the three-dimensional Chern–Simons case, gauge self-interactions in four dimensions cannot be removed by gauge fixing, and it is not a priori clear how they affect the large-N dynamics when matter fields are present. This raises a natural question: can one still find a boson–fermion universality of gauge-invariant observables in such a setting, and if so, what mechanism replaces the gauge-decoupling that operates in three dimensions?

Examples of dualities in four dimensions are typically restricted to supersymmetric settings, as in Seiberg–Witten theory [13] or Montonen–Olive electric–magnetic duality [14]. Non-supersymmetric cases are far less understood, although progress has been made through large-N methods, orbifold/orientifold equivalences, and planar diagrammatic expansions [15, 16].

To address this question, we analyze four-dimensional $SU(N)$ Yang–Mills theory coupled to either massive Dirac fermions or massive complex scalars in the fundamental representation. Keeping the matter fields massive ensures that the effective action obtained after integrating them out admits a well-defined local expansion in powers of the field strength. Focusing on the large-N limit, we find that the gauge dynamics dominate while the matter-induced contributions remain subleading. As a consequence, the gauge-invariant current correlators in both the scalar and fermionic theories coincide at leading order in $1/N$, independent of the 't Hooft coupling. We further find that this correspondence persists when scalar self-interactions are added, showing that the mechanism of universality is robust and extends beyond the free limit. This demonstrates that a non-supersymmetric boson–fermion universality can emerge even in four dimensions, where the gauge sector is fully dynamical rather than topological.

In contrast to the three-dimensional case, where gauge self-interactions vanish in $A_0 = 0$ gauge and matter vertices dominate, the present analysis shows that in four-dimensional Yang–Mills theory the same universality arises from an opposite mechanism — gauge dominance rather than gauge decoupling. This difference highlights a new structural parallel between large-N boson–fermion dualities across dimensions.

MASSIVE FERMIONS VS SCALARS IN 4D YANG–MILLS AT LARGE N

Let us start with a gauge group $SU(N)$ with generators T^a in the fundamental representation, normalized as

$$\mathrm{Tr}(T^a T^b) = T_R\,\delta^{ab}, \qquad T_R = \tfrac{1}{2}.$$

with the Yang–Mills coupling in 't Hooft form $g^2 = \lambda/N$. Next let us define the actions for Dirac fermions and scalars

Fermions:

$$S_\psi = \int d^4x \left[\frac{N}{\lambda}\frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \bar\psi\, i\gamma^\mu\partial_\mu\psi - m_f\bar\psi\psi + \bar\psi_m\gamma^\mu(T^a)_{mn}\psi_n\, A^a_\mu \right]. \tag{1}$$

Scalars:

$$S_\phi \;=\; \int d^4x \,\Big[\frac{N}{\lambda}\frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \partial_\mu\phi_i^\dagger\,\partial^\mu\phi_i - m_s^2\phi_i^\dagger\phi_i + g\,A^a_\mu\,\phi_i^\dagger(T^a)_{ij}\,\overleftrightarrow{\partial^\mu}\phi_j + g^2\,A^a_\mu A^{\mu b}\,\phi_i^\dagger(T^aT^b)_{ij}\,\phi_j\Big]. \tag{2}$$

We next define

$$A'_\mu \equiv \sqrt{\tfrac{N}{\lambda}}\,A_\mu \quad\Rightarrow\quad A_\mu = \sqrt{\tfrac{\lambda}{N}}\,A'_\mu.$$

and the action in terms of A' is
Fermions:

$$S_\psi = \int d^4x\,\Big[\tfrac{1}{2}\,\mathrm{Tr}(\partial_{[\mu}A'_{\nu]}\,\partial^{[\mu}A'^{\nu]}) + \sqrt{\tfrac{\lambda}{N}}\;\bar\psi_m\gamma^\mu(T^a)_{mn}\psi_n\,A'^a_\mu + \bar\psi\,i\partial\!\!\!/\psi + \cdots\Big].$$

