Poincaré invariance and the Unruh effect

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1 Abstract

We show that the Unruh effect, a well accepted theoretical prediction, is actually absent in a rigorously Poincaré-invariant analysis. This result questions the effect's objective existence. An analogy with classical mechanics, which obeys Galilean invariance rather than Poincaré invariance, is illustrative: non-Galilean frames display pseudoforces; incorporating some (e.g., the Coriolis force), while overlooking others (e.g., the centrifugal force) leads to incorrect conclusions. Galilean frames avoid such pitfalls. Similarly, standard derivations of the Unruh effect are Poincaré-violating and thus incomplete. In contrast, the Poincaré-invariant analysis reported here displays no Unruh effect.

2 Introduction

The Unruh effect is the name given to a theoretical prediction that an accelerated observer can detect particles which are unobservable to an inertial observer [1–4]. The effect is often interpreted as a consequence of the intrinsic complexity of the inertial frame vacuum, which is commonly imagined to be populated by pairs of virtual particles that become detectable in an accelerated frame. However, vacuum complexity can be viewed as a pseudoeffect expressed in formalisms which do not manifestly enforce causality [5], such as instant-form (IF) dynamics, a commonly used approach for the quantization of field theories. In contrast, the explicitly causality-enforcing front-form (FF) framework of Dirac [6], has an essentially trivial vacuum, *i.e.*, a structureless Fock space ground state [7].

A question thus arises: Does acceleration complexify the FF vacuum, so that the Unruh effect is also a feature of the FF? If not, then the Unruh effect is itself a pseudoeffect, arising only in approaches that violate causality at intermediate stages of a calculation. This conclusion would have profound implications, since by Einstein's equivalence principle between gravity and acceleration [8], the Unruh and Hawking effects [9] are equivalent. Alternatively, if the Unruh effect occurs in FF dynamics, then the usual interpretation of Unruh and Hawking phenomena, based on particles created from of a supposedly complex quantum vacuum, is mistaken, because the effects would also emerge with a trivial vacuum.

Herein, we discuss the following three distinct cases: (1) IF Minkowski space (*i.e.*, inertial) vacuum, which is complex; (2) FF Minkowski space vacuum, which is trivial – see Section 5 and the Section on FF vacuum A; (3) "Rindler vacuum", associated with an accelerating frame for which "Rindler coordinates" are defined – see Section B.2. The Rindler vacuum is complex because Rindler fields are quantized at equal Rindler time. Importantly, the three vacua are inequivalent because IF, FF and Rindler frames are not related by any Poincaré transformation. Notably, although the Rindler vacuum is complex, it is, nevertheless, the vacuum for a Rindler observer; hence, imperceptible by definition. In this connection, the Unruh effect is a statement that a Rindler (*viz.* accelerated) observer perceives pairs that constitute the IF Minkowski space vacuum.

In section 3, we summarize important features of Dirac's three forms of dynamics [6] and, drawing on an analogy with classical Newtonian dynamics, explain the emergence and character of pseudoeffects in the IF formalism. In Section 4, we explain why an Unruh effect is predicted when fields are quantized using the common IF approach, whereas that is not so when fields viewed from the inertial frame are quantized using the FF. The connection be-
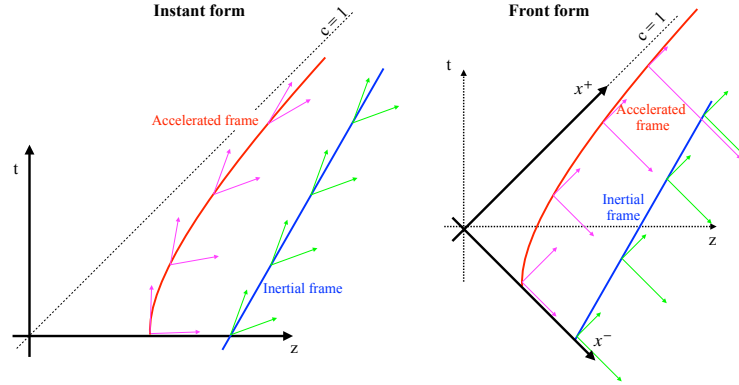
Figure 1: Effect of boosts (blue worldline) and constant accelerations (red worldline) on attached IF (left) and FF (right) frames. Thick arrows show the frame axes of an observer at rest. The 45° dashed line is the trajectory for a particle moving at light speed ($c \equiv 1$ in natural units). The $(-,+,+,+)$ metric is used in this figure, but $(+,-,-,-)$ is used for FF in the main text, following usage in particle physics where FF dynamics is employed. In that case, x^- has the opposite direction, pointing left and upward.

tween a complex vacuum and causality violation in IF dynamics is elucidated in Section 5 on commutation relations. In Section 6, we discuss how the IF and FF findings can be reconciled. This must be possible because natural (observable) phenomena do not depend upon the approach used to explain them. Section 7 supplies concluding remarks.

3   Forms of Dynamics

There is no unique choice of time parameter in relativistic kinematics. Parametrizing the worldline amounts to a specific convention for foliating the four-dimensional (4D) spacetime into 3D space + 1D time. There are three pertinent conventions (forms) fulfilling relativistic invariance, basic causality, and fundamental spacetime symmetry requirements [6]: IF, where time is the usual Galilean time (Fig. 1, left panel); FF, where time is aligned tangentially to the light-cone (Fig. 1, right panel); and Point-Form (PF), where the equal time hypersurfaces are hyperboloids. For future reference, the FF coordinates are

$$\tau \equiv x^+ := t + z \quad \text{(time)}; x^- := t - z \quad \text{(space)}; \tag{1}$$

$$p^- := \omega - p \quad \text{(energy)}; p^+ := \omega + p \quad \text{(momentum)}. \tag{2}$$

The labels here highlight the FF associations, and (ω, p) is the IF energy-momentum. Importantly, distinct forms of dynamics are not related by Poincaré transformations. Furthermore, they are not equivalent: each form of dynamics possesses a different stability group, *viz.* a distinct set of Poincaré generators that are purely kinematical. Such generators operate in constant form-time hypersurfaces, whereas dynamical operators evolve the system from one hypersurface to the next.

The Poincaré group has 10 generators: 4 linear momenta for spacetime translations; 3 angular momenta for space rotations; and 3 Lorentz boosts. The FF stability group is the largest, with 7 generators: the linear momenta; boosts; and one angular momentum. The IF and PF stability groups have only 6 generators: IF – linear and angular momenta; and PF – the boosts and angular momenta.

Besides FF's larger stability group, crucial advantages are offered by the kinematical character of its boosts [10]. For instance, contrary to IF dynamics, popular in textbooks, and to

other forms of dynamics, such as PF and oblique FF [11], FF dynamics manifestly preserves causality and the number of particles in systems related by boosts. These things are key to its success in proving exact results in high-energy physics – see, *e.g.*, Ref. [12].

In stark contrast, IF does not straightforwardly respect causality because it allows for vacuum-induced particle pairs, whose creation events are uncorrelated with the worldline of the system; hence, acausal. Furthermore, the dynamics induced when IF-boosting a system often complicates its description with pseudoeffects [10]. As elaborated in Section 5, since commutation relations are imposed at equal form-time in canonical quantization *i.e.*, along the light-cone in the FF case, the FF vacuum is trivial. (The possible existence of zero-modes [5] is irrelevant to the Unruh effect – see discussion on FF vacuum Section A.)

The conclusion reached in this article is that the Unruh effect is a pseudoeffect. Before demonstrating that, it is useful to elucidate the meaning and significance of this conclusion using an analogy from non-relativistic mechanics.

The symmetry of spacetime in the non-relativistic limit is Galilean invariance. Analyses performed in a frame that violates this basic symmetry, *i.e.*, a non-Galilean (non-inertial) frame induce pseudoeffects, such as Coriolis, centrifugal and Euler forces. In the relativistic case, the spacetime symmetry is Poincaré invariance and analyses in frameworks that violate it likewise generate pseudoeffects. Once spacetime is foliated into space and time, preserving Poincaré invariance in the foliated space requires that there exists a single dynamical operator (the time-evolution operator, typically the Hamiltonian), with all other operators being kinematical, otherwise they would also evolve the system. Plainly, since the Hamiltonian is the quantity that contains the fundamental interactions, evolution generated by other operators is merely an expression of pseudodynamics.

Dirac showed that pseudodynamics arise for the IF foliation because, in addition to the Hamiltonian, the three boost operators are dynamical [6]. Thus, pseudoeffects appear each time the description of a system requires a change of frame, *e.g.*, in particle physics, where particles are frequently boosted from one frame to another and, even more clearly, with the Unruh effect, where several frames enter its description.

However, Dirac also showed that for the FF foliation, there are only 2 dynamical operators besides the Hamiltonian, and that these correspond to two space rotations. The rotation axes are transverse to the spatial coordinate z used to define the FF coordinates, (1), typically the direction of the particle motion. The two rotations are irrelevant in particle physics problems where the only rotational symmetry is around the displacement axis. Thus, FF respects Poincaré invariance for particle physics studies. Likewise, rotational invariance in the transverse directions is unconnected with the Unruh problem, and FF also respects Poincaré invariance in that case.

