

# AllShowers: One model for all calorimeter showers

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## Abstract

Accurate and efficient detector simulation is essential for modern collider experiments. To reduce the high computational cost, various fast machine learning surrogate models have been proposed. Traditional surrogate models for calorimeter shower modeling train separate networks for each particle species, limiting scalability and reuse. We introduce AllShowers, a unified generative model that simulates calorimeter showers across multiple particle types using a single generative model. AllShowers is a continuous normalizing flow model with a Transformer architecture, enabling it to generate complex spatial and energy correlations in variable-length point cloud representations of showers. Trained on a diverse dataset of simulated showers in the highly granular ILD detector, the model demonstrates the ability to generate realistic showers for electrons, photons, and charged and neutral hadrons across a wide range of incident energies and angles without retraining. In addition to unifying shower generation for multiple particle types, AllShowers surpasses the fidelity of previous single-particle-type models for hadronic showers. Key innovations include the use of a layer embedding, allowing the model to learn all relevant calorimeter layer properties; a custom attention masking scheme to reduce computational demands and introduce a helpful inductive bias; and a shower- and layer-wise optimal transport mapping to improve training convergence and sample quality. AllShowers marks a significant step towards a universal model for calorimeter shower simulations in collider experiments.

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## 1 Introduction

28 Simulations of particle detectors in high-energy physics (HEP) experiments incur high computational costs, which are expected to increase beyond available resources in the near future [1, 2]. Fast generative models must substitute the most expensive MC simulation steps to achieve sufficient statistics with the available computing resources. To fully realize the physics potential of new experiments with higher event rates and highly granular calorimeters, more accurate and efficient fast generative models must be developed.

34 Many techniques for fast calorimeter simulation have been explored for existing or similar to existing experiments; generative adversarial networks (GANs) [3–16], variational autoencoders (VAEs) [14, 17–19], classical normalizing flows (NFs) [20–30], auto-regressive models [31], and diffusion and continuous flow models [32–40]. Even the highly granular pixel vertex detector of Belle II has been simulated using a GAN [41]. Future calorimeter designs have also been specifically targeted in various generative modeling projects including; GANs [42], VAEs [43–48], NFs [49, 50] and continuous flow models [47, 51–56].

41 These methods can be seen in comparison in a recent taxonomy of detector simulation [57], and for the simulation of current detector designs, the accuracy and efficiency of many variants was compared in the CaloChallenge 2022 [58].

44 The model put forward in this paper breaks new ground; to the best of our knowledge no previous model has captured the response of both the electromagnetic calorimeter (ECAL) and hadronic calorimeter (HCAL) with such a comprehensive set of particle species, let alone for a highly granular Higgs factory detector.<sup>1</sup> This combination of twelve particle types in a single model would be challenging at current granularities, but is even more challenging with the high granularity expected in future calorimeters.

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<sup>1</sup>When preparing this manuscript for submission, [56] was released. It attempts a similar task, albeit for a smaller set of different particles.

50 A multi-particle model, such as this one, is needed for three reasons: firstly, in production  
51 environments, maintaining code infrastructure uses significant human and computational re-  
52 sources, and by handling twelve particles together we simplify the codebase, reducing the  
53 cost of both validation and maintenance. Secondly, fast calorimeter simulation occurs within  
54 full scale MC simulations, so the models must share local memory resources with many other  
55 components, which quickly becomes a limitation on model size and therefore performance. By  
56 combining particle types, common physics behavior will be shared between particles resulting  
57 in better use of local memory, and so facilitating more accurate modeling. Finally, it is hoped  
58 that fast calorimeter simulation might benefit a wider range of users than just the production  
59 environments at major experiments. These users could save energy and compute time on more  
60 dedicated small scale tasks that might require custom installations. A comprehensive model  
61 for all particle showers reduces the technical overhead of setup and installation, increasing the  
62 utilization and usefulness of the model for these users.

63 To achieve this, we introduce AllShowers, a continuous normalizing flow (CNF) model with  
64 a Transformer architecture. AllShowers is trained on a diverse dataset of simulated showers  
65 in the highly granular calorimeters of the International Large Detector (ILD) [59]. The model  
66 consists of two components: the PointCountFM, which predicts the number of points per layer  
67 conditioned on incident particle information, and the CNF-transformer, which generates the  
68 position and energy of each point additionally conditioned on the layer index of each point.  
69 AllShowers has several significant improvements compared to its predecessor models [47, 54].  
70 Using an embedding layer for the calorimeter layer index allows the model to learn all rele-  
71 vant calorimeter layer properties, such as material budget and distance from the calorimeter  
72 surface, from data. A custom attention masking scheme is employed to reduce computational  
73 demands and introduce a helpful inductive bias, allowing points to attend only to points in  
74 nearby layers. Additionally, a shower- and layer-wise optimal transport mapping is used to  
75 improve convergence during training and sample quality.

76 The layout of this paper is as follows. In the next section, section 2, the dataset is described.  
77 This includes a summary description of the detector chosen as an example of a detector sys-  
78 tem with high granularity calorimeters, the particle gun used for shower generation, and the  
79 data preprocessing. Following this, in section 3, the architecture of the AllShowers model is  
80 presented, along with a description of the training process. Then the results are presented in  
81 section 4. Finally, in section 5, the paper is concluded with a discussion of the findings.

## 82 2 Dataset

83 We used the International Large Detector (ILD) [59] as an example of a detector with highly  
84 granular sampling calorimeters. The ILD was initially designed for the International Linear  
85 Collider (ILC), a proposed electron-positron collider, and could be adapted for other future  
86 colliders. The ILD detector design is optimized for particle-flow algorithms, which reconstruct  
87 particles with high precision by combining information from multiple subdetectors.

88 The ILD calorimeter system consists of a highly granular electromagnetic calorimeter  
89 (ECAL) [60] and a hadronic calorimeter (HCAL). Both of which sit within a superconduct-  
90 ing coil generating a magnetic field of 3.5 T strength. The ECAL is composed of 30 layers with  
91 tungsten absorbers and silicon sensors with about  $5 \times 5 \text{ mm}^2$  pads. For mechanical reasons  
92 and to reduce dead material, two active layers are always mounted on either side of a tung-  
93 sten support. This results in a small modulation in measured energy in even and odd layers.  
94 To improve energy resolution at low energies while preserving good confinement of most EM  
95 showers, two different absorber thicknesses are used: a smaller one for the first 20 layers and  
96 a larger one for the last 10 layers. The HCAL consists of 48 layers with stainless steel absorbers

97 and polystyrene scintillator tiles measuring about  $3 \times 3 \text{ cm}^2$ .

98 As in earlier work [54, 61], we use a regularized readout geometry without insensitive  
99 gaps between calorimeter modules. This broadens the model's applicability to other incident  
100 point locations. Hits, which the model produces in inactive material, will be dropped when  
101 integrated into the full simulation chain.

102 Using Geant4 [62] and the DD4hep [63] framework, we simulated a dataset of four million  
103 showers originating from twelve different incident particle types, namely:  $e^-$ ,  $e^+$ ,  $\pi^-$ ,  $\pi^+$ ,  $K^-$ ,  
104  $K^+$ ,  $K_L^0$ ,  $p$ ,  $\bar{p}$ ,  $n$ ,  $\bar{n}$ , and  $\gamma$ . The incident particle type is randomly chosen for each shower with  
105 equal probability. We cover all incident angles of particles originating at the interaction point  
106 (IP) and reaching the calorimeter barrel region, including magnetic-field effects. This means  
107 that the angular bounds depend on the incident particles charge and energy. The energy of  
108 the incident particles is uniformly distributed between 5 GeV and 130 GeV. A random sample  
109 of 50k showers is used as a validation set. For testing, we simulated several datasets with the  
110 incident particle distributions and statistics given in the results section.

## 111 2.1 Data Representation

112 The calorimeter shower data are represented as a 4D point cloud of energy depositions (Geant4  
113 steps) in active material, where each point is represented as a tuple  $(x, y, z, e)$ . Here,  $x$  and  $y$   
114 denote the local coordinates in millimeters, with  $x$  aligned along the direction of the magnetic  
115 field, and  $z$  indicates the layer index, ranging from 0 to 77 (covering both ECAL and HCAL  
116 layers).  $e$  represents the deposited energy.

