

**Referee report on the paper**  
**”Solution of Baxter equation for the  $q$ -Toda**  
**and Toda<sub>2</sub> chains by NLIE”**  
by **O. Babelon, K. K. Kozlowski and V. Pasquier**

This paper is devoted to the investigation of the solutions of the Baxter  $t-Q$  equations using auxiliary integral equations for the two types of the generalised Toda chains. This research is a further generalization of the approach developed by one of the authors together with J. Teschner in application to the usual Toda chain model. This allows to obtain the quantisation conditions for the spectra of these models in the form of thermodynamic Bethe ansatz equations. The paper is rather technical but is clearly written and present new results on the quantum integrable models.

Taking this into account, I would like to state that the paper ”Solution of Baxter equation for the  $q$ -Toda and Toda<sub>2</sub> chains by NLIE” by O. Babelon, K. K. Kozlowski and V. Pasquier can be published in the SciPost Physics Proceedings if some minor changes listed below will be taken into account.

1. Page 6. It is better to use roman ”i)” mentioning first item in ii) of the procedure described at the beginning of the Section 3.
2. Pages 7 and 8. It seems that the proper citations after formulas (3.7) and (3.11) should be to the formulas (2.6) and (2.7), not (2.12) and (2.13) where certain parameters of the Baxter equations for two models are introduced.
3. Page 10. Before (3.26) - misspelled ”thought”, last letter ’t’ is omitted.
4. Page 10. After (3.26). It seems that better to use notation  $(-)$  instead of  $(-)$  to signify the corresponding terms of the Baxter equation.
5. Starting from the page 15 there is a mess with notations of the functions defined by the equations (3.50) and (3.51). These equations defines the functions  $v_{\uparrow}$  and  $v_{\downarrow}$ , while starting from the beginning of the page 15 authors used notations  $\nu_{\uparrow}$  and  $\nu_{\downarrow}$  in many places. See, for example, pages 16, 20, 23, 26 and 37.
6. Page 16. The set of self-dual Baxter equations are introduced in the paper by the formulas (2.10) and (2.11) while in the text the citation is done only to the formula (2.11). The same at the beginning of the Appendices C.1 and C.2.

7. Page 26. Should be  $e^{-\frac{2\pi}{\omega_2}k\tau_1}$  in the first line, not  $e^{-\frac{2\pi}{\omega_2}\tau_1}$ ?
8. Page 28. Point '.' at end of the page is omitted.