The analogy between the Unruh effect and classical pseudodynamics is now apparent. Nevertheless, a few points may benefit from elaboration. Just as the derivation of, *e.g.*, the centrifugal force in a non-Galilean frame is valid, the derivation of the Unruh effect in IF is valid. We do not suggest otherwise. Rather, the central issue is that the effect originates from pseudodynamics and is therefore neither an objective nor fundamental part of quantum field theory (QFT). The Unruh effect is necessary for the consistency of any IF analysis and it survives if that analysis is incomplete. Drawing again on the non-relativistic analogy, just like overlooking a pseudoforce (*e.g.*, the Coriolis force) would lead to an incorrect conclusion, not accounting for the Unruh effect in an IF analysis will lead to internal inconsistencies. A corollary is that *all* pseudodynamics must be accounted for, which is a non-trivial requirement. Just like analyses in Galilean frames avoid these complications, analyses in the Poincaré-preserving FF do, too. A FF analysis, complete by Dirac's construction, shows that the Unruh effect is not observable because, in contrast to the aforementioned analogy, where non-relativistic pseudoeffects can be experienced by attaching oneself to a non-Galilean frame, the IF or FF frameworks have no

133 material realization. The perception that the Unruh effect is observable thus originates from
134 an incomplete IF analysis.

135   In Section 6, we discuss another pseudoeffect that balances the Unruh effect, ultimately
136 reconciling the IF and FF conclusions. Again, our arguments do not entail that IF analyses
137 which produce an Unruh effect are "wrong", only that such analyses are incomplete. They have
138 omitted a subjective pseudodynamical effect whose contribution is required when one adopts
139 a choice of analysis framework that does not preserve relevant symmetries at intermediate
140 stages of a calculation.

4   Unruh effect

142 Basic discussions of an Unruh effect focus on a free scalar field in (1+1) spacetime dimensions.
143 Here, we also follow this simplification.

144   The Unruh effect is predicted in IF dynamics because in non-inertial IF frames a field ϕ
145 cannot unambiguously be factored into distinct time-dependent and space-dependent parts,
146 namely, separated in a Poincaré-invariant manner [13]. Consequently, in the decomposition
147 of ϕ into what may, for ease of discussion, be called positive and negative frequency modes,
148 f_p, f_p^*:

$$\phi = \int dp \left(\hat{a}_p f_p + \hat{a}_p^\dagger f_p^* \right), \tag{3}$$

149 the assignment of positive or negative frequency mode is frame dependent. ($\hat{a}_p$, $\hat{a}_p^\dagger$ are, re-
150 spectively, annihilation, creation operators. f_p is a mode. A positive (resp. negative) frequency
151 mode is defined as $\partial_t f_p = -i\omega f_p$ (resp. $\partial_t f_p = i\omega f_p$), with t the proper time, a Minkowski metric
152 sign (-.+,+,+) and the frequencies being always positive, $\omega \geq 0$.)

153   In another frame, $\phi = \int dp' \left(\hat{b}_{p'} f_{p'} + \hat{b}_{p'}^\dagger f_{p'}^* \right)$, with $\hat{b}_{p'} = \alpha \hat{a}_p + \beta \hat{a}_p^\dagger$: $\beta \neq 0$ if one of the IF
154 frames is non-inertial. A Bogolyubov transformation [14] expresses this mixing:

$$\begin{bmatrix} \hat{b}_{p'} \\ \hat{b}_{p'}^\dagger \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \beta_{12} \\ \beta_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \hat{a}_p \\ \hat{a}_p^\dagger \end{bmatrix}. \tag{4}$$

155 Since $\hat{a}_p^\dagger$ and $\hat{b}_{p'}^\dagger$ are creation operators, the vacuum of frame 1, satisfying $\hat{a}_p |0\rangle_1 \equiv 0$ by defini-
156 tion, is perceived from frame 2 as containing particles: $\hat{b}_{p'} |0\rangle_1 = (\alpha_{11}\hat{a}_p + \beta_{12}\hat{a}_p^\dagger) |0\rangle_1 = \beta_{12} |p\rangle \neq 0$.

157   Typical formal analyses leading to a prediction of the Unruh effect, involving Rindler
158 frames, are provided pages 12 and 16. Here, we approach the problem by beginning with
159 a boost and observing the effect of making its rapidity time-dependent, so that the boosted
160 frame accelerates. We perform the analysis for IF and FF dynamics in parallel, using normal
161 fonts for IF-specific formulae and **bold fonts** for their **FF equivalents**. The IF analysis em-
162 ploys the *conventional relativistic* metric $(-,+,+,+)$, with which the Unruh effect is usually
163 discussed, whereas the FF derivation follows the *particle physics convention* **(+,−,−,−)**. The
164 latter implies the FF product $\boldsymbol{a^\mu b_\mu = (1/2)(a^+ b^- + a^- b^+)}$ in (1+1)D spacetime. Given the
165 metrics, the positive frequency modes are (recall $\tau = x^+$):

$$f_p \propto e^{i(\omega t - pz)} \text{ (IF)}; \quad \boldsymbol{f_{p^+} \propto e^{-i/2(p^- \tau + p^+ x^-)}} \textbf{ (FF)}. \tag{5}$$

166   For pedagogy, we first discuss the case of inertial frames and then proceed to the acceler-
167 ated frame case.
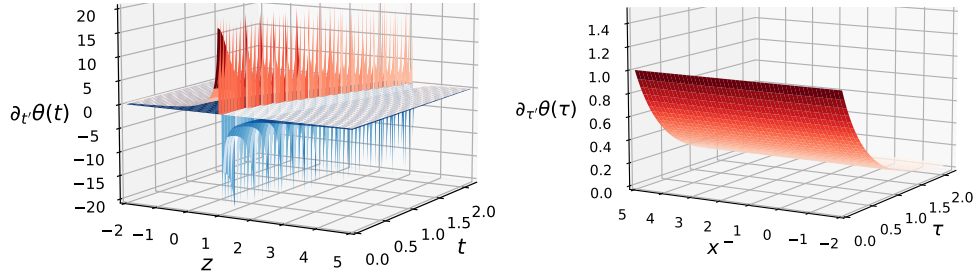
Figure 2: Derivative of the accelerated frame rapidity with proper time. Left panel: IF case $\partial_{t'}\theta(t)$. Right panel: FF case $\partial_{\tau'}\theta(\tau)$. Red indicates positive values and blue, negative values. The swiftly oscillating sign in the IF case reveals the Unruh effect: the inertial frame's positive and negative frequency modes become mixed in the IF accelerated frame. The discrepancy in mode definitions between the IF inertial frame and the accelerated frame entails a non-diagonal Bogolyubov matrix; hence, an Unruh effect. In the FF case, time and space do not mix: $\partial_{\tau'}\theta(\tau)$ is independent of x^-. Consequently, the sign of $\partial_{\tau'}\theta(\tau)$ stays the same, allowing for a consistent definition of positive and negative frequency modes between the FF inertial and accelerated frames. The Bogolyubov matrix is diagonal, precluding an Unruh effect in the FF framework.

4.1 Inertial frame case

It is straightforward to show that a positive (or negative) frequency mode of a wave expressed in an inertial frame remains so in another inertial frame *viz.* one related to the first frame by a Lorentz boost. Figure 1 shows the effect of the boost (blue straight worldline with green coordinate axes): in IF dynamics, it generates a rotation of the coordinate axes through an angle determined by θ, the rapidity, whereas the orientation of the axes is unaffected in FF.