117 To reduce the number of points while preserving geometry independence, the energy de-  
118 positions are binned into a grid that is three times finer in the two transversal dimensions, i.e.,  
119 nine times higher granularity, than the respective readout pads [51]. For each non-empty bin,  
120 a point is created using the  $x$  and  $y$  coordinates of the highest energy deposition within the  
121 bin, and the total energy within the bin.

122 We counteract the incident angle dependence of the shower shape by shifting the  $x$  and  $y$   
123 coordinates of each point such that the incident particle always appears to enter the calorimeter  
124 perpendicularly at the origin. While this transformation does not eliminate all angle depen-  
125 dencies, it significantly simplifies the model's learning task. To further reduce the number of  
126 points, we remove all points with an energy deposit below 10 keV or with a time of over 200  
127 ns (bunch crossing window). The time constructed is already applied before clustering. We  
128 place a quadratic bounding box around the shower core removing all points outside this box.  
129 The side length is chosen to be the side length of the octagon formed by the ECAL surface (c.a.  
130 1500 mm). This will exclude most of the points for which the flat layer assumption breaks.  
131 The excluded energy depositions are far away from the shower core and typically low in en-  
132 ergy. After these preprocessing steps, the average number of points per shower is 2306, with  
133 a maximum of 6006.

134 For preprocessing, the  $x$  and  $y$  coordinates are rescaled to have standard deviation one  
135 and mean zero (Standardization),  $z$  is kept as the discrete layer index, and the logarithm of  
136 the energy is also standardized. Points are zero-padded to a maximum of 6016 points per  
137 shower for batch training. 6016 is a multiple of 128, making the computation of attention  
138 masks easier and more efficient.

## 139 3 Model and Training

140 The AllShowers model consists of two main components: the PointCountFM and the CNF-  
141 transformer, as illustrated in figure 1. The PointCountFM is responsible for generating the  
142 number of points per layer conditioned on the incident particle information (particle type,

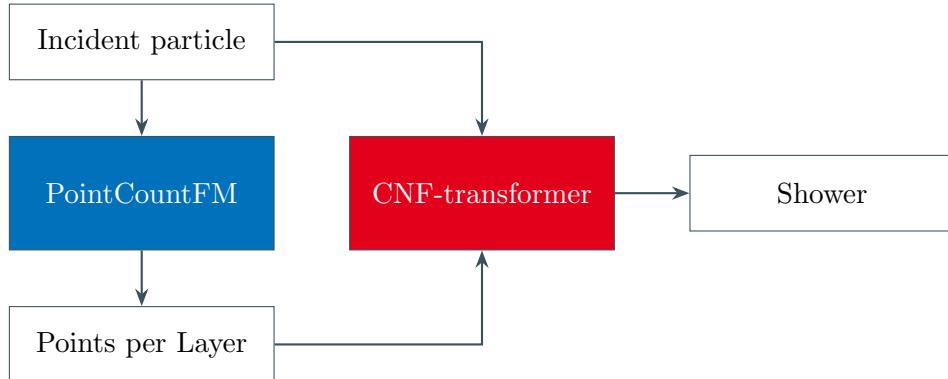


Figure 1: Schematic overview of the AllShowers model architecture. The PointCountFM predicts the number of points per layer, which are then used by the CNF-transformer to generate the full shower. The incident particle information is provided to both models. The point layer index, needed by the CNF-transformer, can be computed from the number of points per layer.

143 energy and angle). After initializing as many points as demanded by PointCountFM, the CNF-  
 144 transformer generates the position within the layer and the energy of each point, again con-  
 145 ditioned on the incident particle information. The layer index is provided as an additional  
 146 condition. Splitting the model into two components in this way is inspired by CaloFlow [20]  
 147 where the layer-wise energy depositions are generated first and has been used in various other  
 148 works.

149 Both models use the continuous normalizing flow (CNF) [64] paradigm as a means to  
 150 model the complex distributions of calorimeter showers. In CNFs, the transformation from  
 151 latent to physics space is modeled as the solution of an ordinary differential equation (ODE)  
 152  $\frac{dx_t}{dt} = v_c(x_t, t)$ , where  $v_c(x_t, t)$  is a neural network that predicts the vector field,  $t$  is the integra-  
 153 tion variable, and  $c$  is the condition.  $x_0$  is the initial condition, a sample from the latent space.  
 154  $x_1$  is a physics space sample. Note that  $x$  denotes the spacial coordinate in the calorimeter  
 155 while  $x_t$  denotes an entire data sample. During generation a numerical ODE solver is used.

156 A likelihood based training of CNFs is possible [64], but computationally inefficient. In-  
 157 stead, we use the recently proposed conditional flow matching (FM) [65] approach. In FM,  
 158 the vector field is constructed as the expectation value of all straight lines connecting physics  
 159 and latent space samples. This mean squared error is evaluated using Monte Carlo integration  
 160 over physics and latent space. FM has been shown to be more efficient than likelihood-based  
 161 training for CNFs [65].

### 162 3.1 PointCountFM

163 The PointCountFM was already introduced in CaloHadronic [47]. It is responsible for gener-  
 164 ating the number of points per layer, 78 integers in total, conditioned on the incident particle  
 165 information (type, energy, angle). The type is given as a one-hot encoded vector, the energy  
 166 is converted to logarithmic scale and standard scaled, and the angle is represented by a vector  
 167 on the unit sphere. This is a generalization of the approach taken in CaloHadronic, where only  
 168 fixed angle and particle type were considered.

169 As an additional improvement, we no longer use dequantization noise during training.  
 170 While dequantization is essential for classical likelihood-based training of flows on discrete  
 171 data, it is not necessary for FM. We found that removing the dequantization noise leads to a  
 172 significant improvement in performance especially for low point counts. Dequantization works  
 173 well when a change by one in the discrete value has no significant effect on the downstream

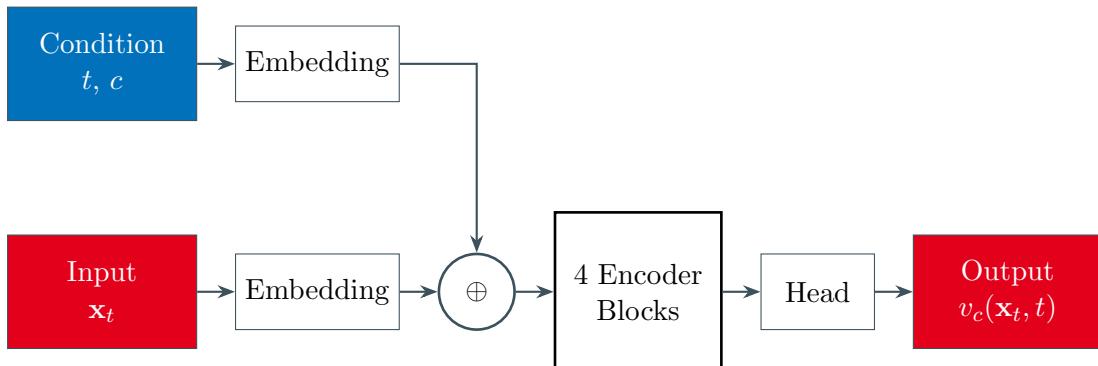


Figure 2: Schematic overview of the CNF-transformer architecture. The input  $\mathbf{x}_t$  for  $t = 0$  is a standard normal sample, for  $t = 1$  it is the preprocessed shower. Since the calorimeter layer is a condition,  $\mathbf{x}_t$  is a three-dimensional point-cloud  $(x_k, y_k, e_k)$ . The condition  $c$  includes the incident particle information and the layer index.  $t$  is the time variable of the neural ODE. The output  $v_c(\mathbf{x}_t, t)$  is the vector field used in the CNF, e.g. the right-hand side of the neural ODE.

174 task. However, in our case, a change from zero to one point in a layer can confuse the CNF-  
 175 transformer significantly, as it has to generate a point in an unexpected layer.

176 A complete list of hyper-parameters can be found in Appendix C.

### 177 3.2 CNF-Transformer

178 After PointCountFM has predicted the number of points per layer ( $n_i$ ), as many latent space  
 179 points as requested are initialized. The first  $n_0$  points are assigned to layer 0, the next  $n_1$   
 180 points to layer 1, and so on. Each point is initialized with a standard normal sample in the  
 181  $x$ ,  $y$ , and  $\log(e)$  dimensions. The layer index,  $z$ , is provided as an additional condition. Then  
 182 CNF-transformer transforms these points into a calorimeter shower.

