A microscopic model of a fractionalized Fermi liquid

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In this short letter we identify a relationship between the Kondo lattice model formulated in Coleman *et.al*, Phys. Rev. Lett. **129**, 177601 (2022) and Ancilla Layer formulation of the Hubbard model recently proposed by Zhang and Sachdev.

The concept of a fractionalized Fermi liquid (FL*), in which a electron and a spin Fermi liquid co-exist, was first proposed as a way to satisfy the Oshikawa sum rule at the large-to-small Fermi surface transition of a Kondo lattice[1]. Recently, this idea has re-emerged in the context of cuprate superconductors[2], to account for the identification of small Fermi-surface pockets in the underdoped compounds. In particular, a recent analysis analysis of the angular dependence of the magnetoresistance[3], which displays a Yamaji effect reveals that the area of these putative Fermi pockets scales with the hole density, rather than the electron density of a conventional Fermi liquid. The persistence of the angle-dependent resistivity to high temperatures appears to rule out the doubling of the elementary cell due to density wave formation. These observations thus suggest the existence of a phase with Fermi surfaces that do not enclose the conventional Luttinger volume, motivating the idea that this is an FL* phase.

Several theoretical proposals [4–9] have posited that an FL* phase involves the co-existence of a charged Fermi liquid and a background spin liquid which supports a sea of neutral fermion excitations that is invisible to ARPES experiments. In this way, the missing portion of the Luttinger volume remains hidden from view. To provide a concrete formulation of the FL* concept, Zhang and Sachdev recently advanced an Ancilla Layer Model (ALM) [10, 11], in which an ancilliary filled band decouples, via a lattice Kondo effect, into a conduction band and a spin liquid: the former subtracts from hole Fermi surfaces, giving them an area proportional to the hole density, while the latter decouples from the Fermi surface count as the hidden component of an FL*[2, 12]. The crux of the issue depends crucially on the existence of a decoupled spin liquid of fermions.

Here we highlight a solvable model, the CPT model[13] which provides a concrete realization of the FL* phase [14] envisioned in the ALM approach. While there are key differences between the two approaches, the salient features have much in common. As in the ALM approach, the CPT model [13] can be regarded as a three-layered set of excitations, consisting of a conduction sea linked to two separate spin layers. The middle layer is described by spin-1/2 moments with a Heisenberg symmetry, which we denote by $\mathbf{S}_1$, while the lower layer describes second set of spin-1/2 moments, $\mathbf{S}_2$ which couple via an anisotropic Kitaev-like Ising interactions. Here we follow the notation used in [10, 11] rather than the orbital-spin notation used in the original CPT model. The resulting Kondo lattice model can be regarded as a three-layer system in which each site contains itinerant electrons (c, c^+) and the two types of spin-1/2 operators, $\mathbf{S}_1$ and $\mathbf{S}_2$ (Fig. 1 a). This model is defined on a hyperoctagon lattice which allows the dynamics of the spin fluid to be exactly solved using classic Kitaev-methods, in this case giving rise to a gapless Yao Lee spin liquid with a Fermi surface, in isolation. The lattice couples a band of conduction electrons to the mid-layer Heisenberg spins $\mathbf{S}_1$ of the Yao-Lee spin liquid via a Kondo interaction. The CPT Hamiltonian thus contains three terms: $H_{CPT} = H_c + H_K + H_{YL}$, where

$$
\begin{aligned}
H_c &= -t \sum_{<i,j>} (c^+_{i\sigma} c_{j\sigma} + \text{H.c.}) - \mu \sum_i c^+_{i\sigma} c_{i\sigma}, \\
H_K &= J_K \sum_i (c^+_i \boldsymbol{\sigma} c_i) \cdot \boldsymbol{S}_{1,i}, \\
H_{YL} &= K/2 \sum_{<i,j>} S^{\alpha_{ij}}_{2,i} S^{\alpha_{ij}}_{2,j} (\boldsymbol{S}_{1,i} \cdot \boldsymbol{S}_{1,j}), .
\end{aligned}
\tag{1}
$$

Here H_c describes the nearest-neighbor hopping of the conduction electrons, H_K couples the conduction sea to the middle-layer of spins $\mathbf{S}_1$ via an antiferromagnetic Kondo interaction while H_{YL} describes the Yao-Lee spin-spin interaction within the bottom two spin layers. The Yao-Lee term involves an anisotropic nearest-neighbor Ising interaction between the $\alpha_{ij} = x, y, z$ components of the orbitals $S_{2,j}$, which is decorated by a Heisenberg interaction between the spins $\boldsymbol{S}_{1,j}$.