The Lorentz boost transformation formulae in the IF and FF frameworks are:

$$
\begin{aligned}
t' &= t\cosh\theta - z\sinh\theta \quad \text{(IF)}; & \tau' &= e^{\theta}\tau & \text{(FF)}, \\
z' &= z\cosh\theta - t\sinh\theta \quad \text{(IF)}; & x^{-\prime} &= e^{-\theta}x^- & \text{(FF)},
\end{aligned}
\tag{6}
$$

or, for energy-momentum:

$$
\begin{aligned}
\omega' &= \omega\cosh\theta - p\sinh\theta \quad \text{(IF)}; & p^{+\prime} &= e^{\theta}p^+ & \text{(FF)}, \\
p' &= p\cosh\theta - \omega\sinh\theta \quad \text{(IF)}; & p^{-\prime} &= e^{-\theta}p^- & \text{(FF)}.
\end{aligned}
\tag{7}
$$

The inverse transforms are straightforwardly obtained by changing $\theta \to -\theta$ and switching $t' \leftrightarrow t$, etc.:

$$
\begin{aligned}
t &= t'\cosh\theta + z'\sinh\theta \quad \text{(IF)}; & \tau &= e^{-\theta}\tau' & \text{(FF)}, \\
z &= z'\cosh\theta + t'\sinh\theta \quad \text{(IF)}; & x^- &= e^{\theta}x^{-\prime} & \text{(FF)}.
\end{aligned}
\tag{8}
$$

In the boosted frame, the sign of a mode frequency can be identified via its time derivative:

$$
\frac{\partial f_p}{\partial t'} = \frac{\partial x^\mu}{\partial t'}\partial_\mu f_p = \left[\frac{\partial t}{\partial t'}\partial_t + \frac{\partial z}{\partial t'}\partial_z\right]f_p \qquad \text{(IF)};
$$

$$
\frac{\partial f_{p^+}}{\partial \tau'} = \frac{\partial x^\mu}{\partial \tau'}\partial_\mu f_{p^+} = \left[\frac{\partial \tau}{\partial \tau'}\partial_\tau + \frac{\partial x^-}{\partial \tau'}\partial_{x^-}\right]f_{p^+} \quad \text{(FF)}.
\tag{9}
$$

Using Eqs. (5) and (8), this becomes:

$$
\frac{\partial f_p}{\partial t'} = i\left[\omega\cosh\theta - p\sinh\theta\right]f_p \quad \text{(IF)};
$$

$$
\frac{\partial f_{p^+}}{\partial \tau'} = -e^{-\theta}\frac{i}{2}p^- f_{p^+} \qquad \text{(FF)},
\tag{10}
$$

180 or, using Eq. (7),

$$\frac{\partial f_p(x^\mu, p^\mu)}{\partial t'} = i\omega' f_p(x^\mu, p^\mu) \qquad \text{(IF)};$$

$$\frac{\partial f_{p^+}(x^\mu, p^\mu)}{\partial \tau'} = -\frac{i}{2} p^{-\prime} f_{p^+}(x^\mu, p^\mu) \quad \text{(FF)}, \tag{11}$$

181 *i.e.*, the frequency in a boosted frame is the boosted frequency, ω' or $p^{-\prime}$. The same holds
182 for negative frequency modes, f_p^* and $f_{p^+}^*$. Using IF, the dispersion relation $\omega = |k|$ implies
183 $\omega' = \omega(\cosh\theta \pm \sinh\theta) = \omega e^{\pm\theta}$. With FF, one has $p^{-\prime} = p^- e^{-\theta}$ ((7)). An exponential func-
184 tion is always positive, so the signs of the mode derivatives are consistent in the initial and
185 boosted frames. Thus, for a time-independent boost, there is no mixing of positive and neg-
186 ative frequency modes; the Bogolyubov transform is diagonal; and the vacua of the original
187 and boosted inertial frames coincide.

188 4.2 Accelerated frame case

189 We now consider transformation to an accelerated frame. The latter can be formalized by
190 making θ vary with time – see Fig. 1. We set $\theta = ut$ or $\theta = v\tau$, with $u \neq 0 \neq v$ and both
191 small, so that acceleration is constant to a good approximation. Importantly, θ is not explicitly
192 space-dependent:

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{(IF)}, \qquad \frac{\partial \theta}{\partial x^-} = 0 \quad \text{(FF)}. \tag{12}$$

193 Using IF dynamics and the appropriate entries in Eqs. (6–8), then (9) yields:

$$\begin{aligned}
\frac{\partial f_p}{\partial t'} &= \big[\big([1 + z'\partial_{t'}\theta]\cosh\theta + t'\partial_{t'}\theta\sinh\theta\big)\partial_t + \\
&\quad \big([1 + z'\partial_{t'}\theta]\sinh\theta + t'\partial_{t'}\theta\cosh\theta\big)\partial_z\big]f_p \\
&= i\big[\omega' + \partial_{t'}\theta(\omega z - tp)\big]f_p \, ;
\end{aligned} \tag{13}$$

194 whereas, using the FF,

$$\begin{aligned}
\frac{\partial f_{p^+}}{\partial \tau'} &= \big[e^{-\theta}(1 - \tau'\partial_{\tau'}\theta)\partial_\tau + e^\theta \partial_{\tau'}\theta x^{-\prime}\partial_{x^-}\big]f_{p^+} \\
&= -\frac{i}{2}\big[p^{-\prime} + \partial_{\tau'}\theta(x^- p^+ - \tau p^-)\big]f_{p^+}\, .
\end{aligned} \tag{14}$$

195 Evidently, the IF and FF expressions are similar: (13) *cf.* (14). Moreover, reviewing Eqs. (6–8),
196 one sees that $\omega z - tp = \omega'z' - t'p'$ and $x^- p^+ - \tau p^- = x^{-\prime}p^{+\prime} - \tau'p^{-\prime}$; the combinations are
197 Poincaré invariant scalar products, which we write as $\tilde{p}^\mu x_\mu = p^\mu \tilde{x}_\mu$:

$$\frac{\partial f_p}{\partial t'} = i\big[\omega' + p^\mu \tilde{x}_\mu \partial_{t'}\theta\big]f_p \qquad \text{(IF)};$$

$$\frac{\partial f_{p^+}}{\partial \tau'} = -\frac{i}{2}\big[p^{-\prime} + p^\mu \tilde{x}_\mu \partial_{\tau'}\theta\big]f_{p^+} \quad \text{(FF)}. \tag{15}$$

198 Compared with the inertial case, (10), a Poincaré-violating term $\partial_{t'}$ or $\partial_{\tau'}$ has appeared.
199 Of course, the appearance of that derivative means the new term violates Poincaré invariance,
200 as was to be expected since inertial and accelerated frames are not related by genuine boosts,
201 for which $\theta = $ constant.

202 Considering the IF case, the time direction changes along the accelerated worldline; so,
203 time and space mix – see Fig. 1. This is expressed in the result

$$\partial_{t'}\theta(t) = u\cosh(ut')[1 + uz' + ut'\tanh(ut')]$$
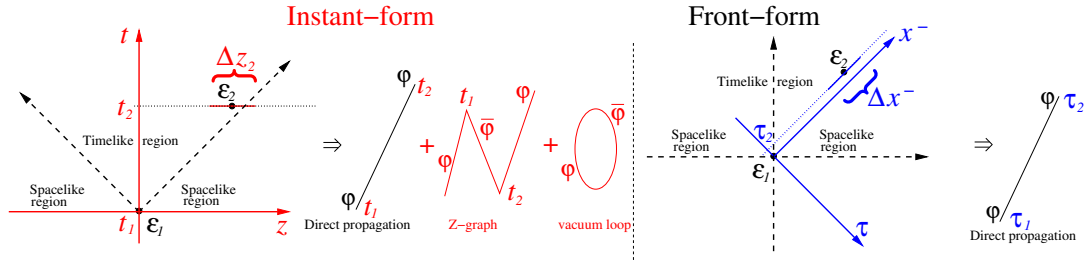
Figure 3: Origin of IF vacuum complexity. A field φ propagates between events $\mathcal{E}_1$ and $\mathcal{E}_2$. Equal-time commutation relations imply uncertainty relations along the fixed-time hypersurface (dotted line). In IF dynamics, timelike events with respect to $\mathcal{E}_1$ may become spacelike owing to the Heisenberg uncertainty Δz_2 (left panel). Thus, the time-ordering of $\mathcal{E}_1$ and $\mathcal{E}_2$ is frame-dependent. When $t_2 < t_1$, causality-violating Z-graphs arise. Their negative probabilities are compensated by vacuum loops, rendering the vacuum complex. The loops' virtual particles may borrow 4-momentum from an accelerated frame and become observable in that frame (Unruh effect). In FF dynamics, commutation relations are set at fixed FF time τ. Thus, the uncertainty Δx^- never accesses the spacelike region: $\mathcal{E}_2$ is always timelike with respect to $\mathcal{E}_1$ (right panel). No Z-graphs arise, virtual loops are forbidden by unitarity, and the vacuum is trivial. Without vacuum loops, no Unruh effect emerges.

$$= u\,\mathrm{sech}(ut)/[1 - uz + ut\tanh(ut)]. \tag{16}$$

The variation of the sign of $\partial f_p/\partial t'$ makes evident that one cannot develop a Poincaré-invariant separation of the field ϕ into time and space components. Since $p^\mu \tilde{x}_\mu = \omega(z \pm t) \geq 0$, the sign of $\partial f_p/\partial t'$ is determined by $\partial_{t'}\theta$, (16). Fig. 2 reveals its rapid oscillation with t and z (or t' and z'). So, what were positive and negative frequency modes in the IF inertial frame mix in the IF accelerated frame: the Bogolyubov matrix is not diagonal and there is an Unruh effect in this development of the IF case.

Using FF, on the other hand, the time direction remains fixed along the accelerated world-line – see Fig. 1. Thus, the time-derivative of θ is solely a function of time:

$$\partial_{\tau'}\theta(\tau) = v\mathrm{e}^{-v\tau'}[1 - v\tau'] = v\mathrm{e}^{-v\tau}/[1 + v\tau]. \tag{17}$$

Consequently, with time and space directions remaining mutually perpendicular, ϕ can still be unambiguously expanded over positive and negative frequency modes, even for accelerated frames. Indeed, the sign of $\partial f_{p^+}/\partial \tau'$ is determined by $\partial_{t'}\theta$, (17), which sign is constant – see Fig. 2, right panel. Therefore, the definition of positive and negative frequency modes in the FF inertial frame and accelerated frame are consistent, the Bogolyubov matrix remains diagonal, and the vacua of frames coincide and are trivial: *there is no Unruh effect in FF dynamics*.