183 An overview of the CNF-transformer architecture is shown in figure 2. The input is the  
 184 point cloud,  $\mathbf{x}_t$ , at ODE time  $t$ . For  $t = 0$ , this is a standard normal sample, and for  $t = 1$ , it is the  
 185 preprocessed shower. The output is the vector field  $v_c(\mathbf{x}_t, t)$  used in the CNF, i.e. the right-  
 186 hand side of the neural ODE. We can split the condition on global conditioning information  
 187 and point-wise conditioning information. The global conditioning information includes the  
 188 incident particle type, energy, and angle, while the point-wise conditioning information is the  
 189 layer index. Input, time, and conditions are embedded and element-wise summed. The result-  
 190 ing representation is processed by four transformer encoder blocks. Finally, a head network  
 191 produces the output vector field.

### 192 3.3 Embeddings

193 The main purpose of the embeddings is to map the different inputs to a common feature space.

194 **Input Embedding** The input  $\mathbf{x}_t$  is a point cloud of shape  $(N, 3)$ , where  $N$  is the number of  
 195 points. We embedded each point independently using a single linear layer going from 3 to 64  
 196 dimensions.

197 **Time Embedding** For the time embedding, we used the standard Fourier feature mapping [66]  
 198 with 3 frequencies, followed by a linear layer going from 6 to 64 dimensions.

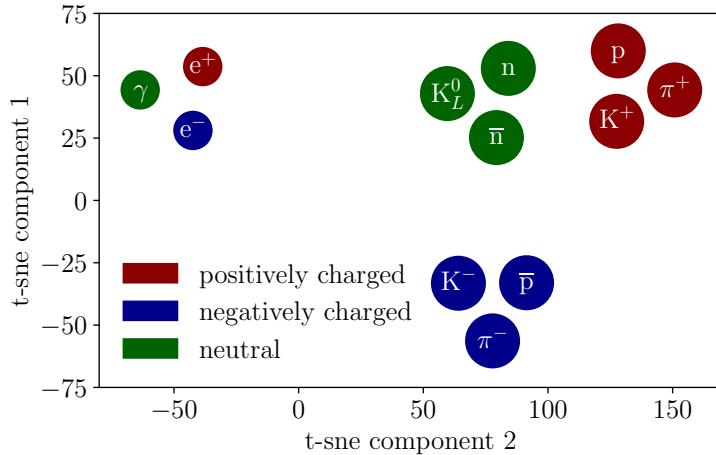


Figure 3: 2D t-SNE [67] visualization of the learned 64 dimension particle type embeddings. Small circles indicate electromagnetic showers, large circles indicate hadronic showers. One can see four distinct clusters: one for electromagnetic showers, one for positively charged hadrons, one for negatively charged hadrons, and one for neutral hadrons.

199 **Condition Embedding** The global conditional information includes the incident particle  
 200 type, energy, and direction. The model explicitly learns 12 embedding vectors, one for each  
 201 particle type. Figure 3 shows a t-SNE [67] visualization of these embeddings. The prepro-  
 202 cessed energy and direction are concatenated and passed through a linear layer going from 4  
 203 to 64 dimensions. Since the point layer is provided as point-wise information, the model has  
 204 an implicit conditioning on the number of points per layer. To make it explicit, we also provide  
 205 the number of points per layer as global information. The number of points per layer is passed  
 206 through a feedforward network with one hidden layer of size 128 with ReLU activation, going  
 207 from 78 to 64 dimensions.

208 **Layer Embedding** The calorimeter layer index is provided as point-wise conditional input.  
 209 The model explicitly learns 78 embedding vectors, one for each of the 78 layers. This allows the  
 210 model to learn layer-specific features like distance from the ECAL surface, material budgets,  
 211 and typical energy deposition.

212 After embedding, the global features are repeated for each point and all features are summed  
 213 element-wise.

### 214 **3.4 Fast Attention Masking**

215 One major drawback of transformers is their quadratic complexity in the number of input to-  
 216 kens. In our dataset, the number of points per shower can be up to 6016, which would require  
 217 more than 36 million attention weights. In the computer science literature, various methods  
 218 for masking attention weights have been proposed. Most notably, the Sparse Transformer [69],  
 219 Longformer [70], and BigBird [71] architectures. However, these methods have been devel-  
 220 oped in the context of natural language processing, where the input is a sequence. In our case,  
 221 the input is a point cloud without any inherent ordering.

222 We developed a custom attention masking scheme that takes advantage of the fact that  
 223 points are grouped by calorimeter layer. We allow points that are up to two layers apart to  
 224 attend to each other. This means that points in layer  $i$  can attend to points in layers  $i-2, i-1,$

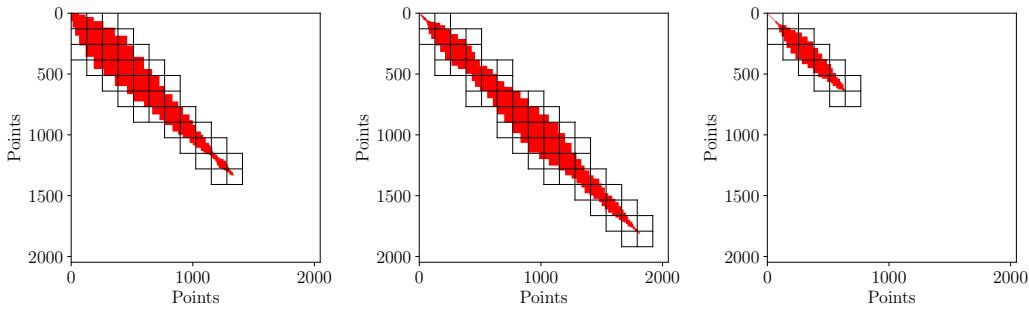


Figure 4: Examples of attention masks for three different showers. Shown is a part of the full  $6016 \times 6016$  attention matrix where each entry indicates whether two points can attend to each other. Red entries indicate allowed attention, white entries indicate masked attention. The black lines indicate the  $128 \times 128$  blocks flex-attention [68] will compute. White entries within these blocks are computed but then masked out.

225  $i$ ,  $i + 1$ , and  $i + 2$ . In combination with padding masking [72], this leads to a high degree of  
 226 sparsity in the attention weights for our dataset. To utilize this sparsity, we used PyTorch's [73]  
 227 built-in FlexAttention [68] module. For this to work efficiently, input points are sorted by layer  
 228 index before starting the training. An example of attention masks for three different showers  
 229 is shown in figure 4.

230 This attention masking scheme leads to a significant speed-up during training and inference-  
 231 ence; roughly a factor of twenty which is roughly considered with the sparsity of the attention  
 232 weights. This allowed us to train the CNF-transformer for more epochs leading to better per-  
 233 formance. We also found improved performance with the same number of training epochs,  
 234 indicating that the inductive bias introduced by the attention masking is beneficial.

### 235 3.5 Layer-wise Optimal Transport Mapping

236 In continuous normalizing flows and diffusion models, the sampling process involves trans-  
 237 forming samples from a simple latent distribution (e.g., a standard normal distribution) to  
 238 match the complex data distribution. To do so, an ordinary or stochastic differential equation  
 239 (ODE or SDE) is solved, which can be resource intensive. The number of function evalua-  
 240 tions (NFE) necessary to get good results is strongly correlated with the curvature of the trajectories  
 241 taken by samples during the transformation  $\kappa = |\ddot{\mathbf{x}}_t|$ , where  $\ddot{\mathbf{x}}_t$  is the second derivative of the  
 242 sample with respect to the integration variable  $t$ .

243 The main reason CNFs have curvature in their trajectories is the random mapping of data  
 244 points to latent points during training. To overcome this problem, batch-wise optimal transport  
 245 (OT) mapping has been proposed [74]. The idea is to approximate the optimal mapping  
 246 between data points and latent points which would lead to straight trajectories. To achieve  
 247 this, the optimal transport problem is solved for each batch during training. However, this  
 248 approach is only feasible for generative problems without or with simple conditioning.

