Report: *Complexity and entanglement for thermofield double states*

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The authors of the paper "Complexity and entanglement for thermofield double states" study the circuit complexity for the thermofield double (TFD) states in the free scalar quantum field theories using the Nielsen's proposal. The main idea is to provide some insights into the CA and CV proposals. Although the results are not sufficient to either confirm or rule out either the CV or CA proposals, the authors believe that the results may suggest some degree of universality. The time-independent part of the article is an extension of the reference [23] using the language of covariance matrices which is easier and more elegant. Here, the authors conclude that the complexity of the formation of the TFD is proportional to the thermodynamic entropy in accordance with the conclusions of the references [52,53]. I am not sure the time-independent section offers many new physical insights into the conclusions of [23,52].

The complexity of the time-dependent thermofield double states has been also studied before in [53]. However, the authors not only extend the results of [52,53] (and also correct some mistakes) they make many detailed calculations for variety of cases and conclude that the complexity of formation of the TFD state does not exhibit the late-time growth characteristic of holographic theories/fast scramblers due to the Gaussian nature of the TFD state for the free scalar theories.

The paper is well-written and although some presented results are not original, it has enough merit to be published in Scipost. In particular, since the paper has a pedagogical style of writing it can be a useful reference for the researchers that just want to start working on this field. However, before recommending the paper for publication, I have a few questions and comments:

1. In page 34 why for small t the growth of $\mathcal{C}_1^{LR}$ is initially linear, while that for $\mathcal{C}_2$ is quadratic?

2. I have similar question for the Entanglement evolution: In the context of harmonic oscillators(HO) it was argued in (Phys. Rev. A 90,062330 (2014)) and later confirmed numerically in (Phys. Rev. B 90, 205438 (2014)) that there is a quadratic initial growth in the EE. For similar conclusions in the context of the holography and CFT see (Phys. Rev. Lett.112, 011601 (2014)) and (J. Cardy, KITP Conference: Closing the

entanglement gap: Quantum information, quantum matter, and quantum fields (2015)). A comment is needed addressing the reason for the absence of this regime in the paper.

3. In the section (6.5) the authors attempt to connect the logarithmic contributions to the evolution of the entanglement to the zero mode. To the best of my reading, they do not isolate the contribution of the zero mode. Of course, the zero mode is there and they find a log behaviour but it is absolutely not clear from the calculation that they are related. I think the authors either need to isolate the zero mode contribution and successfully prove their point or change the claim.

4. At page 63 the authors say that the quasiparticle picture was introduced in 92-97. The LR bound gives an upper bound to the entanglement growth and more than that I think the LR velocity has nothing to do with the quasi-particle velocity. I recommend the authors to be more careful here in the citation.

There are also few typos and minor issues in the paper:

- In the page 3 define t_R and t_L.
- References [74] and [75] are repeated.
- In Figure 4, the vertical label seems to me strange.
- In the equations (196) and (197) I am not sure in the indices the $n = 0$ is defined!