There is a straightforward explanation for arriving at a prediction of the Unruh effect when using IF dynamics but not with the FF. Namely, boost operators are dynamical in the IF – see discussion Section 3: they mix kinematics and dynamics, leading to non-conservation of particle number. In an accelerating frame, this mixing changes with time. Thus, non-stationary dynamics emerges with energy being transferred between systems, which would be an observable effect, unless cancelled by some other process. In contrast, FF boosts are kinematical operators. Hence, no dynamical effects are introduced by acceleration. So, an Unruh detector remains quiet.

5 Connection to commutation relations

Canonical quantization of fields is performed at constant proper time, *i.e.*, with fields defined along the fixed z-direction for the IF inertial frame (Fig. 1, left panel), along the varying z-direction for the IF accelerated frame, and along the fixed $45°$ x^--direction for both inertial and accelerated FF frames (Fig. 1, right panel).

In canonical quantization, the Heisenberg uncertainty principle originates from commutation relations [15]. Since they are imposed at equal time, the uncertainty principle also operates on equal time hypersurfaces. This results in a complex vacuum when time is defined as IF time because events that should be causally linked, *i.e.*, timelike-separated, may become spacelike-separated owing to position uncertainty.

Consider Fig. 3, which shows events $\mathcal{E}_1$ and $\mathcal{E}_2$ at times t_1 and t_2 for the propagation of a massive field. If $\mathcal{E}_1$ and $\mathcal{E}_2$ are spacelike-separated, the time-ordering of $\mathcal{E}_1$ and $\mathcal{E}_2$ is frame-dependent with $t_2 < t_1$ in some frames where they form Z-graphs with event $\mathcal{E}_2$ the spontaneous appearance of a particle-antiparticle pair. The antiparticle annihilates at later time t_1 with the initial particle. This description is acausal since the pair appearance is spontaneous and cannot be caused by the original particle propagation owing to the spacelike interval. Z-graphs contribute negative probabilities to the propagation, so disconnected creation-annihilation diagrams (vacuum loops), balancing the negative probabilities, must be introduced to preserve unitarity [16]. In accelerated frames, the virtual particles in the vacuum loops may borrow 4-momentum from the acceleration process and, unless this is somehow cancelled, become real and observable, leading to the Unruh effect.

An acceleration of a FF frame continuously rescales the axes (Fig. 1, right panel) without reorienting them; hence, the uncertainty principle continues to operate along the light-cone direction x^-. The events $\mathcal{E}_1$ and $\mathcal{E}_2$ remain timelike-separated, causality is preserved, and unitarity forbids vacuum loops [10]. Vacuum simplicity persists in an accelerated FF frame and, without vacuum loops in any type of FF frame, there is no Unruh effect. Consistently, momentum conservation also prohibits vacuum loops: FF particles must have non-negative momenta p^+ [5, 17, 18]; and since $p^+ = 0$ for the vacuum, by definition, [19] one of the particles of the hypothetical vacuum loop would have $p^+ < 0$, which is forbidden.

6 Reconciling Instant and Front Form Descriptions

Physics is independent of any choice of frame or framework. Therefore, IF and FF conclusions must finally agree. Here, we describe how one may reconcile the apparently conflicting pictures drawn above.

Following Ref. [20], which reconciled the IF and FF descriptions of deuteron structure, we consider an Unruh detector [3, 21] made of deuterium atoms. Such a detector would monitor the deuteron distribution between its ground state $|0\rangle$ (the spin-0 singlet state) and its excited state $|1\rangle$ (the spin-1 triplet state). The temperature T is obtained from the population distributions: $N_i = N \exp(-E_i/[k_B T])/Z$, where N is the total number of atoms; E_i, the energy of state $|i\rangle$, k_B, Boltzmann's constant, and Z, the partition function. Irrespective of the Unruh effect, the detector *structure* in the IF case continuously changes with time because of a combination of three facts: (1) an IF boost is dynamical; (2) the rapidity becomes time-dependent for accelerations, $\theta \to \theta(t)$; and (3) a boost is equivalent to a Rindler time translation – see Section C.1. Specifically, the detector structure evolves because the $|0\rangle$ and $|1\rangle$ populations mix owing to an IF-induced spin-orbit force [10, 22] that causes $|1\rangle$ to depopulate in favor of $|0\rangle$, *i.e.*, the detector cools.

Since no cooling occurs in the FF, and physics is independent of the theoretical framework,

an IF source of temperature is required to maintain the initial balance of $|0\rangle$ and $|1\rangle$, despite the continuous depopulation of $|1\rangle$. The ready candidate is the Unruh temperature. The first IF effect – the cooling – has generally been overlooked. Unruh detectors are typically described in terms of energy levels without considering the dynamics of the internal detector structure responsible for those levels [23]. Without such consideration, one cannot account for the IF boost dynamical effect. Overlooking detector cooling, there is no compensation mechanism for the Unruh effect; hence, it seems objectively observable.

In FF dynamics, none of this occurs because boosts are kinematical (no cooling) and the FF vacuum is trivial (no Unruh temperature). Since physics is independent of choice of frame and form, then the two IF effects must compensate each other so that IF and FF agree. It is now evident, however, that since boost-induced dynamics are overlooked in the usual IF analyses that lead to an Unruh effect, it has hitherto been erroneously identified as observable.

7 Discussion and conclusion

The Unruh effect is a widely held to be a feature of QFT. It owes its importance mainly to its equivalence with Hawking radiation, via Einstein's equivalence principle between gravity and acceleration. It is popularly claimed that the Unruh effect needs no empirical verification, beyond that involved in validating Poincaré-invariant QFT as a description of Nature. We have exposed the flaw in that position: FF quantization is a powerful predictive tool in QFT; yet, this approach reveals that prediction of an Unruh effect is an erroneous consequence of causality-violating aspects of conventional IF dynamics.

The crucial point is that any apparent dynamics generated by kinematic transformations, here the Lorentz boosts, is pseudodynamics by definition. In IF, boosts have long been known to generate such effects, while they are purely kinematical in FF. Then, a natural question is whether there should also be a rotational Unruh effect generated by centripetal acceleration. Evidently, the naive answer from the standard approach is *yes*. However, IF rotations are purely kinematical operations and, indeed, even if the IF is used to study a purely rotational Unruh effect, one finds that the effect is absent [**?**, **?**, **?**]. This accords with our present finding: in FF, boosts are kinematical, leading to no Unruh effect, and in IF, rotations are kinematical, leading to no rotational Unruh effect – counterintuitively from the usual IF standpoint. (There is also no rotational Unruh effect with the FF because the two relevant rotations are also kinematical.) This strongly supports the conclusions arrived at herein.

Properly, the Unruh effect is cancelled in IF dynamics by another pseudoeffect, namely, cooling of the Unruh detector owing to dynamical evolution under a succession of IF boosts. Consequently, notwithstanding popular perspectives, one is left with a final point that should not be lightly dismissed; namely, neither the Unruh effect nor any of its supposed corollaries can be considered real without empirical verification.

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A Front Form Vacuum

No heating — That virtual particle loops are absent from the front form (FF) vacuum is firmly established. Nevertheless, the role of zero-momentum modes in the FF vacuum is a topic of intense discussion, *e.g.*, such issues as whether, in flat spacetime, the vacuum energy can be renormalized away are widely canvassed [24, 25]. Though important in themselves, these questions are irrelevant to the Unruh effect because $p^+ = 0$ modes cannot transfer momentum (equally, kinetic energy) to the particles forming the Unruh detector/thermometer: zero-modes cannot heat thermometers.

Event horizons do not split zero modes — The popular interpretation of the Unruh and Hawking effects suggests an intuitive explanation of why zero-modes are irrelevant in this connection. In that interpretation, one of the particles of the vacuum loop falls beyond the event horizon (for the Unruh effect, the Killing horizons are the x^+ and x^- axes); so, cannot annihilate with its partner. Such a process requires space separations between the two particles and evolution, while zero-modes have no space or time extension, making them irrelevant. Thus, either zero-modes are immaterial; or the vacuum pair interpretation is erroneous; or it is instant form (IF) specific, hence can play no role in an objective explanation.

Constraints from General Relativity — Considering that, by Einstein's equivalence principle, accelerated frames are equivalent to curved spacetime, one arrives at another argument for the irrelevance of zero-modes, as well as a new perspective on the Unruh effect.

Indeed, it is known that the definition of positive and negative frequency modes is ambiguous in a curved spacetime/non-inertial frame, in contrast to Minkowski spacetime. This is the formal reason for the Unruh effect, see Section 4. These features apparently contradict the basic principle of general relativity that one must tend to a Minkowski spacetime in the limit of vanishing distances. In other words, when distance scales are negligible in comparison with the curvature radius, space appears flat, with a unique vacuum.