249 Instead of mapping data and latent points, we map physics point-cloud points to latent  
 250 point-cloud points exploiting the permutation invariance. Since the calorimeter layer condi-  
 251 tioning breaks permutation invariance, the OT mapping is only applied per shower and layer.  
 252 We solve the OT problem using the Python Optimal Transport (POT) library [75]. The cost  
 253 function is the Euclidean distance in the 3D space of preprocessed points.

254 The layer-wise OT mapping leads to shorter trajectories, faster training convergence, and  
 255 better results.

### 256 3.6 Training Details

257 We trained the CNF-transformer using the Lookahead optimizer [76] with RAdam [77, 78]  
258 as the inner optimizer and decoupled weight decay [79]. We found this combination, also  
259 known as Ranger [80], to be especially robust against training instabilities, leading to reliable  
260 convergence in our experiments. We wrote a custom Ranger implementation in PyTorch to fit  
261 our needs. As learning rate scheduler, we used a cosine annealing schedule. Since RAdam has  
262 an integrated warm-up phase, we did not use an additional warm-up schedule. We trained  
263 the model with a batch size of 256 for 200 epochs. The training took less than 24 hours on 16  
264 Nvidia A100 GPUs. A complete list of hyper-parameters can be found in Appendix C.

### 265 3.7 Energy Calibration

266 After training, we found that the total energy per shower generated by the CNF-transformer  
267 was too low by approximately 3.3% on average. While a simple rescaling of the point energies  
268 could fix this, it would influence other distributions in a negative way. Instead, we rescale the  
269 incident energy we provide as condition to the CNF-transformer by a factor of 1.033 during  
270 inference. This simple calibration step fixes the total energy per shower without negatively  
271 impacting other distributions.

## 272 4 Results

273 In the following section, the performance of the model is presented on multiple levels. Beginning  
274 at the single event scale, in section 4.1, the ability of the model to generate realistic de-  
275 tailed events is shown. Secondly, in section 4.2, the ensemble-level distributions of the model  
276 are compared to the targets they seek to replicate. Following this, in sections 4.3 and 4.4, the  
277 ensemble-level distributions are compared to other models with similar objectives, and finally,  
278 section 4.5 looks at the inference speed of this model.

279 In order to render all comparisons fair, the same post processing is applied to the output of  
280 all models. Hits produced by the models are clustered into regular grids intended to resemble  
281 the granularity of the calorimeter in question; so in the ECAL, hits have been clustered into  
282 cells of  $5 \times 5$  millimeters, and in the HCAL, into cells of  $30 \times 30$  millimeters. Each model has  
283 used its own conventions for training data preprocessing, and we do not wish the relationship  
284 between the grid in post processing and any grids imposed on the training data to introduce  
285 artifacts, therefore we add a random offset to the post processing grid in each event. Finally,  
286 cells with energy below half the energy deposited by a Minimum Ionizing Particle (MIP) are  
287 conventionally removed before reconstruction to reduce electronics noise, so we remove these  
288 cells in the post processing as well.

### 289 4.1 Individual Showers

290 One of the more exciting features of a high granularity calorimeter is how distinctly it resolves  
291 particle showers from different particle types. In figure 5, we can see examples of six particle  
292 types, each shown once as simulated by Geant4 (upper) and once by AllShowers (lower). The  
293 direction and energy chosen is the same for each model, and the number of points per layer  
294 is fixed to be that chosen by the Geant4 simulation, that is to say, for AllShowers, only the  
295 CNF-transformer is used, PointCountFM does not run. Thus the two models are compelled to  
296 generate events with similar depth for each shower, and the results are directly comparable.  
297 In each image, a gap can be seen at about  $z = 2015$  mm where the ECAL ends and the HCAL  
298 has yet to start.

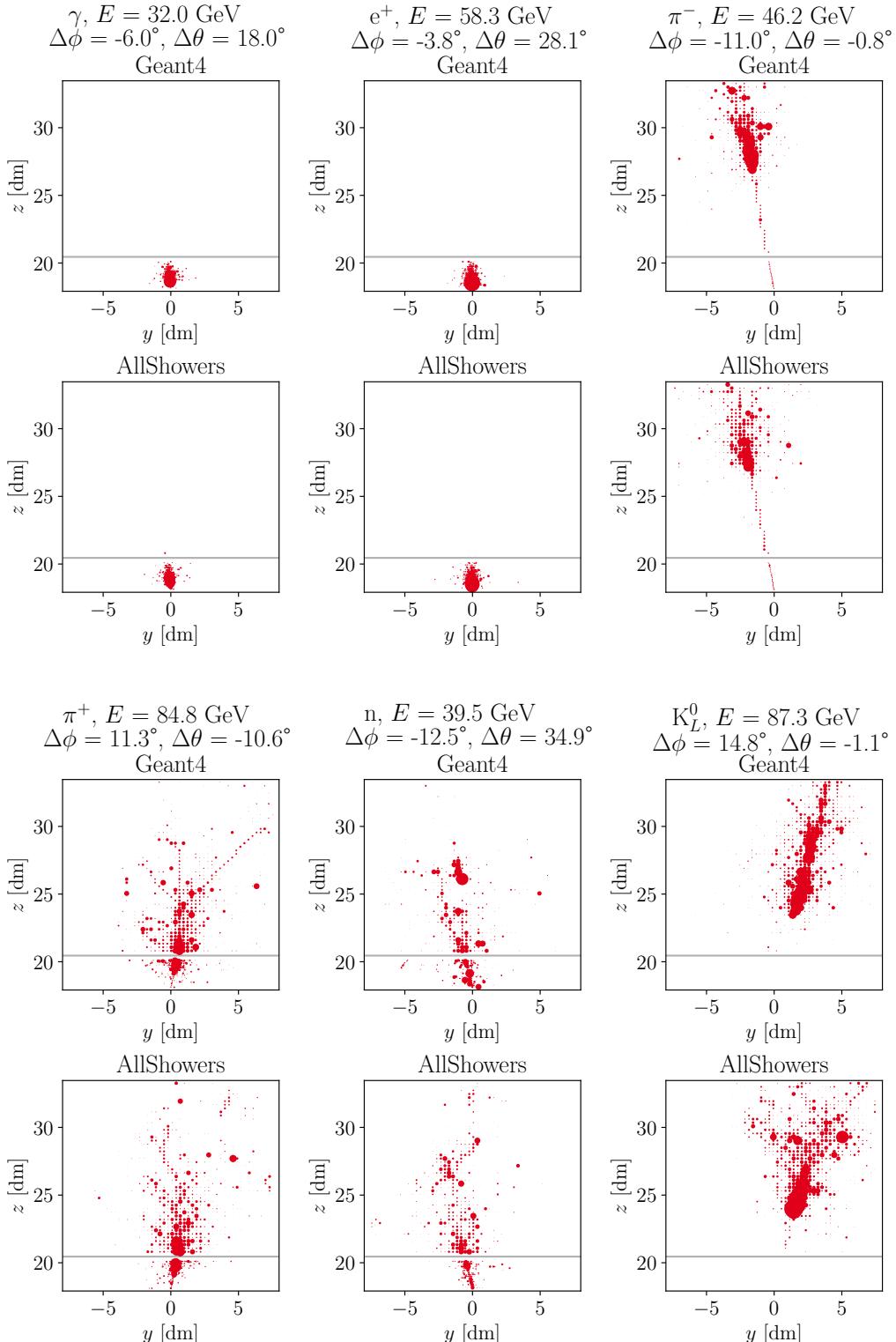


Figure 5: Comparison of individual showers simulated with Geant4 and with All Showers for different incident particles, energies and angles. The point size indicates the energy of each hit. For these showers, the number of points per layer was taken from the Geant4 simulation rather than generated by the PointCountFM to allow for a more direct comparison of the spatial and energy distribution of hits.

