The physical interpretation of the fine-structure constant $\alpha \approx \frac{1}{137.036...}$

Luca Varani
IES, Univ Montpellier, CNRS,
860 rue St Priest, F-34095 Montpellier cedex 5, France

Federico Intini
Department of Engineering, "Enzo Ferrari", University of Modena and Reggio Emilia, Via P. Vivarelli, 10, I-41125 Modena, Italy

Lino Reggiani
Mathematical and Physical Department, "Ennio De Giorgi", Salento University, via Monteroni, I-73100 Lecce, Italy

Correspondence should be addressed to Luca Varani at luca.varani@umontpellier.fr
(Dated: December 4, 2025)

The fine-structure constant α is a fundamental, dimensionless constant in physics, introduced by Sommerfeld in 1916, that is still an empirical parameter in the Standard Model for physical particles, the most advanced theory of modern physics. Its numerical value 1/137 is continuosly receiving wide attention in respect of the accuracy of its value, now claimed to stay at 11 digits in fair agreement with the Sommerfeld expression.

This number was physically interpreted as a way of quantifying the strength of the electromagnetic interaction. Since then, several physicists (like Dirac, Pauli, Feynman and others) expressed their unsatisfaction of this situation waiting for a more direct interpretation of this number.

Here we address this problem by giving a direct microscopic meaning to 137. We show the existence of a one-to-one correspondence between 137 and the variance of the photon average number $\overline{N}$ inside a black-body of given volume and temperature. The explicit expression writes: $\overline{\delta N^2} = \gamma \overline{N} = \frac{1}{\alpha}$ with the variance and the average determined by the Planck's law applied to a cubic cavity of side $L = 3.3$ cm exact and an equilibrium temperature of 0.516 K corresponding to the equivalent energy of the measured fine-structure line of about 45 μeV, with $\gamma = 1.37$ being the Fano factor.

As a broader perspective we propose that any atomic spectral line be associated with a structure-constant that provides the statistical information on the photon gas inside the corresponding black-body. From this point of view, the expression of the Sommerfeld constant is wieved as a particular case that coincides exactly with our results when the emission line is of 45 μeV.

Present findings open new experimental possibilities to measure the statistical properties of thermal photons and in particular of the fine-structure constant by using a recently developed counting-statistics technique.

Keywords: fine structure constant, quantum-relativistic statistics, black-body properties, generalized state equation of a quantum gas.

Introduction - Spectra of matter radiation-emission are known to exhibit a finite series of narrow lines in an atomic rarefied gas and, by contrast, a continuous spectrum in highly compressed gases, liquids and solids. In the special case of a macroscopic black-body (bb) cavity, with given temperature and volume, the emission spectrum is also continuous and described by Planck's distribution law containing all the photon energies existing in the universe with the proper probability weight to keep their average number finite at a given temperature and volume of the bb. Thus, the bb can be thought as the color table of the universe with the painter being a divinity or the nature itself.

Since the discovery by Michelson and Morley in 1886 [1] of a fine structure emission line from the Hydrogen atom in the far-infrared region, the unsolved question arises whether it exists a relation between these different types of fine-structure spectra and, in the positive case, what kind of relation one could expect going beyond the vague answer: the fine-structure constant associated with the fine-structure line of the Hydrogen spectrum quantifies the strength of the electromagnetic interaction between elementary charged particles in the atom.

Here, the two physical quantities of interest are given by two rational numbers that relate, respectively, (i): the fine structure line (FSL), a

$$a = \alpha_{line} \tag{1}$$

(generalizing the fine structure constant (FSC) of the Sommerfeld definition) and, (ii): the variance of the photon number, A

$$A = \overline{\delta N^2}_{\rm line} = \gamma \overline{N}_{\rm line} \tag{2}$$

with γ the Fano factor and described by Planck's state equation of the photon gas inside the corresponding bb at thermal equilibrium by the reciprocity relation:

$$A = \frac{1}{a} \tag{3}$$

where for the FSC of Sommerfeld [2] it is: $1/\alpha \approx 137$.