Naturally, naive and classical vacua have no special scales and one may, unhindered, reach the small distance limit. However, in the IF framework, the vacuum is complex, with a characteristic distance scale set by the size of the vacuum loops. Such a scale forbids one from achieving the small-distance limit, keeping the definition of positive and negative modes ambiguous, and allowing for an Unruh effect. This perspective makes the crucial role of a nontrivial vacuum clear [26]. Conversely, the FF vacuum is trivial; hence, without distance scales that would prevent approach to the Minkowski limit. So, positive and negative modes are well defined, precluding an Unruh effect.

This discussion also elucidates why zero-point energy cannot contribute to an Unruh effect; namely, by definition, it is associated with an operator expectation value at a single point [25], hence, devoid of a characteristic distance scale.

Nonperturbative vacuum — We note that studies of the vacuum in QFT often employ a perturbative formalism; thus, one can consider whether the nonperturbative FF vacuum could be nontrivial. This is also irrelevant to our subject because no field coupling or other expansion parameter is active in predictions of the Unruh effect, which is derived from kinematical considerations alone. If such were not the case, then one could always choose a small coupling for which a perturbative treatment is sufficient, since typical analyses find the Unruh effect to occur universally, irrespective of coupling strength. Additionally, discussions of the Unruh effect are often set for simplicity in (1+1)D [4, 13], where the triviality of the nonperturbative FF vacuum is clear: quantum chromodynamics has been solved exactly in (1+1)D, with the meson spectrum recovered in the number of colors $N_c \rightarrow \infty$ limit using FF [27] or IF [28]. Both studies involve obtaining the Hartree-Fock solution for its vacuum, but with the FF calculation being considerably simpler owing to vacuum simplicity. Finally, vacuum structure, whether trivial or complex, is not invoked: the demonstrations only use coordinate definitions

of the forms of dynamics, the consequent quantization conditions, and generic properties of QFT. Only the interpretation of the result invokes the vacuum (perturbative) structure as it provides a seemingly intuitive picture.

Infinite momentum frame — Often, FF results are formally recovered by boosting IF results to the infinite momentum frame (IMF) [29]. This is also the case here, since in the IMF, the Rindler, IF and FF times coincide – see Fig. 1. From Ref. [30], this implies the same vacua and, in turn, no Unruh effect. Yet, one also needs to allow for the possibility of zero-modes in the IF-IMF vacuum. Some of these modes correspond to real particles – rather than the null-momentum vacuum loops ("tadpole graphs") responsible for zero-point energy – and might contribute to the static properties of bound states [31]. However, consistent with the FF result, even these particular IF-IMF zero-modes cannot lead to the dynamics underlying the Unruh effect for several reasons. First, such modes manifest themselves only within bound states, potentially contributing inside to long-distance field correlations (sometimes called "condensates") without support outside a bound state [10, 32–34]. Second, IF-IMF vacuum loops or condensates cannot produce pairs of physical particles for the same reason as for the FF (see Section 5): all particles must have nonnegative IF-IMF momenta. Since the vacuum or zero-modes have null momenta by definition, and since momentum is conserved, this excludes loops because one of the particles would have a negative momentum. Considering the IMF limit offers a simple interpretation of the absence of the Unruh effect: in the IMF the acceleration must fall to zero; thus, so must the Unruh temperature.

Summary — Herein, FF vacuum triviality refers to the absence of loops of particles with non-zero momenta, without considering zero-modes, which might play a role in some physics but are irrelevant to the Unruh effect. Furthermore, vacuum triviality is not employed in the FF analyses pages 4, 12 and 17: an Unruh effect is precluded in FF quantization for the same reason that the vacuum is trivial; so, the latter need not be assumed. Discussing vacuum triviality is useful chiefly in interpreting the FF results.

B Standard derivations of the Unruh effect

B.1 Analyticity Argument

Here, we follow a standard IF approach to the Unruh effect [13] and simultaneously present the analogue in FF dynamics. Herein and in the subsequent sections, we use red fonts for IF-specific formulae, which are taken directly from Ref. [13], and blue fonts for their FF equivalents. Since the Unruh effect is usually discussed in the "relativity convention" metric, *viz.* $(-, +, +, +)$, while the FF is generally discussed in the "particle physics convention" $(+, -, -, -)$, then to preserve the familiar formulae, we use $(-, +, +, +)$ in the IF case and $(+, -, -, -)$ in FF.

Following Ref. [13], we consider massless scalar particles in (1+1)D spacetime for simplicity. The spacelike line element is thus:

$$ds^2 = -dt^2 + dz^2 \quad \text{(IF)}; \qquad ds^2 = d\tau dx^- \quad \text{(FF)}. \tag{B.1}$$

A uniformly accelerated observer follows a trajectory:

$$t(\rho) = \frac{1}{\alpha}\sinh(\alpha\rho)\text{(IF)}; \; \tau(\rho) \equiv t(\rho) + z(\rho) = \frac{1}{\alpha}e^{\alpha\rho}\text{(FF)},$$

$$z(\rho) = \frac{1}{\alpha}\cosh(\alpha\rho)\text{(IF)}; \, x^-(\rho) \equiv t(\rho) - z(\rho) = \frac{1}{\alpha}e^{-\alpha\rho}\text{(FF)}, \tag{B.2}$$
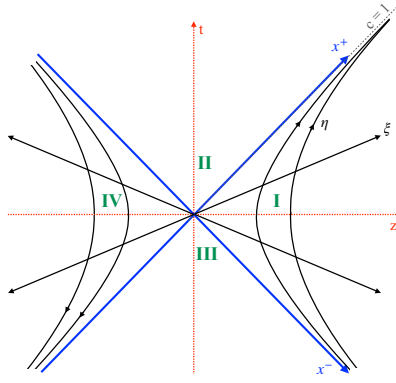
Figure 4: Coordinate frames: Rindler (black), Minkowski IF (red), and FF (blue). As in Fig. 1, we use the $(-,+,+,+)$ metric for both IF and FF.

where α is the acceleration and ρ a parameter. The trajectory describes a hyperboloid in the IF and a hyperbola in the FF:

$$t^2(\rho)=z^2(\rho)-\alpha^{-2} \quad \text{(IF)}; \quad \tau(\rho)=\frac{1}{\alpha^2 x^-(\rho)} \quad \text{(FF)}. \tag{B.3}$$

Working with these kinematics, Rindler coordinates η, ξ can be defined, along with the Rindler frame. Their relation to the Minkowski (*i.e.*, inertial) frame is:

$$t=\frac{1}{a}e^{a\xi}\sinh(a\eta) \quad \text{(IF)}; \quad \tau=\frac{1}{a}e^{a(\eta+\xi)} \quad \text{(FF)},$$

$$z=\frac{1}{a}e^{a\xi}\cosh(a\eta) \quad \text{(IF)}; \quad x^-=\frac{1}{a}e^{a(-\eta+\xi)} \quad \text{(FF)}, \tag{B.4}$$

with the metric $ds^2 = e^{2a\xi}(-d\eta^2 + d\xi^2)$. Thus, η can be identified as the Rindler proper time and ξ as the Rindler space coordinate.

Equations (B.4) provide a coordinate system only on the right spacelike quadrant of the Minkowski space delimited by $|t|=z, z\geq 0$ and denoted quadrant **I** in Fig. 4. Coordinates for the left spacelike quadrant ($|t|=z, z\leq 0$, quadrant **IV**), are obtained by flipping Minkowski coordinate signs. This is because the Rindler space coordinate ξ is a ray originating at $z=0$ rather than a line spanning $[-\infty, +\infty]$. Therefore, in quadrant **IV**, ξ points left and the Rindler time is reversed: η flows downward. The reasoning applies to IF and equally to FF since the FF Killing field is

$$\partial_\eta = \frac{\partial\tau}{\partial\eta}\partial_\tau + \frac{\partial x^-}{\partial\eta}\partial_{x^-} = a(\tau\partial_\tau - x^-\partial_{x^-}), \tag{B.5}$$

with components $(2a\tau, -2ax^-)$. Thus, ∂_η is orthogonal to the $\tau=0$ hypersurface, as in the IF case and, like the IF, ∂_η defines positive- and negative-modes for a Fock basis. Consequently, in quadrant **IV**,

$$t=-\frac{1}{a}e^{a\xi}\sinh(a\eta) \quad \text{(IF)}; \quad \tau=-\frac{1}{a}e^{a(\eta+\xi)} \quad \text{(FF)},$$

$$z=-\frac{1}{a}e^{a\xi}\cosh(a\eta) \quad \text{(IF)}; \quad x^-=-\frac{1}{a}e^{a(-\eta+\xi)} \quad \text{(FF)}. \tag{B.6}$$