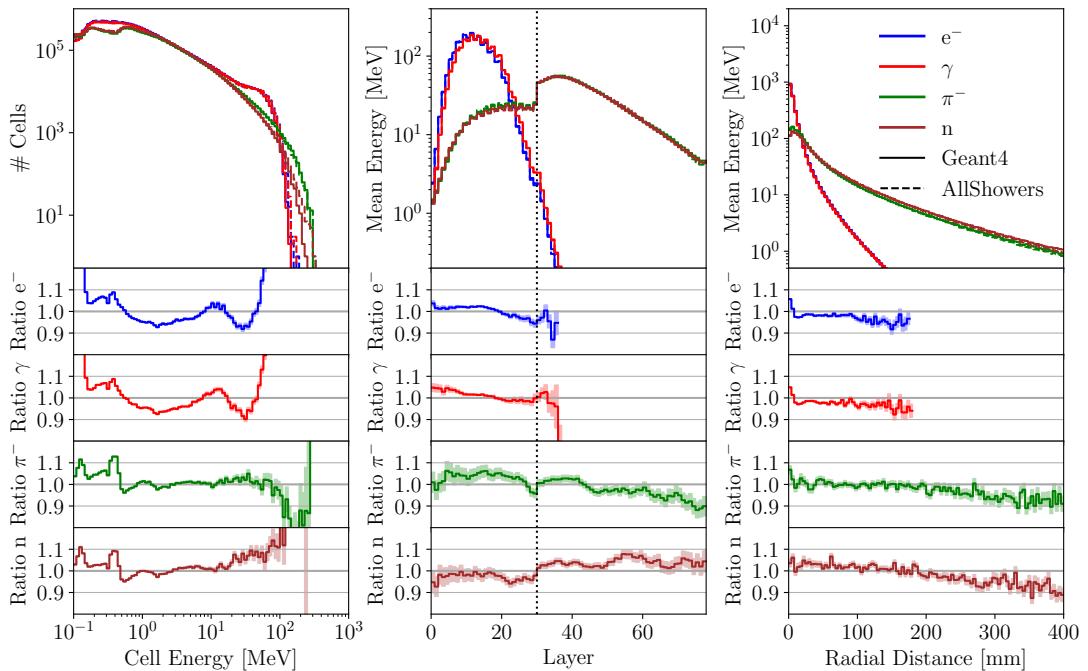


Figure 6: Histograms comparing Geant4 and AllShowers for different incident particle types and angles. The incident energy is fixed to 100 GeV. From left to right: cell energy spectrum, longitudinal energy distribution, and radial energy distribution. The solid lines represent Geant4 and the dashed lines AllShowers. The lower panels of each plot show the ratio of AllShowers over Geant4. For the cell energy spectrum, the Poisson error and for the longitudinal and radial energy distributions, the standard deviation of the mean is shown as error bars. Per particle type and generator, 10k showers were simulated/generated.

299 It is clear from these images that the embeddings (see section 3.3) used to encode the  
 300 particle type are sufficient for AllShowers to produce appropriately tailored behavior in each  
 301 shower. By eye, not only is each particle type distinct, but the features align well with those  
 302 seen in the Geant4 simulation. The electromagnetic showers of the  $\gamma$  and  $e^+$  each have the  
 303 typical cloud-like distribution, well contained in the ECAL. The charged pions ( $\pi^+$  and  $\pi^-$ )  
 304 each have well defined MIP tracks in AllShowers, which correctly point to the start of the  
 305 shower. For the  $\pi^-$ , this requires traversing right through the ECAL into the HCAL. Each pion  
 306 shows a mild bend of the MIP track in opposite directions to account for the response to the  
 307 magnetic field. Both pions then shower, with AllShowers displaying marginally fewer defined  
 308 secondary tracks than Geant4, but still providing some, and displaying a very plausible shower  
 309 pattern. The neutron ( $n$ ) event produced by AllShowers also replicates the overall shower  
 310 cone well, again perhaps showing fewer secondaries. Finally, as appropriate for a neutral  
 311 particle, AllShowers does not generate a MIP track for the  $K_L^0$  particle. The fetcher of neutral  
 312 hadrons is hidden for the neutron shower shown here since it starts showering immediately  
 313 upon entering the calorimeter. The  $K_L^0$  shower develops in AllShowers with good substructure,  
 314 including visible internal secondary tracks, and a correct funnel shape. The shower start is  
 315 marginally less aggressive in AllShowers than in Geant4, but it is a very plausible  $K_L^0$  shower.

## 316 4.2 Distributions

317 While having visually credible individual showers is clearly an asset, almost all physics anal-  
 318 ysis happens on the ensemble-level. In figure 6, we present histograms comparing kinematic

319 behavior of Geant4 and AllShowers for selected particles;  $e^-$ ,  $\gamma$ ,  $\pi^+$  and  $n$ . Other particles  
 320 have similar accuracy, but are omitted for clarity in the plots.

321 In the top panels, both AllShowers (dashed line) and Geant4 (solid line) are shown for each  
 322 of three quantities: cell energy spectrum, longitudinal energy distribution and radial energy  
 323 distribution. For most values, the top plot shows no observable difference between Geant4  
 324 and AllShowers. In the lower plot, ratios of AllShowers to Geant4 are shown.

325 The most striking aspect of these plots is the clear dimorphism of electromagnetic showers  
 326 ( $e^-$  and  $\gamma$ ) and hadronic showers ( $\pi^+$  and  $n$ ). This bimodal behavior is well known, and that  
 327 AllShowers accurately captures both variants demonstrates its flexibility.

328 In the cell energy spectrum on the left, AllShowers is within 10% of Geant4 for most of  
 329 the range in all particle types. It makes a good replication of the MIP peak near  $10^{-1}$  GeV,  
 330 and does not significantly deviate until we reach the sparsely populated tails of the spectrum.  
 331 The high energy tails tend to be somewhat overpopulated in AllShowers, the poor modeling  
 332 is likely due to scarcity of this region in the training data.

333 In the longitudinal energy distribution in the centre, the same dimorphism between elec-  
 334 tromagnetic and hadronic showers is clear. AllShowers's behavior here is strongly influenced  
 335 by the performance of the PointCountFM, and the agreement with Geant4 is within 10% for  
 336 all but the extreme tails. This plot emphasizes the value of also modeling the HCAL for elec-  
 337 tromagnetic showers:  $\gamma$  showers in particular are not always well contained to the ECAL, and  
 338 AllShowers manages to capture the tail that bleeds into the HCAL.

339 Finally, we see the dimorphism again in the radial distribution on the right. This radial  
 340 distribution shows remarkably good agreement for the bulk of the shower. At the innermost  
 341 core, some deviation is visible; but still within 10% of Geant4 for all particles. While there is  
 342 more deviation in the tails, there are very few particles in these regions to work with, so it is  
 343 expected that model performance may not be optimal here.

### 344 4.3 Comparison to CaloClouds3

345 For the case of photons only, we can compare the performance to the performance of the  
 346 CaloClouds3 model [54]. CaloClouds3 is a fast generative diffusion model, specialized to  
 347 only photon showers, trained on the ECAL only. As current generative models would not be  
 348 applied in regions where different layer orientations meet, we also restrict the comparison  
 349 data to photon showers, with  $45^\circ < \theta < 135^\circ$  and  $79^\circ < \phi < 109^\circ$ . An energy range is chosen  
 350 such that it sits comfortably inside both models training regions; 10 to 90 GeV.

351 In figure 7 the standard three kinematic profiles are shown for both fast models and Geant4.  
 352 On the left, the cell energy spectrum for AllShowers is notably better aligned with Geant4 than  
 353 CaloClouds3, in particular, AllShowers has a well formed replication of the MIP peak near  
 354  $10^{-1}$  GeV. Neither model quite fits the high energy tail, but with very few data points, this is a  
 355 challenging region to learn.

356 In the centre, CaloClouds3 and AllShowers perform equally well on the longitudinal energy  
 357 distribution. CaloClouds3 is a little better at replicating the alternating layer pattern, but tends  
 358 to overpopulate the start and end of the shower. On the right hand side of this plot, a grey  
 359 band indicates the HCAL, for which only AllShowers has training data. This region is about as  
 360 populated as the smallest bin in the ECAL, so its contribution is not negligible, and AllShowers's  
 361 capacity to capture this information would be valuable in advanced reconstructions.