Integrating out ψ gives a functional determinant (the Yang Mills parts are not written below):

$$Z_\psi[A'] \;=\; \int D\psi\,D\bar\psi\;e^{iS_\psi} = \exp\left\{ -\,i\,\mathrm{Tr}\,\ln\Big((i\partial\!\!\!/ - m_f)\,\delta_{ij} + \sqrt{\tfrac{\lambda}{N}}\;\gamma^\mu(T^a)_{ij}\,A'^a_\mu\Big)\right\}.$$

We can expand the logarithm as

$$-\,i\,\mathrm{Tr}\,\ln((i\partial\!\!\!/ + m_f)\,\delta_{ij}) - i\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left(\sqrt{\tfrac{\lambda}{N}}\right)^n\mathrm{Tr}\big[\big((i\partial\!\!\!/ - m_f)^{-1}\,\gamma^\mu(T^a)_{ij}A'^a_\mu\big)^n\big]\,. \tag{3}$$

Now we note that each vertex insertion contributes a factor $\sqrt{\lambda/N}$. Also, each trace is over color in the fundamental, so yields $\mathrm{Tr}(T^{a_1}\cdots T^{a_n})$ and in the large-N is of $O(1)$.
Thus we have the following

Quadratic term ($n=2$): coefficient $\sim$ (λ/N),
Cubic term ($n=3$): coefficient $\sim$ $(\lambda/N)^{3/2}$,
Quartic term ($n=4$): coefficient $\sim$ $(\lambda/N)^2$,
etc.

Scalars: For the scalar case we had

$$S_\phi = \int d^4x\;\phi_i^\dagger\Big(-(\partial^2 + m_s^2)\,\delta_{ij} + \sqrt{\tfrac{\lambda}{N}}\,A'^a_\mu(\partial^\mu\delta_{ij})(T^a)_{jk} + \tfrac{\lambda}{N}A'^a_\mu A'^{\mu b}(T^aT^b)_{ij}\Big)\phi_j.$$

So the quadratic operator is

$$\mathcal{O}_{ij} = -(\partial^2 + m_s^2)\delta_{ij} + V_{ij}, \qquad V_{ij} = \sqrt{\tfrac{\lambda}{N}}\,A'^a_\mu(\partial^\mu\delta_{ij})(T^a)_{jk} + \tfrac{\lambda}{N}A'^a_\mu A'^{\mu b}(T^aT^b)_{ij}.$$

Then

$$S_{\mathrm{eff}}[A'] = -\,i\,\mathrm{Tr}\,\ln\mathcal{O} = -\,i\,\mathrm{Tr}\,\ln(-(\partial^2 + m_s^2)\delta_{ij}) - i\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{Tr}\big[\big((-\partial^2 - m_s^2)^{-1}\,V\big)^n\big]\,. \tag{4}$$

An n-point term from expanding $\ln\det$ scales as

$$\left(\sqrt{\tfrac{\lambda}{N}}\right)^n \sim (\lambda/N)^{n/2},$$

with a single color trace $\sim O(1)$. Thus A'^2 is leading ($\sim\lambda/N$), $A'^3 \sim (\lambda/N)^{3/2}$, etc.

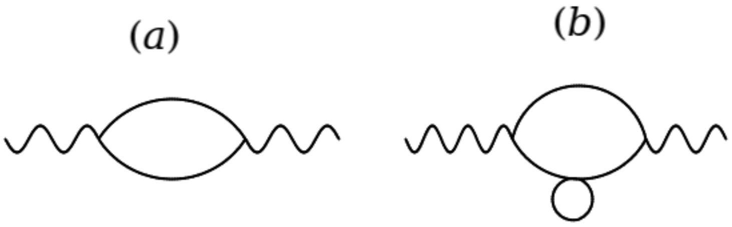

FIG. 1. Example of contributions to the A'^2 term after integrating out matter. Each gauge–matter vertex scales as $\sqrt{\lambda/N}$, so the diagrams in (a) are of order λ/N. In (b), the quartic $(\phi^\dagger\phi)^2$ insertion necessarily appears inside a closed fundamental loop and therefore introduces one boundary ($B = 1$), but this does not change the overall $\mathcal{O}(\lambda/N)$ scaling of the diagram.