In Rindler coordinates, the Klein-Gordon equation for massless free fields is:

$$\Box\phi = \frac{1}{a}e^{-2a\xi}(-\partial_\eta^2 + \partial_\xi^2)\phi = 0. \tag{B.7}$$

419 (The extension to massive fields is straightforward and without qualitative import.) Solutions
420 are given by planes waves: $g_p = (4\pi\omega)^{-1/2}e^{-i\omega\eta+ip\xi}$, with $\omega = |p|$. In Minkowski frames, the
421 Klein-Gordon equation is:

$$\Box\phi = (-\partial_t^2 + \partial_z^2)\phi = 0 \quad \text{(IF)}; \Box\phi = (\partial^+\partial^-)\phi = 0 \quad \text{(FF)}. \tag{B.8}$$

422 Here, $\partial^+ \equiv 2\partial/\partial x^-$, $\partial^- \equiv 2\partial/\partial\tau$. They are the operators associated with FF momentum,
423 $p^+ = \omega + p$, and FF energy, $p^- = \omega - p$, respectively. The plane wave solutions of Eqs. (B.8)
424 are:

$$f_p \propto e^{-i\omega t+ipz}, f_p^* \quad \text{(IF)}; f_{p^+} \propto e^{-i/2(p^+x^-)}, f_{p^+}^* \quad \text{(FF)}. \tag{B.9}$$

425 There is no $e^{-i/2(p^-\tau)}$ component in the FF wave function because massless particles are also
426 zero-energy in FF dynamics: $p^- \equiv \omega - p = 0$.

427   Since in quadrant **IV**, future Minkowski time (IF or FF) flows in the "$-\eta$" direction – see
428 Fig. 4, two sets of positive frequency Rindler modes must be introduced: one for quadrant **I**
429 and the other for quadrant **IV**,

$$g_p^{(I)} = \begin{cases} \dfrac{1}{\sqrt{4\pi\omega}}e^{-i\omega\eta+ip\xi} & \text{I} \\ 0 & \text{IV} \end{cases}, \tag{B.10a}$$

$$g_p^{(IV)} = \begin{cases} 0 & \text{I} \\ \dfrac{1}{\sqrt{4\pi\omega}}e^{+i\omega\eta+ip\xi} & \text{IV} \end{cases}, \tag{B.10b}$$

430 so that $\omega \geq 0$ in both quadrants. The modes $g_p^{(I)}$ and $g_p^{(IV)}$ can be analytically continued
431 outside their respective quadrant, which enables ϕ to be expressed in terms of four Rindler
432 creation/annihilation operators:

$$\phi = \int dp\big(\hat{b}_p^{(I)}g_p^{(I)} + \hat{b}_p^{(I)\dagger}g_p^{(I)*} + \hat{b}_p^{(IV)}g_p^{(IV)} + \hat{b}_p^{(IV)\dagger}g_p^{(IV)*}\big). \tag{B.11}$$

433   In contrast, Minkowski space not being partitioned, two creation/annihilation operators
434 are sufficient:

$$\phi = \int dp\big(\hat{a}_p f_p + \hat{a}_p^\dagger f_p^*\big) \quad \text{(IF)};$$
$$\phi = \int dp^+\big(\hat{a}_{p^+}f_{p^+} + \hat{a}_{p^+}^\dagger f_{p^+}^*\big) \quad \text{(FF)}. \tag{B.12}$$

435   The annihilation operators of Eqs. (B.11(and (B.12) define the Rindler and Minkowski
436 vacua, respectively:

$$\hat{b}_p^{(I)} |0_R\rangle = \hat{b}_p^{(IV)} |0_R\rangle = 0 \quad \text{(Rindler)};$$
$$\hat{a}_p \ |0_M\rangle = 0 \quad \text{(IF)};$$
$$\hat{a}_{p^+} |0_M\rangle = 0 \quad \text{(FF)}. \tag{B.13}$$

437   Exposing an Unruh effect hinges on showing that $\hat{b}_p^{(I,IV)}$ cannot be expressed with $\hat{a}$ only
438 but must also include $\hat{a}^\dagger$. Then, a Rindler observer will detect particles from the Minkowski
439 vacuum, $\hat{b}_p^{(I,IV)} |0_M\rangle \neq 0$.

440   The expressions of $\hat{b}_p^{(I,IV)}$ in terms of $\hat{a}$ and $\hat{a}^\dagger$ can be obtained by finding Rindler modes
441 $h_p^{(I,IV)}$ that have the same vacuum as the Minkowski modes $f^{(I,IV)}$ [35]. Using Eqs. (B.4) and
442 (B.6), the relationship between the phases of the Rindler and Minkowski modes in the two

443   spacetime quadrants is (recall the different metric signatures for IF and FF):

$$
\begin{aligned}
e^{-a(\eta-\xi)} &= \begin{cases} -a(t-z) & \text{(IF);} & ax^- & \text{(FF)} & \text{I} \\ a(t-z) & \text{(IF);} & -ax^- & \text{(FF)} & \text{IV} \end{cases} \\
e^{a(\eta+\xi)} &= \begin{cases} a(t+z) & \text{(IF);} & a\tau & \text{(FF)} & \text{I} \\ -a(t+z) & \text{(IF);} & -a\tau & \text{(FF)} & \text{IV} \end{cases}
\end{aligned}
\tag{B.14}
$$

444   Above, for the sake of generality, we wrote $a\tau$, but for a (1+1)D massless scalar field, contri-
445 butions involving τ vanish because their phase is $p^-\tau$, with $p^- = 0$. Recall also that a is the
446 acceleration and should not be confused with the operators $\hat{a}$, $\hat{a}^\dagger$.

447   We can now identify which Rindler modes match the spacetime dependence of the Minkowski
448 modes f_p or f_{p^+}. Using Eqs. (B.10) and (B.14) and the Rindler dispersion relation, $\omega = p$, the
449 Rindler mode in quadrant **I** that has the same phase as f_p or f_{p^+} is:

$$
\begin{aligned}
\sqrt{4\pi\omega}\, g_p^{(I)} &= a^{i\omega/a}(-t+z)^{i\omega/a} \quad \text{(IF);} \\
\sqrt{4\pi\omega}\, g_p^{(I)} &= (ax^-)^{ip^+/2a} \quad\quad\quad \text{(FF).}
\end{aligned}
\tag{B.15}
$$

450   The available Rindler mode in quadrant IV, (B.10b), does not have the same phase as in
451 (B.15). To match the phase, we must consider the complex conjugate of $g_p^{(IV)}$ with negative
452 momentum $k := -p < 0$:

$$
\begin{aligned}
\sqrt{4\pi\omega}\, g_{-p}^{(IV)*} &= a^{i\omega/a} e^{\pi\omega/a}(-t+z)^{i\omega/a} \quad \text{(IF);} \\
\sqrt{4\pi\omega}\, g_{-p}^{(IV)*} &= e^{\pi p^+/2a}(ax^-)^{ip^+/2a} \quad\quad \text{(FF).}
\end{aligned}
\tag{B.16}
$$

453   The Rindler dispersion relation, $\omega = |k|$, and the definition, $p^+ \equiv \omega + k$, impose $p^+ = 0$ on
454 the domain $k = -p \le 0$. Thus, in the FF case, there is no available Rindler mode in quadrant
455 **IV**. (B.16) then becomes:

$$
\begin{aligned}
\sqrt{4\pi\omega}\, g_{-p}^{(IV)*} &= a^{i\omega/a} e^{\pi\omega/a}(-t+z)^{i\omega/a} \quad \text{(IF);} \\
\sqrt{4\pi\omega}\, g_{-p}^{(IV)*} &= \text{constant} = 0 \quad\quad\quad\quad \text{(FF).}
\end{aligned}
\tag{B.17}
$$

456   The rightmost result follows because g_{-p} must be a plane wave, by definition of the Fourier
457 decomposition of ϕ, (B.11), and a nonzero value for the constant would violate this constraint.

458   With Eqs. (B.15) and (B.17), the Rindler modes can now be expressed in terms of the
459 Minkowski modes:

$$
\begin{aligned}
\sqrt{\pi\omega}\left(g_p^{(I)} + e^{-\pi\omega/a} g_{-p}^{(IV)*}\right) &= a^{i\omega/a}\left(-t+z\right)^{i\omega/a} \quad \text{(IF);} \\
\sqrt{4\pi\omega}\, g_p^{(I)} &= (ax^-)^{ip^+/2a} \quad\quad\quad\quad \text{(FF).}
\end{aligned}
\tag{B.18}
$$

460   Plainly, only the Rindler mode $g_p^{(I)}$ matches the phase of the Minkowski FF modes f_{p^+} – see
461 (B.9), and not $g_{-p}^{(IV)*}$; the Bogolyubov matrix is diagonal. Hence, there is no Unruh effect in
462 the FF framework. Notwithstanding this, concluding details are included below.