362 Finally, in the radial distribution on the right, AllShowers is significantly better than Calo-  
 363 Clouds3. It maintains a flat ratio to Geant4 right out into a long distribution tail, and only  
 364 marginally misrepresents the centre of the shower. CaloClouds3 is unable to keep a flat ratio,  
 365 and deviates significantly from Geant4 towards the tail. While the deviation of CaloClouds3  
 366 in the tail here seems very large, it should be noted that the comparison of machine learning  
 367 models ultimately will have to be done on physics observables, computed after a full event

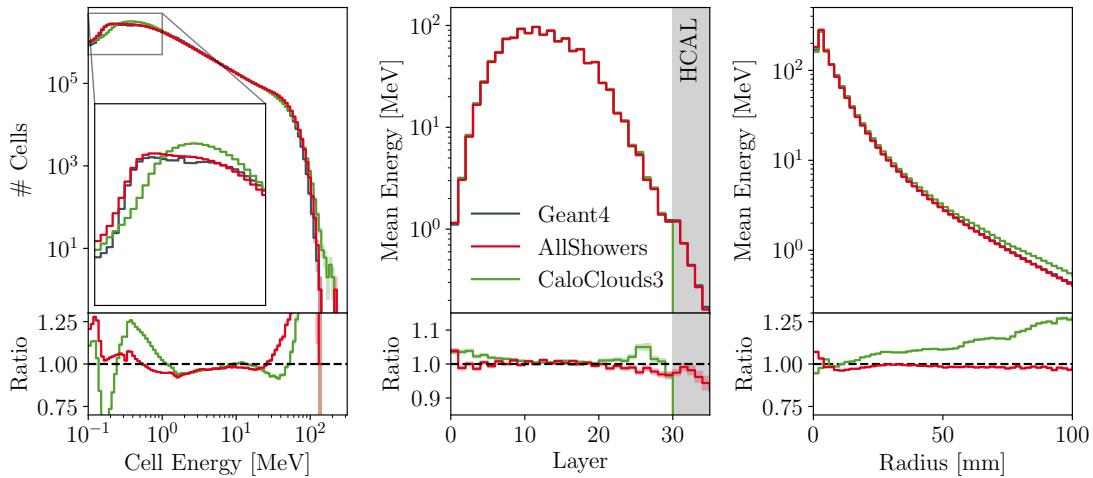


Figure 7: Comparison of AllShowers and CaloClouds3 on photon showers with incident energies uniformly distributed between 10 GeV and 90 GeV. Incident angles are distributed over the intersection of the respective training regions. From left to right: cell energy spectrum, longitudinal energy distribution and radial energy distribution. Per generator, 50k samples are used.

reconstruction has been applied, and that as shown in [61] the CaloClouds3 model performs reasonably well on  $\pi^0$ -reconstruction.

Conceptually, the key distinction between AllShowers and CaloClouds3 is that the diffusion model in CaloClouds3 generates points which are independent and identically distributed (iid). There are longitudinal correlations, imposed by the normalising flow component of CaloClouds3, but these are not known to the diffusion model, and they only describe the macro features, energy per layer and points per layer. This means that CaloClouds3 cannot capture point-to-point correlations. By contrast, AllShowers entertains correlations between points themselves. All these distributions demonstrate that even for a photon shower, the ability to capture subtle substructure can substantially improve the performance of the model.

The linearity of the reconstructed energy is always a key feature for a calorimeter, and must be well replicated in simulations. In figure 8 the linearity of photons as simulated by AllShowers and CaloClouds3 is plotted against a Geant4 reference. The simplified energy reconstruction is a linear sum of the energy deposits, with different scaling factors for sections of the calorimeter with different properties. Three scaling factors are chosen; one for the energy sum of the first 20 ECAL layers, then a second for energy sum of the last 10 ECAL layers, and finally a factor for the energy sum of the HCAL. All factors are chosen to minimize the mean squared error of the reconstructed Geant4 energies and then applied to both the Geant4 and the two ML model data.

In the reconstructed energy on the left AllShowers produces agreement with Geant4 on most points, however, some energies show significant deviations. AllShowers does not make energy predictions in PointCountFM, there is only a single energy correction factor applied, see section 3.7. This correction factor can raise or lower all points collectively, but cannot alter the relative height. By contrast, CaloClouds3 is performing very well across the whole range. Two elements contribute to this, the basic flat profile is achieved by the normalising flow in CaloClouds3, which predicts energy per layer for the model. Then in order to obtain the best mean value for all points, a single correction factor is applied, in the same way as for AllShowers.

For the energy resolution on the right, the range of reconstructed energies from AllShowers simulations is significantly too wide. This results in higher values (more variance) in the

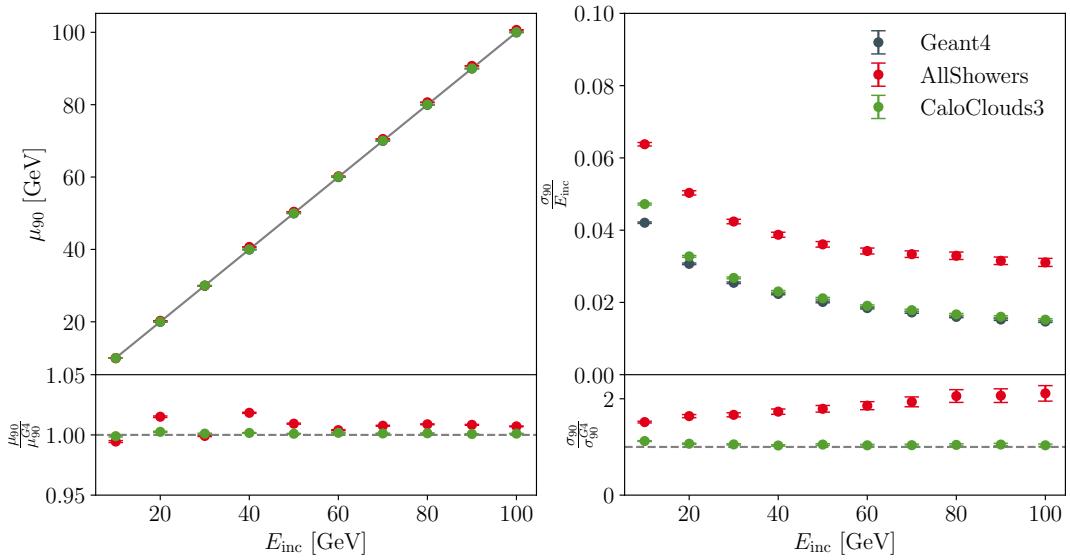


Figure 8: Linearity of rescaled energy sum for photon showers. Incident energies are chosen in steps of 10 GeV between 10 GeV and 100 GeV. Per energy and generator 10k samples are used. Incident angles are distributed over the intersection of the respective training regions.

398 resolution plot. By comparison, CaloClouds3 has a slight deviation in the lower energies,  
 399 but is otherwise well matched to Geant4. The strong performance here is produced by the  
 400 dedicated energy per layer predictions made by the normalising flow in CaloClouds3, which  
 401 models the variations in energy accurately.

402 To improve the linearity and energy resolution of AllShowers it would be possible to add an  
 403 energy per layer prediction to PointCountFM, in the same manner as is done in CaloClouds3.  
 404 As AllShowers includes hadronic showers, it is desirable to retain correct energies for points in  
 405 MIP tracks, and so a simple rescaling of the energy from PointCountFM would be detrimental.  
 406 It is possible to conceive of various schemes that could rescale the energy per layer, while  
 407 leaving the energy of MIPs intact, but we leave the exploration of these options to a future  
 408 work.

#### 409 4.4 Comparison to CaloHadronic

410 Another specialized model, which offers a comparison point for  $\pi^+$  showers, is  
 411 CaloHadronic [47]. CaloHadronic is trained only on  $\pi^+$  that enter the calorimeter at a per-  
 412 pendicular angle, so both models will be asked for perpendicular incident angles. The energy  
 413 range chosen is the full range that CaloHadronic was trained on: 10 to 90 GeV.

414 In figure 9 the standard three kinematic profiles are shown for both fast models and Geant4.  
 415 On the left, in the cell energy spectrum, both fast models make a reasonably good approxima-  
 416 tion of the two MIP peaks (one in the ECAL and one in the HCAL). CaloHadronic significantly  
 417 overestimates the high energy tail of the cell energy spectrum, while AllShowers manages to  
 418 maintain a closer fit to Geant4 for significantly more of the distribution.

419 In the centre, the longitudinal energy distribution for AllShowers is notably better aligned  
 420 with Geant4 than CaloHadronic. AllShowers can accurately capture the alternating layer pat-  
 421 tern, and also shows better replication of the initial layers of the HCAL. Conditioning on the  
 422 layer index and allowing to learn the layer properties in an embedding vector (see section 3.3)  
 423 likely helps here. Overall, the longitudinal distribution created by AllShowers is remarkably  
 424 well modelled.

