Referee report on "Reconstructing the gluon" by A. K. Cyrol, J. M. Pawlowski, A. Rothkopf and N. Wink

The paper "Reconstructing the gluon" discusses the reconstruction of the gluon spectral function in a special gauge from numerical (lattice) data for the gluon propagator. In order to do so, first the asymptotic behavior of the spectral function at low frequencies is derived. Then, based on the analytic properties of the propagator, a suitable functional basis is used which allows for the reconstruction procedure.

It is somewhat surprising that such a general relation between the behavior of the bosonic spectral function at low frequencies and the Euclidean correlator in the infra-red has not been recognized before. Nevertheless, the derivation of (6) is convincing and the given example is illustrative.

To apply (6) requires, of course, a sufficiently well established knowledge about the infra-red Euclidean propagator. For the latter different scenarios are discussed. In particular, sources for the $p^2\ln p^2$ behavior in the decoupling solution are highlighted and it is made clear that while the general form might be well motivated, details are less clear e.g. because of limitations in the Euclidean data. Since with currently available lattice data this is not possible, the authors advertise a combined lattice-functional approach with improved analytic methods. I very much appreciate the critical discussion of the subject. Eventually (only) existing numerical data for the gluon propagator in the scaling scenario are used to reconstruct the gluon spectral function.

For the reconstruction, an ansatz for the retarded propagator in the entire complex plane is made, an appropriate basis is selected such that the asymptotics and the general functional form of ρ is respected, and unknown parameters are determined within the analysis. The advantage of the proposed method becomes clear from Fig. 9 and is explained by a mock example in an Appendix which highlights that prior knowledge of the underlying analytic structures (including the criteria mentioned in Appendix C) is actually crucial for the reconstruction.

This is an excellent and exciting study. The reference list is extensive and gives a fair account of the existing literature. I look forward to the future studies mentioned as an outlook. I recommend publication in SciPost Physics but propose the following optional things to take into consideration:
1) It would be nice if the authors could leave a sentence or two about the physical consequences of the "positivity violation" in the spectral function.
2) The meaning of κ could already be explained together with (17) and (18).
3) Maybe this is sufficiently discussed already but how general is the structure of the ansatz in (36). Could one imagine more?
4) How are the error estimates in Fig. 6 (left) obtained? How are the systematics estimated? To what extent does this contain the fact that the actual analysis was done with a reduced set of allowed structures in (36)?
5) What remains somewhat unclear: Does it turn out that the logarithmic corrections (as summarized on p.7) are crucially important for the reconstruction? Does this have consequences for the "positivity violation"?
6) Minor things: Below (10) probably "low frequencies" is meant. Below (12) "negative" is missing. Before (17) a reference might be handy.