Referee Report

Title: *Replica Bethe Ansatz solution to the Kardar-Parisi-Zhang equation on the half-line*,
by Alexandre Krajenbrink, Pierre Le Doussal

The authors study the KPZ equation on the positive half-line with a Neumann boundary condition at $x = 0$, $\partial_x h(0,t) = A$, and droplet initial condition. Inspired by a very recent theorem of Parekh, the authors solve the equivalent problem of KPZ on the half-line with Dirichlet boundary condition at $x = 0$ but in which the initial condition is a Brownian motion with drift $A+1/2$. A full solution is obtained for positive drift. The central result is an exact formula for the Laplace transform of the distribution of the exponential of the height at 0, valid at all times. This result is expressed as a (Fredholm) Pfaffian of a matrix kernel. The authors also show (in section 5) that their formula can also be written as the square-root of a (Fredholm) Determinant of a scalar kernel. The large time asymptotics is also investigated : the generic case leads to the GSE TW distribution; the limiting case $A+1/2 = 0$ gives the GOE. The crossover is also studied in details, leading to a new expression for the cross-over kernel K^ϵ which is conjectured to be equivalent to another kernel K_{cross} that has appeared in various recent works.

The strategy used by the authors is based on the replica Bethe Ansatz, a technique one that one the authors – with different collaborators – successfully applied a few years ago to solve the KPZ equation (simultaneously with other groups). The main difficulty being to determine the overlap of the Bethe states with the initial condition. An exact formula is obtained in Appendix A, based on a generalization of a "magic formula" derived by Imamura and Sasamoto.

This paper presents leading edge results and is extremely well written. The logical line followed by the authors is very clearly stated. The authors are very explicit and very honest about their assumptions and the (present) limitations of their results. All important details are given and, with some patience, it is possible to understand and to reproduce the calculations of the manuscript. I have no specific remarks apart from:

(i) a possible typo in eq. (23): I do not understand why A depends on the ϵ_i's.

(ii) It took me time to understand that in eq. (28) what is written is a product of two fractions (and not three terms in the numerator and three terms in the denominator). I had to go to ref [24]. May be you could add some space between the two fractions.

(iii) In the exponential in eq. (29): $m_j \to m_p$.

(iv) May be it could be useful to add a few words to explain the continuum limit $\sum_{k_j} \to m_j \int_{\mathbb{R}} \frac{dk}{2\pi}$.

(v) A crucial step is given in eqs (39) and (40). However, no reference and no proof of the Schur Pfaffian are given. I checked it with Mathematica for small n_s but did not find a proof. It could be nice to have some clue here.

(vi) The authors use the Mellin-Barnes summation formula before eq. (45). It would be useful to explain where and how they overcome the barrier $A > (n-1)/2$.

I wholeheartedly recommend this manuscript for publication. This is an impressive work, beautifully presented.