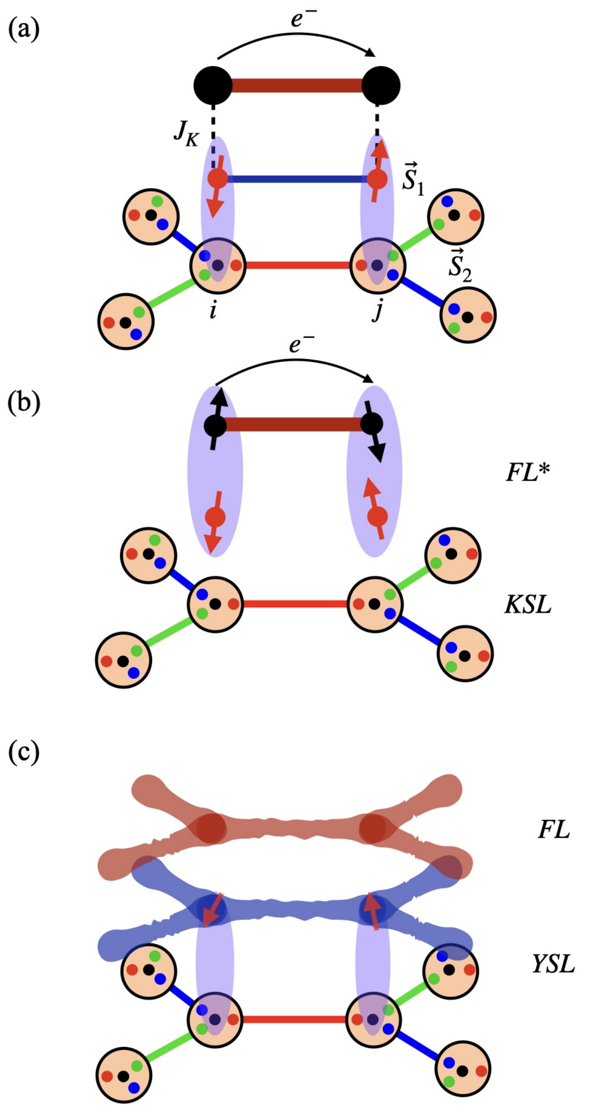

FIG. 1. a.) In the CPT model (1) each site is trivalent, hosting conduction electrons, shown here as black circles, localized spins-1/2 $\mathbf{S}_1$ linked by Heisenberg exchange and $\mathbf{S}_2$, linked by color-coded Kitaev interactions. The frustrated Kitaev interactions induce a Majorana fractionalization of spins, which are then Kondo-coupled to the conduction electrons on the same lattice. b.) FL* phase. At large Kondo coupling spins $\mathbf{S}_1$ are "absorbed" into the electron band contributing to its Luttinger volume. Then spins $\mathbf{S}_2$ create Kitaev spin liquid - a Majorana thermal metal with a Fermi surface. c.) FL phase. Zero Kondo coupling. The spins do not contribute to the Luttinger volume of the conduction band. They create Yao-Lee spin liquid instead.

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Although both CPT and ALM models support the FL* phase, the global phase diagrams are different. The most prominent difference is the presence of a superconducting

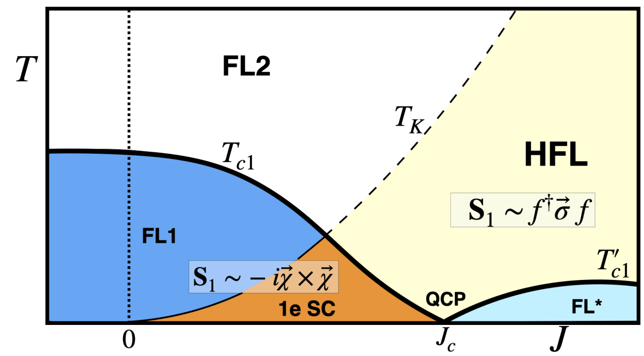

FIG. 2. Phase diagram of the CPT model Eq.(1) and Fig. 1a at one electron per site [14]. In $\mathrm{FL}_{1,2}$ phases the spin and electron subsystems are decoupled (Fig. 1c). Below the Kondo temperature T_K they are coupled . At $J < J_c$ the ground state is an exotic (fractionalized) superconductor. At $J > J_c$ the ground state is a fractionalized Fermi liquid FL* (Fig. 1b). The transition lines T_{c1}, T'_{c1} mark phase transitions in, respectively, the Yao-Lee and Kitaev spin liquids. Below these transitions the visons become rare.