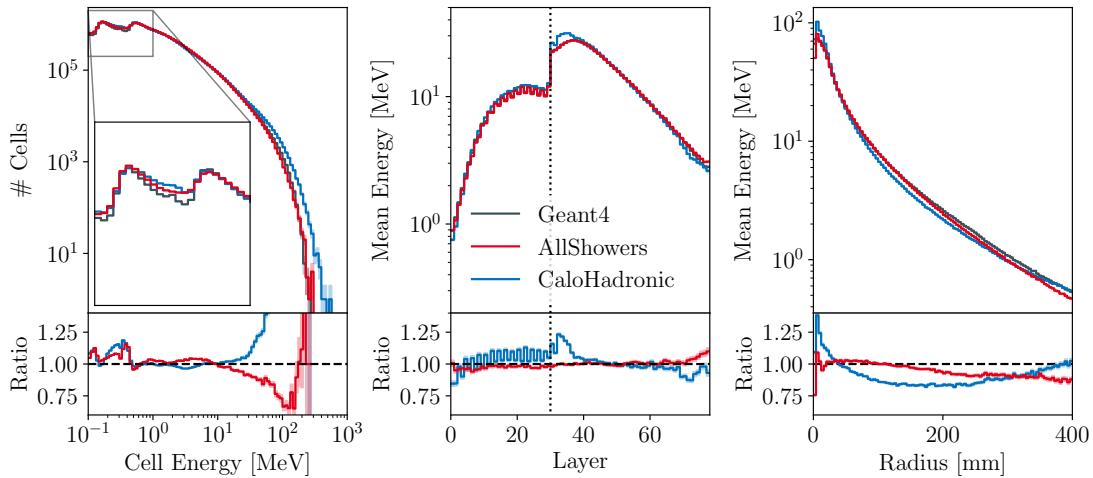


Figure 9: Comparison of AllShowers and CaloHadronic on  $\pi^+$  showers with incident energies uniformly distributed between 10 GeV and 90 GeV. All pions enter the calorimeter perpendicularly. From left to right: cell energy spectrum, longitudinal energy distribution and radial energy distribution. Per generator, 50k samples are used.

425 Finally, on the right, we compare the radial distribution for AllShowers and CaloHadronic.  
 426 In this distribution, CaloHadronic and AllShowers are more closely matched. CaloHadronic  
 427 slightly overestimates the centre of the shower, and underestimates a significant portion of  
 428 the bulk. AllShowers performs well in the centre, with only minor fluctuations in the first few  
 429 bins, then tends to underestimate the tail.

430 Reconstructed energy of pions offers important insight into the relationship between en-  
 431 ergy deposits in the ECAL and HCAL for individual showers. In figure 10, we show the linearity  
 432 and resolution of the reconstructed energy of  $\pi^+$  showers generated by the two fast models and  
 433 Geant4. The energy reconstruction is performed in the same way as for photons in section 4.3.

434 AllShowers offers a good reconstructed energy, marginally underestimating the energy of  
 435 low energy  $\pi^+$  showers, whereas CaloHadronic consistently overestimates the pion energy.  
 436 Looking at the resolution of the reconstructed energies, neither model is performing well. In  
 437 both cases, the distribution of the reconstructed energies is too wide at all incident energy  
 438 points. AllShowers's performance is better, being within 50% of Geant4 for all incident ener-  
 439 gies, but both models leave a lot to be desired in this metric.

440 Overall, AllShowers clearly provides better kinematic descriptions of  $\pi^+$  showers, with  
 441 both the performance of the CNF-transformer in the radial direction and the combination of  
 442 the PointCountFM and the CNF-transformer in the longitudinal direction demonstrating un-  
 443 precedented accuracy on  $\pi^+$  showers.

#### 444 4.5 Timing

445 When comparing the timing of AllShowers to other models, we provide both the time for  
 446 the execution of all 32 function evaluations used in the current version of the model, and a  
 447 speculative time needed for a model with only 1 function evaluation. A reasonable future  
 448 investigation for AllShowers would be to distill the model. A distilled model could require as  
 449 little as a single function evaluation to attain similar performance, but at this point we have  
 450 yet to achieve this optimization. So the timings for 32 function evaluations correspond to the  
 451 current performance, and the timings for 1 function evaluation are speculative, but provide a  
 452 good estimate of what timing performance might be attained by the next likely optimization.

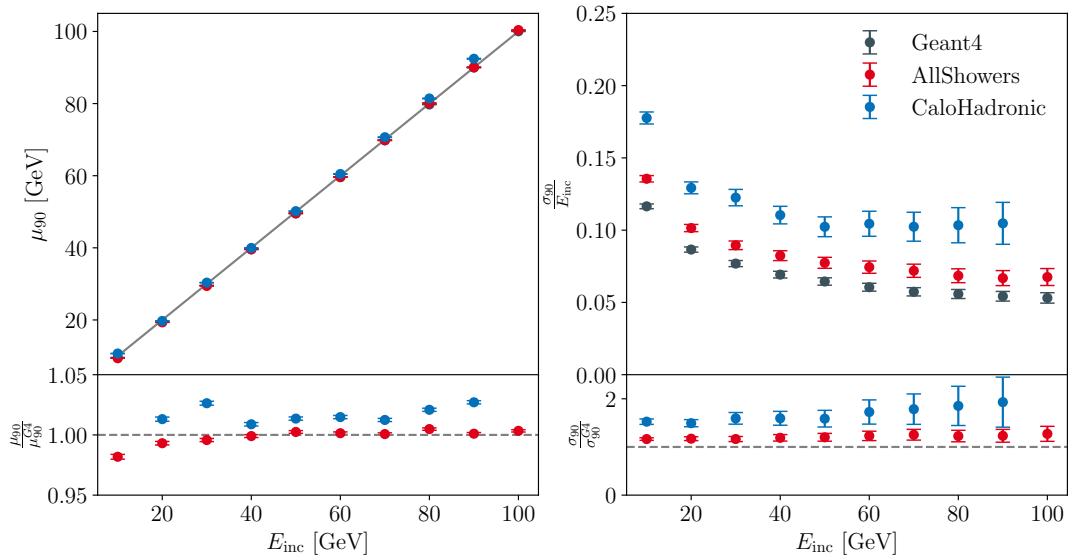


Figure 10: Linearity of rescaled energy sum for  $\pi^+$  showers. Incident energies are chosen in steps of 10 GeV between 10 GeV and 100 GeV. Per energy and generator 10k samples are used. All pions enter the calorimeter perpendicularly.

453 To get the timing for 1 function evaluation, we generate showers with a single euler step.

454 In table 1, we compare the timing of AllShowers to CaloClouds3 and Geant4 on photon  
 455 showers. Timings are measured for photons at 9 fixed incident energies; from 10 GeV to 90  
 456 GeV in steps of 10 GeV. The photons are all fired perpendicular to the calorimeter, and 100  
 457 batches are simulated per incident energy. Time of the first two batches is discarded, as the  
 458 warm-up step may have longer, and more erratic timing dependant on memory allocation and  
 459 “just in time” compilation. Times are only for the model’s point generation itself, they do not  
 460 include any overhead for moving data to or from the GPU or projecting hits into the detector  
 461 geometry. This slightly complicates the comparison to Geant4, which by design places hits in  
 462 sensitive cells only, and inherently incurs the overhead of the full detector geometry. To allow  
 463 for a fair comparison, we force our models to use a single computational thread on CPU, as  
 464 Geant4 does not support parallelism within a single event simulation.

465 All CPU timings are performed on a single core of an AMD EPYC 7402 processor with  
 466 512GB RAM. All GPU timings are performed on NVIDIA’s A100.

467 CaloClouds3 is a fully distilled model, so 1 function evaluation is all that is ever used.  
 468 It has also been aggressively optimized for the specific case of photons, including leveraging  
 469 photon specific behaviors, such as the lack of significant substructure in the showers. With  
 470 an iid assumption on the points, larger batch sizes become particularly efficient. AllShowers’s  
 471 current format prohibits specialized treatment of photons, and being the first generation of  
 472 this model design, it has not undergone such significant optimization as CaloClouds3, so it  
 473 is expected that AllShowers cannot compete in inference time with CaloClouds3. Indeed,  
 474 CaloClouds3 is at least two orders of magnitude faster on CPU. On the GPU the difference  
 475 is less dramatic, but overall it is seen that CaloClouds3 will remain significantly faster until  
 476 AllShowers is distilled or otherwise optimized.