We hence see that expanding the matter determinants produces a tower of gauge–field interactions:

$$A'^2 \sim \frac{\lambda}{N}, \qquad A'^3 \sim \left(\frac{\lambda}{N}\right)^{3/2}, \qquad A'^4 \sim \left(\frac{\lambda}{N}\right)^2, \ \ldots$$

When integrating over A', contractions of gauge indices can multiply these by factors of N, but we must compare them with the vertices already present in the Yang–Mills action.

$$S_{\text{eff}}[A'] \;\sim\; \Delta S^{(2)}_{\text{matter}}[A'] \;+ \tfrac{1}{2}(\partial A')^2 + \;\sqrt{\tfrac{\lambda}{N}}\,(\partial A')[A', A'] \;+\; \tfrac{\lambda}{N}\,[A', A']^2 \tag{5}$$

The effective action relevant for integrating over A' in the large-N limit is where $\Delta S^{(2)}_{\text{matter}}$ is action obtained after integrating out matter. The Yang–Mills cubic and quartic vertices, scaling respectively as $\sqrt{\lambda/N}$ and λ/N, dominate over any corresponding matter-induced vertices.

For comparison, the leading local operator induced by integrating out heavy matter is the A'^2 term, i.e. $\text{Tr}(F_{\mu\nu}F^{\mu\nu})$, which appears at order (λ/N) in Eq. (4). Its one–loop Wilson coefficient differs for fermions and scalars,

$$\Delta\mathcal{L}^{(F)}_{\text{eff}} = \frac{\lambda}{N}\,\frac{1}{24\pi^2}\ln\!\left(\frac{M^2}{\mu^2}\right)\text{Tr}(F_{\mu\nu}F^{\mu\nu}), \qquad \Delta\mathcal{L}^{(S)}_{\text{eff}} = \frac{\lambda}{N}\,\frac{1}{192\pi^2}\ln\!\left(\frac{M^2}{\mu^2}\right)\text{Tr}(F_{\mu\nu}F^{\mu\nu}), \tag{6}$$

so that the ratio of the Wilson coefficients is $c_F/c_S = 8$.

To probe current correlators, we couple the current terms in the action of both fermion and scalar theories to a classical background gauge field $\mathcal{B}$. Integrating over the matter terms produces a tower of gauge–field interactions (schematically, ignoring derivatives for now):

$$(A' + \mathcal{B})^2 \sim \frac{\lambda}{N}, \qquad (A' + \mathcal{B})^3 \sim \left(\frac{\lambda}{N}\right)^{3/2}, \qquad (A' + \mathcal{B})^4 \sim \left(\frac{\lambda}{N}\right)^2, \ \ldots \tag{7}$$

Current correlators are obtained by differentiating with respect to $\mathcal{B}$ as many times as required, then integrating over the dynamical gauge field A' and finally setting $\mathcal{B} = 0$. One can instead redefine $A' \to A' - \mathcal{B}$, so all dependence of $\mathcal{B}$ is absorbed in the Yang Mills portion of the effective action.