phase separating the normal FL and FL* phases of the CPT model. In [14], we presented the phase diagram of the CPT model at a filling corresponding to one electron per site, where the electron subsystem forms a compensated metal. We identified three distinct phases. For $J_K < 0$ (the left side of the phase diagram), the electron subsystem is decoupled from the spin liquid (SL). This phase is a conventional Fermi liquid (FL), though with a subtlety: it features two Fermi surfaces—electron and hole—of equal volume (compensated metal).

Different phases of the CPT model correspond (see Fig. 2) to different patterns of the spin fractionalization. Spin 1/2 is unique in the sense that it can be fractionalized as a product of two Majorana fermions, as in the Kitaev model [15]. The tensor product of two spins can be fractionalized as Majorana fermion bilinears of six Majoranas which allows to fermionize the Yao-Lee model [16]. This fractionalization pattern works in the exotic fractionalized superconducting phase which exists in the CPT model at small J_K [13]. The FL* phase appears at strong positive J_K (the right side of the phase diagram Fig. 2). The corresponding fractionalization pattern here is more conventional:

$$\mathbf{S}_1 = \frac{1}{2} f^+_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta, \quad \sum_\alpha f^+_\alpha f_\alpha = 1, \tag{2}$$

which allows one to decouple the interaction as

$$J_K \sum_i (c_i^+ \boldsymbol{\sigma} c_i) \cdot \boldsymbol{S}_{1,i} \to \Phi^2/2J_K + \Phi(c^+ f + f^+ c), \tag{3}$$

(Ref. [11] uses an alternative representation where the spinon operators are represented by Nambu spinors and the corresponding Higgs field is a complex 2×2 matrix).

In the saddle point approximation the Higgs field Φ acquires a finite expectation value $\Phi \sim t\exp[-1/\rho(\epsilon_F)J_K]$ and both itinerant electrons and holes hybridize with the f-operators, that is with $(\mathbf{S}_1)$ spins, resulting in a Kondo insulator—effectively two Kondo insulators, one for electrons and one for holes. By projecting the Yao-Lee Hamiltonian (1) onto the ground state of this Kondo insulator, we obtain the Kitaev spin liquid for the $(\mathbf{S}_2)$ spins:

$$\langle 0_{KI}|H_{YL}|0_{KI}\rangle = K/2\sum_{<i,j>} S_{2,i}^{\alpha_{ij}}S_{2,j}^{\alpha_{ij}}\langle(\boldsymbol{S}_{1,i}\cdot\boldsymbol{S}_{1,j})\rangle = \\ K^*\sum_{<i,j>} S_{2,i}^{\alpha_{ij}}S_{2,j}^{\alpha_{ij}} = \mathrm{i}K^*\sum_{<i,j>} u_{ij}\chi_i\chi_j, \tag{4}$$

where $u_{ij} = \pm 1$ is the Z_2 static gauge field and χ_i are Majorana fermions. On hyperoctagonal lattice the Majorana fermions of the Kitaev spin liquid have a Fermi surface. As is mentioned above, a distinct feature of our model is the intermediate phase between FL and FL* - the fractional superconductor. This feature is not universal; its existence is guaranteed only in the presence of nesting between the electron and Majorana FS's.

In our subsequent paper [17] we discussed finite doping. The difference is minimal. The superconducting phase may become Pair Density Wave one. The strong J_K limit becomes heavy fermion metal, the orbital Kitaev liquid remains intact.

An interesting question is whether one can generalize the above construction for square lattice. We believe it is possible, but there is price to pay. Namely, the corresponding Yao-Lee model will include not two spins, but three. This will be a subject of our future publication.

Conclusions. This paper has emphasized how a Yao-Lee spin liquid embedded within a Kondo lattice, the CPT model, provides a tractable model of a fractionalized Fermi liquid or FL^*, establishing an interesting connection to the Ancillar Layer Model(ALM) approach to the Hubbard model. The Z_2 character of the FL^* in this approach may be an interesting feature to examine in the context of the ALM approach.

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