477 In table 2, a similar timing comparison is shown for pion showers. This time CaloHadronic  
 478 is used as a comparison point, and for CaloHadronic, the NFE is also a tunable parameter.  
 479 In the table, timings are shown for both a hypothetical distilled version with one function  
 480 evaluation and for the number of function evaluations used in the current versions of the  
 481 models. As with the photon timings,  $\pi^+$  showers are all fired perpendicular to the calorimeter,

Hardware	Model	NFE	Batch Size	Time / Sample [s]	Speed-factor
CPU	Geant4	-	1	2.88	1.0x
			1	0.014	194.3x
	CaloClouds3	1	16	0.0041	654.5x
			1	0.17	16.7x
		32	16	0.16	17.6x
			1	5.0	0.6x
GPU	CaloClouds3	1	1	0.014	208.3x
			16	0.00088	3256. x
	AllShowers	1	1	0.014	209.3x
			16	0.0010	2806. x
		32	1	0.045	64.3x
			16	0.0050	581.6x

Table 1: Timing comparison between Geant4, CaloClouds3, and AllShowers on photon showers.

482 and 9 fixed energies are simulated between 10 and 100 GeV. Here the comparison is closer, as  
 483 both models are compelled to deal with substructure in hadronic showers.

484 As CaloHadronic was a pilot model, designed to demonstrate the potential to combine  
 485 ECAL and HCAL simulation, its code was never restructured to allow compilation. Thus, if  
 486 CaloHadronic were timed including preprocessing of input data and postprocessing of gen-  
 487 erated outputs, it would be unrealistically slow. Instead, only the evaluation of the PyTorch  
 488 model itself was timed, as this would dominate the timing in a more realistic deployment. Due  
 489 to the omission of all other elements from the timing, the times for CaloHadronic can be re-  
 490 garded as mildly optimistic. Despite this, AllShowers comes out as faster than CaloHadronic,  
 491 both at a single function evaluation, and with the NFE that is customary for the model. This  
 492 shows all round more efficient use of resources, including good GPU performance.

## 493 5 Conclusion

494 We have presented AllShowers, a novel generative model for high-granularity calorimeter  
 495 shower simulation. AllShowers is the a unified generative model capable of generating multi-  
 496 ple particle types, encompassing both electromagnetic and hadronic showers, within a single  
 497 architecture. This can help reduce the memory footprint, a significant bottleneck in large-scale  
 498 Monte Carlo production, by allowing loading a single model for all particle types. Moreover,  
 499 the model is conditioned on incident angle and energy, enabling broad applicability, and it can  
 500 simultaneously simulate energy depositions across both ECAL and HCAL, thereby enabling  
 501 end-to-end calorimeter response generation.

502 AllShowers shows strong agreement with Geant4 across a range of individual-shower fea-  
 503 tures, including aspects of the fine spatial structure accessible with highly granular calorime-  
 504 ters, as well as for ensemble-level distributions spanning multiple particle species. In com-  
 505 parisons at the shower level, its performance is competitive with specialized baselines —  
 506 CaloClouds3 for photons and CaloHadronic for pions — often yielding closer agreement on

Hardware	Model	NFE	Batch Size	Time / Sample [s]	Speed-factor	
CPU	Geant4	-	1	2.09	1.0x	
			1	0.59	3.5x	
		59	16	0.73	2.8x	
			1	34.8	< 0.1x	
		AllShowers	16	43.3	< 0.1x	
	CaloHadronic		1	0.12	16.7x	
			16	0.12	18.0x	
			32	3.5	0.6x	
			16	3.6	0.6x	
	GPU	CaloHadronic	1	0.0086	243.0x	
			16	0.0033	633.3x	
		59	1	0.40	5.3x	
			16	0.15	13.7x	
		AllShowers	1	0.013	157.3x	
			16	0.0010	1990.5x	
			32	0.044	47.7x	
			16	0.0047	447.7x	

Table 2: Timing comparison between Geant4, CaloHadronic, and AllShowers on pion showers. Geant4 and CaloHadronic times are taken from [47].

several observables. For photons, the absence of an iid assumption leads to slower generation than CaloClouds3, while potentially capturing additional correlations in the shower development. However, a definitive assessment of the trade-off between computational performance and physics fidelity for these models ultimately requires evaluating realistic physics observables after full detector reconstruction. For pions, AllShowers achieves comparable or improved agreement relative to CaloHadronic, while also providing faster sampling.

Looking forward, we aim to improve the energy resolution of AllShowers either by generating layer-wise energy deposits similar to CaloClouds3 or by applying a postprocessing step. Additionally, we plan to distill the model to reduce the number of function evaluations (NFE) required at sampling time and to extend AllShowers to additional detector geometries, further broadening its applicability to high-energy physics simulation workflows.

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 532 Federal Ministry of Research, Technology and Space (BMFTR) in the ErUM-Data action plan.

## 533 A Code and Data Availability

534 The code written for this work is available in the following Git repositories:

535       CNF-transformer: <https://github.com/FLC-QU-hep/AllShowers>  
 536       PointCountFM: <https://github.com/FLC-QU-hep/PointCountFM>  
 537       Ranger-Light optimizer: <https://github.com/FLC-QU-hep/ranger-lite>  
 538       Collection of shower IO utilities: <https://github.com/FLC-QU-hep/ShowerData>

539 The training datasets simulated for this work are available at:

540       AllShowers Dataset: <https://doi.org/10.5281/zenodo.18020348>

## 541 B Number of Trainable Parameters

Model	Layer-level model	Point-level model	Total
AllShowers	351,822	263,491	615,313
CaloClouds3	6,026,520	69,640	6,096,160
CaloHadronic	349,905	1,784,724	2,134,629

Table 3: Number of trainable parameters for AllShowers, CaloClouds3, and Calo-  
 Hadronic.

542 In table 3, we compare the number of trainable parameters for AllShowers, CaloClouds3,  
 543 and CaloHadronic. Shown are the number of parameters in the layer-level model (Point-  
 544 CountFM for AllShowers and CaloHadronic, and the normalizing flow for CaloClouds3), the  
 545 point-level model (CNF-transformer for AllShowers, and the diffusion models for CaloClouds3  
 546 and CaloHadronic), and the total number of parameters. It is evident that AllShowers has  
 547 a significantly smaller total number of parameters compared to both CaloClouds3 and Calo-  
 548 Hadronic.

## 549 C Hyper-Parameters

550 All hyper-parameters used to train PointCountFM can be found in table 4 and those used to  
 551 train the CNF-transformer in table 5.

Type	Parameter	Value
Data Preprocessing	incident particle type	one-hot encoding
	incident energy	Standard Scaling of $E_{\text{inc}}$
	incident angle	unit sphere representation
	point per layer	Standard Scaling of $\log(0.5 + N_i)$
Model	hidden layers	5
	hidden dims	128, 256, 512, 256, 128
	activation	ReLU
Training	optimizer	Adam
	learning rate scheduler	OneCycle
	maximum learning rate	$10^{-3}$
	batch size	1024
	epochs	1000
Sampling	ODE solver	Heun
	NFE	100

Table 4: Hyper-parameters used for the PointCountFM model.

Type	Parameter	Value
Data Preprocessing	point $x, y$	Standard Scaling
	point energy	Standard Scaling of $\log(E)$
	incident energy	Standard Scaling of $\log(E_{\text{inc}})$
	flow time	faure embedding with 3 frequencies
	incident angle	unit sphere representation
	OT mapping	layer-and-shower-wise
Model	embedding dim	64
	transformer encoder blocks	4
	attention heads	4
	feedforward dim	256
	attention masking	custom calorimeter-layer-based
Training	optimizer	Ranger (Lookahead + RAdam)
	learning rate scheduler	cosine annealing
	initial learning rate	$10^{-3}$
	weight decay	$10^{-2}$
	gradient clipping	0.2
	batch size	256
	epochs	200
Sampling	ODE solver	midpoint
	NFE	32

Table 5: Hyper-parameters used for the CNF-transformer model.

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