463   The IF combination in (B.18) is not yet properly normalized. The normalized IF mode is
464 (the FF mode remains unchanged):

$$
\begin{aligned}
h_p^{(I)} &= \frac{1}{\sqrt{2\sinh(\pi\omega/a)}}\left(e^{\pi\omega/2a} g_p^{(I)} + e^{-\pi\omega/2a} g_{-p}^{(IV)*}\right) \quad \text{(IF);} \\
g_p^{(I)} &= \frac{1}{\sqrt{4\pi\omega}}\left(ax^-\right)^{ip^+/2a} \quad\quad\quad\quad\quad\quad \text{(FF).}
\end{aligned}
\tag{B.19}
$$

The normalized Rindler modes, h_p, have corresponding creation/annihilation operators $\hat{c}_p$, which, by the construction above, act on the Minkowski vacuum as $\hat{c}_p |0_M\rangle = 0$ and $\hat{c}_p^\dagger |0_M\rangle = |p\rangle$. Using (B.19) and analogous reasoning for $\hat{b}_p^{(IV)}$, one obtains the relationship between $\hat{c}_p$, $\hat{c}_p^\dagger$ and the original Rindler operators:

$$
\begin{aligned}
\hat{b}_p^{(I)} &= \frac{1}{\sqrt{2\sinh(\pi\omega/a)}}\big(e^{\pi\omega/2a}\hat{c}_p^{(I)} + e^{-\pi\omega/2a}\hat{c}_{-p}^{(IV)\dagger}\big) \quad \text{(IF)};\\
\hat{b}_p^{(I)} &= \hat{a}_{p^+} \quad \text{(FF)},\\
\hat{b}_p^{(IV)} &= \frac{1}{\sqrt{2\sinh(\pi\omega/a)}}\big(e^{\pi\omega/2a}\hat{c}_p^{(IV)} + e^{-\pi\omega/2a}\hat{c}_{-p}^{(I)\dagger}\big) \quad \text{(IF)};\\
\hat{b}_p^{(IV)} &= \hat{a}_{p^-} \quad \text{(FF)}.
\end{aligned}
\tag{B.20}
$$

Acting now with, *e.g.*, $\hat{b}_p^{(I)}$ on the Minkowski vacuum, one finds:

$$
\begin{aligned}
\hat{b}_p^{(I)}|0_M\rangle &= \frac{1}{\sqrt{2\sinh(\pi\omega/a)}}\big(e^{-\pi\omega/2a}|-p\rangle\big) \quad \text{(IF)};\\
\hat{b}_p^{(I)}|0_M\rangle &= \hat{a}_{p^+}|0_M\rangle = 0 \quad \text{(FF)}.
\end{aligned}
\tag{B.21}
$$

Evidently, in a Rindler frame, following this reasoning, one can observe particles from the IF Minkowski vacuum but not from the (trivial) FF vacuum.

The crucial point is the transition between Eqs. (B.15) and (B.18): modes with negative momentum, $k = -p$, are introduced. This is allowed in the IF: it simply means waves moving toward "$-z$" (for quadrant I); and they can be detected by an observer in that quadrant. In the FF, on the other hand, $p^+ > 0$ always; waves with $p^+ \leq 0$ would propagate backward in IF time; thus violating causality. Therefore, they do not exist; so, neither do Unruh particles. In other words, because, formally, $f_{-p^+} = f_{p^+}^*$, one must set $f_{-p^+} = 0$ or one would double count in the Fourier expansion, (B.12). (Note that $f_{p^+}^*$ cannot be invoked for (B.16) since the procedure is to match Rindler modes to f_{p^+}, not to $f_{p^+}^*$.)

The demonstration provided in this Appendix is formal. To apprehend the difference between the IF and FF demonstrations, it is helpful to interpret the central result, (B.19). Let us first consider the usual mode expansion of a field operator, (B.12)]. Modes f_p and f_p^* correspond to particles and antiparticles since $\hat{a}_p$ removes a quantum of the field and $\hat{a}_p^\dagger$ adds one. Let us choose to call f_p the antiparticle mode and f_p^* the particle mode (B.19) for IF, *viz.* $h_p^{(I)} \propto \big(e^{\pi\omega/2a}g_p^{(I)} + e^{-\pi\omega/2a}g_{-p}^{(IV)*}\big)$, means that for an individual mode h_p, particles of momentum $-p$ and antiparticles of momentum p co-exist, as in the disconnected pair-creation diagrams of vacuum loops. This is the standard interpretation of the Unruh effect: the IF vacuum structure, made of disconnected loops of particles/antiparticles of opposite momenta, becomes observable. In the FF case, the term $e^{-\pi\omega/2a}g_{-p}^{(IV)*} = (ax^-)^{ip^+/2a}$ does not contribute because its power, $p^+ := \omega + (-p) = 0$, and a constant mode normalizes to 0. The absence of $g_{-p}^{(IV)*}$ does not permit the simultaneous creation of a particle and an antiparticle with opposite momenta, *i.e.*, of disconnected vacuum loops, as is well-known for FF.

B.2 Direct derivation of the Bogolyubov transformation

As discussed in Section B.1, the Bogolyubov transformation was derived following the original analyticity argument [35]. Here we discuss the direct derivation – see, *e.g.*, Ref. [4]. The steps need not be fully detailed because the argument precluding the Unruh effect in FF is the same as in the method discussed Section B.1.

498 The Bogolyubov transformation is

$$
\begin{aligned}
g_p^{(\mathrm{I})} &= \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}}\big(\hat{\alpha}_p^{(\mathrm{I})} f_p + \hat{\beta}_p^{(\mathrm{I})} f_p^*\big), \\
g_p^{(\mathrm{IV})} &= \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}}\big(\hat{\alpha}_p^{(\mathrm{IV})} f_p + \hat{\beta}_p^{(\mathrm{IV})} f_p^*\big),
\end{aligned}
\tag{B.22}
$$

499 with $\hat{\beta}_p^{(\mathrm{IV})} = -\mathrm{e}^{-\pi\omega/a}\hat{\alpha}_p^{(\mathrm{I})*}$, $\hat{\beta}_p^{(\mathrm{I})} = -\mathrm{e}^{-\pi\omega/a}\hat{\alpha}_p^{(\mathrm{IV})*}$.

500 For IF, these coefficients are non-zero; hence, the Unruh effect is predicted. In contrast, for

501 FF, $\hat{\beta}_p^{(\mathrm{IV})} = \hat{\alpha}_p^{(\mathrm{IV})*} = 0$ because no FF modes are permitted in quadrant **IV**. Hence, $\hat{\beta}_p^{(\mathrm{IV})} = \hat{\beta}_p^{(\mathrm{I})} = 0$

502 and no Unruh effect is perceived.

C The Unruh effect in thermal quantum field theory

504 Before examining the Unruh effect using thermal QFT, it is worth recalling the correspondence

505 between Rindler time translations and Minkowski boosts, and the implications for Rindler

506 Hamiltonian definition in FF dynamics.

C.1 Relation between Rindler time translations and boosts: implications for the Hamiltonian

509 For IF and FF, a Rindler time translation corresponds to a Minkowski boost. This is readily

510 seen using Eqs. (B.4) and making a Rindler time translation $\eta \to \eta + \Delta$, which leads to:

$$
\begin{aligned}
t' &= \frac{1}{a}\mathrm{e}^{a\xi}\sinh(a\eta + a\Delta), \\
&= \tfrac{1}{a}\mathrm{e}^{a\xi}\big(\sinh(a\eta)\cosh(a\Delta) + \cosh(a\eta)\sinh(a\Delta)\big) \quad (\mathrm{IF}); \\
\tau' &= \frac{1}{a}e^{a(\eta+\Delta+\xi)} \quad (\mathrm{FF}), \\
z' &= \frac{1}{a}\mathrm{e}^{a\xi}\cosh(a\eta + a\Delta), \\
&= \tfrac{1}{a}\mathrm{e}^{a\xi}\big(\cosh(a\eta)\cosh(a\Delta) + \sinh(a\eta)\sinh(a\Delta)\big) \quad (\mathrm{IF}); \\
x^{-\prime} &= \frac{1}{a}\mathrm{e}^{a(-\eta-\Delta+\xi)} \quad (\mathrm{FF}).
\end{aligned}
\tag{C.1}
$$

511 Hence,

$$
\begin{aligned}
t' &= t\cosh(a\Delta) + x\sinh(a\Delta) \quad (\mathrm{IF}); \quad \tau' = \mathrm{e}^{a\Delta}\tau \quad (\mathrm{FF}), \\
z' &= z\cosh(a\Delta) + t\sinh(a\Delta) \quad (\mathrm{IF}); \quad x^{-\prime} = \mathrm{e}^{-a\Delta}x^- \quad (\mathrm{FF}),
\end{aligned}
\tag{C.2}
$$

512 which are the usual boost formulae with rapidity $a\Delta$.

513 Now, since FF boosts are kinematical, the above correspondence makes, in FF dynamics, the

514 Rindler time translation a kinematical operator. Therefore, it cannot serve as the Hamiltonian.

515 In Euclidean space, where thermal QFT is used to study the Unruh effect, the dilation operator

516 defines the FF Hamiltonian, as explained in the next section.