Without loss of generality, let us call the generic value of elements in A as A_i and of elements in a as a_i with the reciprocity condition $A_i \times a_i = 1$, Then, it follows that $A_i = 1/a_i$ for any i, and in particular for the Sommerfeld FSC

$$\overline{\delta N^2_\alpha} = \frac{1}{\alpha} = 137 \tag{4}$$

Notice that, by construction the above relation is exact and thus independent of the number of digits taken for the numerical value of α. As a consequence, the claimed mystery concerning the meaning of the number 137, is here interpreted as a numerical result implied by the Planck's photon state-equation (2). Furthermore, even of more importance, the above relation holds for any line of the spectrum, thus generating an infinite number of LSCs, in a one-to-one correspondence with each photon energy. In other words, the Sommerfeld definition of the FSC that includes also the well-known physical expression:

$$\alpha = \frac{e^2}{2h\epsilon_0 c} = \frac{1}{137.0360} \tag{5}$$

being e the unit charge, h the Planck's constant, ϵ_0 the vacuum permittivity, c the light velocity in vacuum, and with an accuracy of at least seven digits. Indeed, Sommerfeld obtained this result starting from the physical interpretation of the FSC as a relativistic effect on the electron energy associated with its orbital motion around the proton. Successively, Dirac in 1928 [3], more exactly included also the spin effect on the electron orbit (indeed, to date the FSC is interpreted as due to spin-orbit interaction).

By contrast, in the present paper we interpret the FSC starting directly from the photon properties described by the proper quantum-relativistic statistics for the specific bb cubic cavity.

Theory and application - The numerical results interrelating $\overline{\delta N^2}$ with $1/\alpha$ are obtained as follows. By recalling that for a cubic bb of side L at temperature T from Planck's law it is [4, 5]:

$$\overline{\delta N^2} = \frac{1}{3}\left(\frac{2\pi k_B}{ch} LT\right)^3 = 2.776118 \times 10^7\,(LT)^3 \tag{6}$$

with k_B the Boltzmann constant and LT given in (m K), we specialize the above equation to the relation $\overline{\delta N^2} = 1/\alpha$, and find:

$$\frac{1}{\alpha} = \overline{\delta N^2} = \gamma\overline{N} = 997.5828\left(T_{\frac{1}{\alpha}}\right)^3 \tag{7}$$

where $T_{\frac{1}{\alpha}} = 0.516\ K$ and, to fit $\frac{1}{\alpha}$, $L_{\frac{1}{\alpha}} = 0.033$ m.

Equation (7) is the central result of the paper, it gives the LSC of the considered line energy ϵ_{line} to be the same of the thermal energy $k_B T$ of the bb with volume given by $L^3_{\frac{1}{\alpha}}$ thus expressing the LSC within a quantum-relativistic thermodynamic approach based on the Planck's distribution law.

Figure 1 reports the LSCs pertaining to the full Hydrogen spectrum obtained within the present approach. It illustrates the main result of our theoretical model by showing the variance of the photon-number fluctuations (left scale) and the average number of photons (top scale) as functions of the energy of the considered emission line taken to be the same of the thermal energy of the bb (bottom scale) in agreement with Eq. (7). As relevant example we can cite the Hydrogen hyperfine structure constant (HSC) whose origin is attributed to the interaction between nuclear and electron spins and the Lamb shift structure constant (LS) related to vacuum fluctuations [6]. The measured value of the frequency

