

Referee report on the resubmitted paper "Survival probability in Generalized
Rosenzweig-Porter random matrix ensemble."

by G. De Tomasi, M. Amini, S. Bera , I. M. Khaymovich and V. E. Kravtsov

The paper contains a detailed analysis of decay of a prepared wavepacket whose dynamics is driven by a random matrix Hamiltonian of (generalized) Porter-Rosenzweig model. The model attracted considerable attention in recent years as, possibly, the simplest nontrivial toy model for a system with non-ergodic multifractal eigenstates. Depending on the control parameter γ eigenstates also can be fully extended (a.k.a. ergodic) or localized. This gives an opportunity to probe all regimes in a unified one-parameter framework.

The main ingredient of the analysis is the *Ansatz* Eq.(8) which postulates that in all three phases the eigenstates profile is Lorentzian, with the effective widths $\Gamma(N)$ reflecting absence or presence of ergodicity by its dependence on the system size N . The ensuing analysis of the wavepacket dynamics is relatively straightforward, though I would like to praise the authors for their accurate and careful description of the procedure and illuminating discussion of various dynamical regimes. The main practical message is that it is much more reliable to extract the multifractal exponents from the exponential decay of survival probability in the multifractal phase rather than from eigenfunction moments. This is an interesting observation, and may be of essential practical utility. Altogether, the paper is certainly a useful addition to the growing literature on Porter-Rosenzweig model, and I can recommend it for publication, after the authors take into account the following remark.

My main concern is that when discussing the main Ansatz Eq.(8) the authors

- (i) only very grudgingly mention the paper [21] by C. Monthus. As far as I can see, it would be more fair to explicitly state that the Ansatz was first proposed in that paper, which arrived to it by semi-heuristic Wigner-Weisskopf type arguments, and only then to give references to much more recent [28] and to yet unpublished [18].
- (ii) completely disregard the the paper by A. Ossipov & K. Truong (EPL 116 (2016) 37002) which contains results highly relevant to this formula.

In fact I would dare to say that in certain sense that work is presently the best analytical justification of the above Ansatz. Indeed, it was shown there (although

not in the most explicit form) that the Local Density of States for the same model is given precisely by Lorentzian form with the same width $\Gamma(N)$, and further the expression for $\Gamma(N)$ is derived in the multifractal phase explicitly, with all due prefactors.

Indeed, Eq.(7) in A. Ossipov & K. Truong for the moments of the eigenvectors contains the Lorentzian (and for $q = 1$ it gives essentially an expression for the LDOS). The width of the Lorentzian, denoted there as s , can be found from Eq.(11) of that paper in the case of the random diagonal part (relevant to the Rosenzweig-Porter model). In that case, it was shown that the parameter s becomes self-averaging and can be replaced by its mean $\langle s \rangle$.

When the variance of the diagonal matrix elements (denoted *sigma* in that paper), becomes large, then $\langle s \rangle$ can be calculated asymptotically. The corresponding result is given by their Eq.(17). For the Rosenzweig-Porter model, $\sigma^2 = N^{\gamma-1}$, which is large for $\gamma > 1$. Substituting $\sigma = N^{(\gamma-1)/2}$ into Eq.(17), one obtains for $E/\sigma \ll 1$

$$\langle s \rangle = \sqrt{\pi/2} N^{-(\gamma-1)/2}.$$

which precisely gives an asymptotic expression for the width of the Lorentzian (including a prefactor!). This expression was actually used further in that work to derive Eqs.(20)-(21) for the eigenfunction moments.

To summarize, the work by A. Ossipov & K. Truong contains valuable information corroborating with the main Ansatz and must be added to the list of references and included into the discussion. The paper by C. Monthus ought to be given full credit for introducing the Ansatz itself.