C.2 Thermal QFT

518 For a scalar field, the thermal QFT derivation of the IF Unruh effect [36] utilizes the periodicity

519 in Euclidean time (*viz.* imaginary time $\tilde{t} := it$) of a QFT at finite temperature $T = 1/\beta$,

520 $0 \le \tilde{t} \le \beta$.

Consider the two-point Schwinger function of field $\phi(\tilde{t}, z)$:

$$G_\beta(\tilde{t}, z) = \frac{1}{Z}\text{Tr } e^{-\beta H}\mathcal{T}[\phi(\tilde{t}, z)\phi(0, 0)], \tag{C.3}$$

with Z the partition function and $\mathcal{T}$ the imaginary-time ordering operator. In the Boltzmann factor $e^{-\beta H}$, H is the operator providing energy eigenvalues, *not necessarily the time-evolution operator*, although it is both in IF dynamics. This implies $e^{-\beta H}\phi(\tilde{t}, z)e^{\beta H} = \phi(\tilde{t} - \beta, z)$, which, with the cyclicity property of a trace and (C.3), yields – see, *e.g.*, Ref. [37, Eq. (2.86)]:

$$G_\beta(\tilde{t}, z) = G_\beta(\tilde{t} - \beta, z). \tag{C.4}$$

Using Eqs. (C.2), a shift in imaginary time $\tilde{t} \to \tilde{t} + i\Delta$, $i\Delta = 2\pi/a$, is seen to yield $t' = t$ and $z' = z$ or $\tau' = \tau$ and $x^{-'} = x^-$. Namely, after a Wick rotation, the Rindler metric makes a succession of boosts that correspond to a single rotation in imaginary time; and the shift is just a change in temperature. On the other hand, in FF dynamics, as shown in Section C.1, the Rindler Hamiltonian is not the time translation operator; so, $e^{-\beta H}\phi(0, 0)e^{\beta H} \neq \phi(\beta, 0)$ and there is no cyclic relation in FF.

Evidently, only in IF does the formal equivalence between β and periodicity yield an Unruh temperature $\beta^{-1} = T = a/2\pi$.

The Rindler frame identifies with Dirac's Point-Form (PF, see Section 3) after switching time and space, which is nugatory in Euclidean space because the axes are indistinguishable. The PF Hamiltonian being the dilation operator [38], as evinced by the continuous dilation of the axes attached to the red wordline in Fig. 1, right panel, one may choose it as the FF Rindler Hamiltonian. One then recovers (C.4), but the Rindler frame is no longer equivalent to a cyclic frame under dilation factors $d_{1,2}$: $\tau' = \frac{1}{a}e^{a(d_1\eta + d_2\xi)} \neq \tau$ except for the trivial case $d_1 = 1 = d_2$. Consequently, the thermal QFT approach reveals no Unruh effect in FF dynamics.

D Dynamical derivation

Consider the Lagrangian of a free scalar field φ in (1+1)D:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2, \tag{D.1}$$

with m the field mass. Then change the perspective to a frame in which the field appears shifted,

$$\varphi \equiv \phi + a/m^2. \tag{D.2}$$

In this frame, after subtracting a (physically immaterial) constant, $a^2/[2m^2]$, $\mathcal{L}$ becomes:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + a\phi, \tag{D.3}$$

where a acts as an external source term and provides a linear potential, *viz.* a constant acceleration. This model was studied in Refs. [39, 40], albeit not in the context of the Unruh effect. To see that $a\phi$ in (D.3) produces a constant force, consider the classical mechanics equivalent, $\mathcal{L} = \frac{1}{2}m\dot{z}^2 - \frac{1}{2}kz^2 - az$, where k is the harmonic oscillator constant. The potential term az generates a constant force a in the (1+1)D case [41].

Suppose a detector sensitive to ϕ is attached to the new frame. It may be pictured as being composed of fermions that couple to ϕ. The field waves $\partial_\mu\phi\partial^\mu\phi$ induce transitions in the detector, making it sensitive to ϕ, and the $a\phi$ term accelerates the detector.

To avoid infrared issues, suppose space is of finite extent, with length L and circular boundary conditions. Then decompose the two fields as follows (the p_n^μ spectrum is discrete because L is finite):

$$\phi = \frac{1}{\sqrt{2L}} \sum_n \left(\hat{A}_{p_n} e^{ip_n^\mu x_\mu} + \hat{A}_{p_n}^\dagger e^{-ip_n^\mu x_\mu} \right), \tag{D.4a}$$

$$\varphi = \frac{1}{\sqrt{2L}} \sum_n \left(\hat{a}_{p_n} e^{ip_n^\mu x_\mu} + \hat{a}_{p_n}^\dagger e^{-ip_n^\mu x_\mu} \right). \tag{D.4b}$$

The relation between $(\hat{A}_{p_n}, \hat{A}_{p_n}^\dagger)$ and $(\hat{a}_{p_n}, \hat{a}_{p_n}^\dagger)$ is obtained by expressing (D.2) in terms of a translation operator, U:

$$\varphi = U^\dagger \phi U, \quad U = e^{-ia\mathsf{P}/m^2}, \tag{D.5}$$

where P is the momentum integrated over a time ΔT characterizing the (Unruh) detector response. P is obtained from the momentum density, *viz.* the canonical conjugate of φ:

$$\mathsf{p} = \frac{-i}{\sqrt{2L}} \sum_n \sqrt{\frac{\omega_n}{2}} \left(\hat{a}_{p_n} e^{ip_n^\mu x_\mu} - \hat{a}_{p_n}^\dagger e^{-ip_n^\mu x_\mu} \right). \tag{D.6}$$

Focusing now on IF dynamics (red fonts):

$$\mathsf{P} = \int_{\Delta T, \pm L} d^2 x\, \mathsf{p} = -i \sum_n \sqrt{\omega_n L}\, \Delta T \left(\hat{a}_{p_n} - \hat{a}_{p_n}^\dagger \right). \tag{D.7}$$

(Factors of π are omitted here as they are immaterial to the discussion.) Equations (D.5) and (D.7) give

$$U = \exp \frac{a\Delta T}{m^2} \sum_n \sqrt{\omega_n L} \left(\hat{a}_{p_n} - \hat{a}_{p_n}^\dagger \right), \tag{D.8}$$

which becomes, upon using the Baker-Campbell-Hausdorff formula and the commutation properties of $\hat{a}_{p_n}$ and $\hat{a}_{p_n}^\dagger$:

$$U = \prod_n \exp \left[\frac{(a\Delta T)^2}{2m^4} \omega_n L \right] \exp \left[-\frac{a\Delta T \sqrt{\omega_n L}}{m^2} \hat{a}_{p_n} \right]. \tag{D.9}$$

The Unruh effect is a statement that although the vacuum of an inertial field (here φ) is, by definition, without real particles, so that $\hat{a}_{p_n} |0_M\rangle \equiv 0$, an accelerated field (here ϕ) is not: $\hat{A}_{p_n} |0_M\rangle = U \hat{a}_{p_n} U^\dagger |0_M\rangle \neq 0$. This again suggests that the IF predicts an observable Minkowski vacuum. However, the detector involved in the observation process should also be included in the analysis. Such would require incorporation of fermion fields into the Lagrangian, (D.1), and subsequent study of the bound states constituting the detector. The discussion Section 6 then indicates that the interaction of $a\phi$ with the detector, *i.e.*, the dynamical effect of the time-dependent boost, will cancel the Unruh effect.

For the FF, however, the momentum density is a simple space derivative: $\mathsf{p} = \partial^+ \varphi \equiv \partial \varphi / \partial x^-$. Therefore:

$$\mathsf{P} = \int_{\Delta T, \pm L} d\tau\, dx^- \frac{\partial \varphi}{\partial x^-} = \int_{\Delta T} d\tau \left[\varphi \right]_{-L}^{+L} = 0, \tag{D.10}$$

owing to the periodic boundary conditions. Thus, $U = \mathbb{1}$ and the trivial vacuum of φ in Minkowski space $\hat{a}_{p_n} |0_M\rangle = 0$ remains such for the accelerated field: $\hat{A}_{p_n} |0_M\rangle = \hat{a}_{p_n} |0_M\rangle = 0$. There is no Unruh effect in FF.

One may interpret the IF and FF results as follows.

(*i*) IF. p is the time derivative of the field, $\mathsf{p} = \partial \varphi / \partial t$. Thus, it gives the difference between φ

on two infinitesimally separated hypersurfaces, *i.e.*, its dynamical evolution.

(*ii*) FF. p is a space derivative of the field, $p = \partial \varphi / \partial x^-$. It operates on a single hypersurface without dynamical evolution.

Consequently, the conclusions of the previous sections are recovered: (*i*) the IF treatment *dynamically* affects a seemingly nontrivial vacuum, whereas (*ii*) the FF treatment does not impact upon a trival vacuum. These qualities trace back to the dynamical vs. kinematical character of IF *cf*. FF boosts.

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