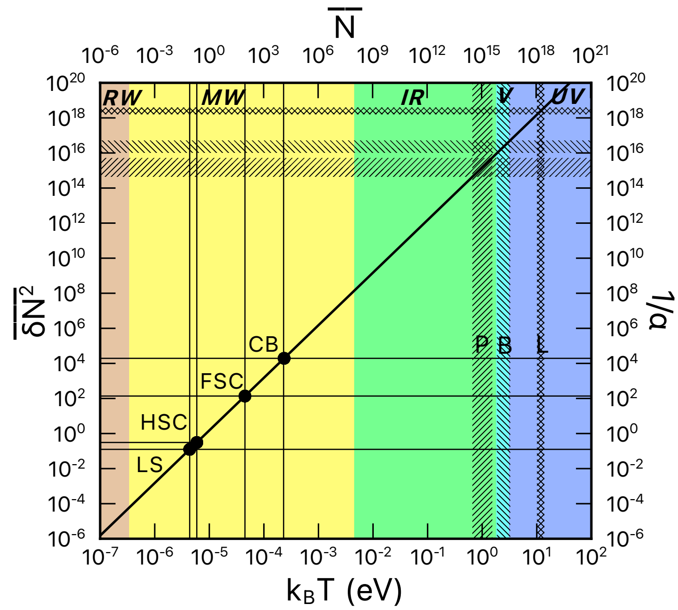

FIG. 1. Variance of the photon number fluctuations $\overline{\delta N^2}$ (left scale) and inverse of the fine-structure line $1/\alpha$ (right scale) as functions of the thermal energy $k_B T$ with the equivalent temperatures of the bb and of the emitted line coinciding (bottom scale) and the average photon number $\overline{N}$ (top scale). The volume of the bb is taken to be constant and given by $0.033^3\ m^3$. The points refer to: CB (Cosmic Background), FSC (Fine-Structure Constant), HSC (Hyperfine-Structure Constant), LS (Lamb Shift). The straight line represents the equation $\overline{\delta N^2} = \gamma\overline{N}$. The shaded areas correspond to some of the Hydrogen spectral series: L (Lyman), B (Balmer), P, (Paschen). Other series are not included for the sake of clarity. The colored areas represents the different bands of the electromagnetic spectrum: RW (Radio-Waves), MW (Micro-Waves), IR (Infra-Red), V (Visible) and UV (Ultra-Violet).

of the emitted radiation corresponding to the HSC is $f = 1.420406 \times 10^9$ Hz [7] ($\Delta E = 5.874326 \times 10^{-6}$ eV) that correspond to a value

$$\left(\frac{1}{\alpha}\right)_{\rm HSC} = 0.3048498 \tag{8}$$

Analogously, for the LS the measured frequency is $f = 1.057830 \times 10^9$ Hz [8] ($\Delta E = 4.374829 \times 10^{-6}$ eV) that correspond to a value

$$\left(\frac{1}{\alpha}\right)_{\rm LS} = 0.1212096 \tag{9}$$

Figure (2) reports the LSCs extended to the atomic and cosmological spectrum of the photons emitted by all the atoms (from Hydrogen to Uranium and limited to the ionization energy of the 1s electron state for simplicity) and from the bb at a given temperature, i.e. practically all the lines that can be found in the entire universe since the initial Bing Bang (BB) and even before if needed (to this porpose the time scale go back to 10^{-47} s), crosses the present days, at about 10^{17} s and includes the near future up to about 10^{25} s when the sun will be already switched off.

The figure can be coupled to the thermal evolution of the universe by replacing the energy scale with the time scale describing the universe expansion at the light speed starting from the BB on the right hand corner of the figure at about 10^{-39} s, when the universe average thermal energy was of about 10^{32} eV, up to a future time on the left hand corner at about 10^{25} s where the universe average thermal-energy will be of about 10^{-8} eV. We remark that the present model does not include the so called inflation period centered at about 10^{-35} s.

Conclusions - The paper investigated the physical meaning of the fine-structure constant, introduced by Sommerfeld in 1916 to explain the origin of the line at about 45 μeV discovered in 1886 by Michelson and Morley [1] in the emission spectra of atomic Hydrogen. Since to date the interpretation of the value of $\alpha \approx 1/137$ independently from its value is considered to be a mistery waiting for a solution. Here we propose to shed light on the mistery by using a statistical relation between the thermal energy and the avrage number of photons in an appropriate black-body cubic cavity. Indeed, the value of α follows from the state-equation of the photon gas inside this cavity with the edge of length $L = 3.3$ cm exactly at an equilibrium temperature of equivalent energy $k_B T \approx 45$ μeV as dictated by the Planck's law and given in Eq. (7). We notice that the photon gas state-equation can be written in full analogy with that of the ideal classical-gas equation as

$$PV = 0.9\overline{N} k_B T \tag{10}$$

with P the pressure exerted by the photon gas on the internal walls, the factor 0.9 being related to the super-Poissonian property of the Bose-Einstein distribution