To show how these matter induces vertices do not contribute to evaluation of current correlators, let us begin with the $SU(N)$ Yang–Mills action written in 't Hooft normalization (we work with A instead of A' now),

$$S_{\text{YM}} = \frac{N}{\lambda}\int d^4x\,\text{Tr}\Big[\tfrac{1}{2}(\partial A)^2 + (\partial A)[A, A] + [A, A]^2\Big], \qquad A_\mu = A^a_\mu T^a, \quad \text{Tr}(T^aT^b) = T_R\delta^{ab}.$$

We introduce a background gauge field $\mathcal{B}_\mu$ by shifting $A_\mu \to A_\mu - \mathcal{B}_\mu$. The cubic term $(\partial A)[A, A]$ produces the three–gluon interaction

$$S_3 = \frac{N}{\lambda}\int d^4x\; f^{abc}(\partial_\mu A^a_\nu)A^{b\mu}A^{c\nu},$$

so the corresponding vertex is of order (N/λ) and carries one factor of the structure constant f^{abc}.

The gluon propagator obtained from the quadratic term is color–diagonal and contributes a factor (λ/N) for each internal line. The color algebra $f^{acd}f^{bcd} = C_A\,\delta^{ab} = N\,\delta^{ab}$ completes the scaling.

a. Two–point function from Yang–Mills vertices. The background two–point diagram constructed from two cubic vertices and two propagators scales as

$$\left(\tfrac{N}{\lambda}\right)^2 \left(\tfrac{\lambda}{N}\right)^2 \times N = N.$$

Hence, the pure Yang–Mills contribution to the two–point kernel is of order $O(N)$.

b. Including a single–trace quartic insertion . We organize the matter–induced interactions in a gauge–invariant, background–field expansion

$$S_{\text{eff}}^{\text{matter}}[A] = \int d^4x \Big[c_2 \ \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + c_{DF} \ \mathrm{Tr}(D_\rho F_{\mu\nu} D^\rho F^{\mu\nu}) + c_4 \ \mathrm{Tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\sigma F_\sigma{}^\mu) + \cdots \Big], \qquad c_i = O(1). \tag{8}$$

The ... also include interactions between B_μ and A_μ which appear because of the seagull terms. Their inclusion leads to same conclusions as below.

Expanding $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$ about a slowly varying background generates, in particular, a local four–gluon vertex from the c_4 term. Insert this *single–trace quartic* once in the current two–point diagram, together with two Yang–Mills cubic vertices, the color contraction yields

$$f^{acd}\ \mathrm{Tr}(T^cT^dT^eT^f)\, f^{bef} = -\,\frac{T_R}{4}\, C_A^2\, \delta^{ab} = -\,\frac{T_R}{4}\, N^2\, \delta^{ab}, \tag{9}$$

providing the overall N^2 dependence.

In this diagram the inclusion of the quartic vertex introduces two additional internal propagators, giving a total of four. Each propagator contributes a factor (λ/N) from the Yang–Mills kinetic term, while each of the two cubic vertices contributes (N/λ). The quartic matter vertex itself is $O(1)$. Combining these factors with the color factor (9) gives

$$\underbrace{\left(\tfrac{\lambda}{N}\right)^4}_{\text{4 propagators}} \times \underbrace{\left(\tfrac{N}{\lambda}\right)^2}_{\text{2 YM cubic vertices}} \times \underbrace{N^2}_{\text{color factor}} = O(1) \quad \text{(for fixed 't Hooft coupling } \lambda).$$

Hence a single–trace quartic insertion reduces the leading large–N scaling of the two–point function from $O(N)$ (pure Yang–Mills exchange) to $O(1)$.

One can easily generalize and conclude that in the 't Hooft limit all higher matter vertices are negligible. This explains why the current correlators of scalar and fermion matter coincide at leading order in $1/N$:

$$\left\langle J^{a_1}_{\mu_1}(x_1)\cdots J^{a_n}_{\mu_n}(x_n)\right\rangle_{\text{scalar}} = \left\langle J^{a_1}_{\mu_1}(x_1)\cdots J^{a_n}_{\mu_n}(x_n)\right\rangle_{\text{fermion}} + \mathcal{O}\!\left(\tfrac{1}{N}\right).$$

