Disentangling or entangling matter from gauge field using local unitary transformations.

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In these notes I review the standard mapping between a perturbed LGT and a LGT including matter as originally described in Fradkin and Shenker [1], see also Fradkin and Susskind [2], Fradkin [3], Tupitsyn *et al.* [4]

I will work in a Z_2 gauge theories [5], but everything that follows can be generalized to at least Abelian gauge theories (and possibly also to non-Abelian) Kogut and Susskind [6], Kogut [7], Creutz [8, 9]. I work in 2D but again this can be generalized to higher D. We start with a $2D$ lattice with spin one half d.o.f. on every link l, sites are denoted with s.

The usual Kogut-Susskind Hamiltonian for the pure gauge theory is

$$H_{ks} = -J_m \sum_p B_p + h_x \sum_x \sigma_l^x, \tag{1}$$

where $B_p = \prod_{l \in p} \sigma_z^l$, p are the plaquettes of an oriented lattice and σ are the Pauli matrices. The gauge symmetry is generated by

$$A_s = \prod_{l/s \in \delta l} \sigma_l^x, \tag{2}$$

since $[H, A_s] = 0$, $\forall s$. We can thus decide to pin-point a specific gauge sector (a choice of the eigenstates for any A_s) by modifying the Hamiltonian to

$$H_g = H_{ks} + \sum_s J_s^e A_s. \tag{3}$$

The choice of $J_s^e = -1$, $\forall s$ leads to the standard Z_2LGT, while chosing e.g. $J_s^e = +1 \, \forall s$ leads to the "odd" Z_2 LGT Moessner and Sondhi [10].

We are free to add to the above system a system of spins living on the sites of the lattice s. In order to differentiate the Hilbert space of the sites from that of the links we will use τ for the Pauli matrices acting on the sites.

$$H_m = -\sum_s n_s \tau_s^x, \tag{4}$$

Here n_s can be freely chosen as the filling factor for the matter. For fully filled matter $n_s = 1$, $\forall s$, but other choices are possible. Here we will consider fully filled matter.

At this stage, gauge fields (on the links) and matter fields (on the sites) are completely disentangled. We will later entangle them. However before doing this, we perturb the Hamiltonian H_g by explicitly breaking the Z_2 invariance

$$H_{gb} = H_g + \sum_l \lambda_l \sigma_l^z. \tag{5}$$

Even though the symmetry is now explicitly broken due to the stability of the phase the system is still in the same phase for sufficiently small $\lambda_l \forall l$ Hastings and Wen [11]. If we consider the two Hamiltonians together

$$H = H_m + H_{gb} \tag{6}$$

we can immediately see that H_m is diagonal and define new sectors, for H given by the matter content.

We can map via a local unitary transformation the above Hamiltonian to the standard Abelian-Higgs Hamiltonian Fradkin and Shenker [1].

The building blocks are the symmetric entangler/disentangler (called also U operators in the standard LGT literature Creutz [8], Tagliacozzo *et al.* [12]). In the Z_2case they are just the C-not gates acting on two qubits and defined as

$$U_{ij} = |0\rangle\langle 0|_i \mathbb{I}_j + |1\rangle\langle 1|_i \sigma_j^x, \tag{7}$$

With the important property that

$$U_{ij}(\sigma_i^z \otimes \sigma_j^z)U_{ij}^\dagger = \mathbb{I}_i \otimes \sigma_j^z \tag{8}$$

$$U_{ij}(\sigma_i^x \otimes \sigma_j^x)U_{ij}^\dagger = \sigma_i^x \otimes \mathbb{I}_j \tag{9}$$

It is an easy exercise to check that if we define as

$$U = \prod_s E_s \tag{10}$$

with

$$E_s = \prod_{l/s\in\delta l} U_{s,l} \tag{11}$$

$$H' = UHU^\dagger = -J_m \sum_p B_p + \sum_s J_s^e A_s + h_x \sum_x \sigma_l^x + \sum_l \lambda_l \tau^z_{s_l^-} \sigma_l^z \tau^z_{s_l^+} - \sum_s n_s \tau_s^x A_s, \tag{12}$$

where we have introduced $\tau^z_{s_l^-}, \tau^z_{s_l^+}$ as the operators acting on a site s on the left $(-)$or on the right $(+)$ of the link l.

Now we need to notice that the last piece still commutes with everything else. Fixing e.g. $n_s = 1$everywhere we thus obtain

$$\tau_s^x A_s = \mathbb{I}_S, \tag{13}$$

that we can interpret, as a constraint. The last piece thus acts as a new Gauss law. By restricting H' to act only in the subspace of states fulfilling the new Gauss-law we get, that inside the physical Hilbert space, we obtain

$$H_p' = -J_m \sum_p B_p + \sum_s J_s^e \tau_s^x + h_x \sum_x \sigma_l^x + \sum_l \lambda_l \tau^z_{s_l^-} \sigma_l^z \tau^z_{s_l^+}, \tag{14}$$

exactly an Abelian-Higgs gauge theory.

We can thus locally entangled matter and gauge d.o.f. using the product of local unitary transformations.

The opposite is also obviously true, we can disentangle matter from gauge field in any dimension by just using a local unitary transformation.

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