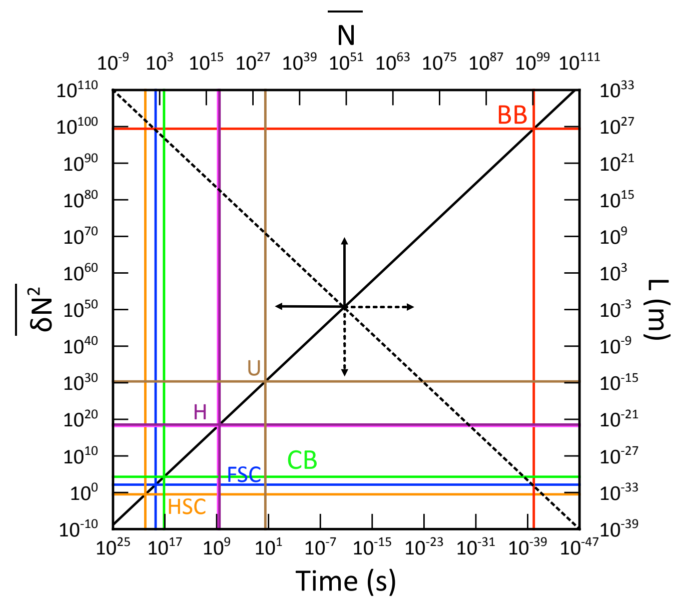

FIG. 2. The figure summarizes the photon statistics properties: variance of photon number, average photon number, universe expansion length $L = c$ time with the age of the universe taken within the range from 10^{-47} s to 10^{25} s when going from high to low thermal energy values. The perpendicular and parallel lines in the graph intercept the line of the photon state-equation (continuous diagonal) at important points like Hydrogen and Uranium atoms ionization energies, respectively. The dashed diagonal line shows the linear expansion of the universe with increasing time. CB is the present microwave Cosmic Background energy, remembering the preceding BB, FSC and HSC the fine and hyperfine structures energies of H, respectively. The continuous and dashed arrows indicate the axis corresponding respectively to the continuous and dashed diagonals.

function and $\overline{N}$ playing the role of the Avogadro number with a finite accuracy controlled by experimental measurements, as dictated by the unavoidable line-width in detecting the energy of the radiation-emission. The above state-equation generalizes the concept of fine structure constant (FSC) to that of a line structure-constant (LSC) being common to all the photon lines present in the universe since the Big Bang, and Fig. (2) reports the main features of the LSCs from before about the Big-Bang, at $\approx 10^{-47}$ s), up to the next future ($\approx 10^{25}$ s) when the solar system is expected to have already taken over. These results could open a large variety of considerations that we prefer to leave to further investigations.

We want to stress, that the concept of fine-structure constant introduced by Sommerfeld in 1916 to interpret the corresponding fine-structure line at about 45 μeV found by the pioneer experimental measurements of the emission spectrum of atomic Hydrogen by Michelson and Morley in 1986 is here interpreted in terms of Planck statistical model based on a black-body thermodynamic system for the photon gas. Present results assert the coincidence of the Sommerfeld inverse fine structure constant with the variance of the photon number associated with this spectroscopic line by the exact expression

$$\frac{1}{\alpha_{\rm line}} = \overline{\delta N^2_{\rm line}} = \frac{1}{137.0360} \quad (11)$$