DISCUSSION

The analysis presented here shows that in the large-N limit, four-dimensional $SU(N)$ Yang–Mills theories coupled to fundamental scalars or fermions display an identical gauge-sector response, independent of the matter content or the 't Hooft coupling. The equality of gauge-invariant current correlators arises because, at large N, the dynamics is dominated by the universal Yang–Mills self-interactions while matter-induced vertices remain subleading. This represents a distinct mechanism of boson–fermion universality compared with the $(2+1)$-dimensional Chern–Simons–matter dualities, where gauge fixing $(A_0 = 0)$ removes self-interactions and the equivalence is driven by *gauge decoupling*. Here, by contrast, the universality emerges from *gauge dominance*: the Yang–Mills self-interactions control the leading large-N dynamics, enforcing an identical effective behavior for both bosonic and fermionic matter.

It should be emphasized that this equivalence relies on the locality of the effective gauge theory obtained after integrating out the matter. For *massless* matter fields, the polarization tensor behaves as $\Pi(p^2) \sim \log(p^2/\mu^2)$, producing nonlocal terms in the effective action of the form

$$\int d^4x\, d^4y\, F_{\mu\nu}(x) \frac{1}{(x-y)^4} F^{\mu\nu}(y),$$

which invalidate any derivative expansion. Finite matter masses regulate these infrared singularities and render the determinant analytic in p^2/m^2, ensuring that the large-N analysis proceeds within a local, renormalizable effective theory. Hence, the universality established here pertains to the *infrared-regulated* theory, where the Yang–Mills backbone remains dominant.

The conclusion therefore holds for generic renormalizable Yang–Mills–matter theories, suggesting a broader universality class in four dimensions that parallels the role played by Chern–Simons dualities in three. It would be interesting to explore whether such gauge-dominated universality connects to large-N orbifold/orientifold equivalences or to the planar limits underlying AdS/CFT correspondence.

We note that even though calculations were done assuming integrating single scalar and fermion the derived result apply in the 't Hooft limit where number of scalars/fermions $N_f = \mathcal{O}(1)$ with massive matter (ensuring a local derivative expansion). It does not persist in the Veneziano limit $N_f \sim N$.

CONCLUSION

We have shown that four-dimensional $SU(N)$ Yang–Mills theories coupled to either massive fermionic or scalar matter possess a universal large-N structure in their current correlators. After integrating out the matter fields, the Yang–Mills kinetic and interaction terms dominate the effective action, while the matter-induced vertices are suppressed by powers of λ/N. Consequently, the leading gauge dynamics, and hence the current–current correlators, are identical for fermionic and scalar theories and remain independent of the 't Hooft coupling. This equivalence arises not from a duality transformation but from the hierarchy of interaction strengths that emerges once matter is integrated out, ensuring the universality of large-N behavior in four dimensions. In contrast, for three-dimensional Chern–Simons–matter systems, gauge fixing to $A_0 = 0$ removes nonlinear Yang–Mills–type interactions, allowing matter-induced vertices to dominate and produce non-trivial λ-dependent dynamics.

A natural extension of this work is to examine subleading $1/N$ corrections and the inclusion a ϕ^4 coupling to test the robustness of this universality. Indeed, since the present arguments rely only on the relative large-N scaling of gauge and matter vertices, introducing a ϕ^4 coupling merely modifies the merely renormalizes internal loops without modifying the large-N hierarchyreinforcing the generality of this gauge-dominated universality as depicted in Fig.1(b) .

Since these correlator are spin–blind at leading order, our result predicts that, for fixed t'Hooft coupling and identical mass parameters, the ratio $\langle JJ\rangle_{\mathrm{fermion}}/\langle JJ\rangle_{\mathrm{scalar}}$ approaches unity up to $\mathcal{O}(1/N)$ corrections. One can check this either through lattice simulations in the heavy–flavor regime or within holographic models in which adjoint currents couple universally to bulk gauge modes.

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