where the accuracy of the numerical value is taken within 7 digits. One of the relevant consequences of this finding stems from the fact that the statistical theory generalizes the interpretation of all the lines of the spectrum not only for the case of Hydrogen but of all the atoms in the periodic table of elements, and further. In other words, we introduce the value of the fine-structure line in a one-to-one correspondence with the variance of the corresponding photon number according to the Planck's law of the photon spectrum of the bb cavity. Therefore, the Sommerfeld result and its physical expression in terms of five constants (5) and the physical meaning as evidence of the electromagnetic field-strength remains acceptable for the case of the fine-structure constant but is no longer of physical meaning for all the other lines of the spectrum that now are taken to represent the statistical property of the photon gas itself and associated with the bb radiation model. Indeed, from a statistical point of view the new $\frac{1}{\alpha_{line}}$ introduced here, being the inverse of a variance, represents the so called precision (or accuracy) in the corresponding statistical approach based on the inverse domain of positive rational numbers with respect to the direct domain of the positive rational numbers associated with the variance definition. We would like to stress the importance of the Sommerfeld expression for α (5) that at present represents the only available data that allows us to fix the value of the volume of the bb cavity, thus obtaining the whole ensemble of α_{line} starting from their energy values. The possibility to obtain the variance, and/or the average photons number directly from counting statistical experiments should be expected as a possible opportunity in near future following recent experimental achievements [10, 11].

ACKNOWLEDGEMENTS

Prof. Dr. Tilmann Kuhn from Münster University is warmly thanked for the very valuable comments he provided on the subject. Prof. Dr. Luca Peliti, Deputy Director Santa Marinella Research Institute (Roma) is acknowledged for valuable comments on the quantum diffusion model and for providing us with a copy of Ref. [12].

[1] A. Michelson and E. Morley, On a method of making the wave-length of sodium light the actual practical standard of length, *American Journal of Science* **34**, 427 (1887). DOI: 10.2475/ajs.s3-34.204.427

[2] A. Sommerfeld, Zur Quantentheorie der Spektrallinien, *Annalen der Physik* **51**, 1–52 (1916). DOI: 10.1002/andp.19163561702

[3] P.A.M. Dirac, The quantum theory of the electron, *Proc. Royal Soc. A: Mathematical, Physical and Engineering Sciences* **117**, 610 (1928). DOI: 10.1098/rspa.1928.0023

[4] H. Leff, Fluctuations in particle number for a photon gas, *Am. J. Phys.* **83**, 362 (2015). DOI: 10.1119/1.4904322

[5] L. Reggiani, and E. Alfinito, Fluctuation dissipation theorem and electrical noise revisited, *Fluct. Noise Lett.* **18**1, 930001 (2019). DOI: 10.1142/S0219477519300015

[6] J. W. Rohlf, *Modern Physics from α to Z^0* (Wiley, New York, 1994)

[7] P.J. Mohr et al., *Rev. Mod. Phys.* **88** (2016), https://doi.org/10.1103/RevModPhys.88.035009

[8] N. Bezginov et al., *Science* **365** (2019) 1007, https://doi.org/10.1126/science.aau7807

[9] L. Reggiani, E. Alfinito and T. Kuhn, Duality and reciprocity of fluctuation-dissipation relations in conductors, *Phys. Rev. E* **94**, 032112 (2016). DOI: 10.1103/PhysRevE.94.032112

[10] S. Virally, J. O. Simoneau, C. Lupien, and B. Reulet, Discrete photon statistics from continuous microwave measurements, *Phys. Rev. A* **93**, 043813 (2016). DOI: 10.1103/PhysRevA.93.043813

[11] S. Beaudoin, E. Pinsolle and B.Reulet, Counting statistics of ultrabroadband microwave photons, *Phys. Rev. Res.* **6**, 043238 (2024). DOI: 10.1103/PhysRevResearch.6.043238

[12] L. Peliti and P. Muratore-Ginanneschi, R. Fürth's 1933 paper "On certain relations between classical statistics and quantum mechanics" ["Über einige Beziehungen zwischen klassischer Statistik und Quantenmechanik", Zeitschrift für Physik, 81 143–162], *The European Physical Journal H* **48:4**, (2023); DOI: 10.1140/epjh/s13129-023-00052-5.