Report on *Extreme boundary conditions and random tilings*

1 Overview

Random tiling (or dimer) model have a long history in Statistical Physics and probability theory, and to this day the object of intense interest. Thanks in particular to specific solvability properties (representation as free fermions or, in another language, determinantal point process), one can access a number of features (limit shapes, fluctuations in the bulk and near the boundary, ...) believed to hold for more general height models, such as the six-vertex model.

In these lecture notes, the author surveys results, heuristics and conjectures for tilings and related models, and explains how to derive some of them, mostly from the perspective of free fermions. More discussion of other approaches or formulations would certainly be useful; for example, much is known on dimers with Yang-Baxter weights, lattice realizations of fermions, use of algebraic combinatorics, non-intersecting path representations, etc. There is a fair amount of typos, some listed below, and the manuscript would also benefit from proofreading for language. Otherwise, this is a generally clear, interesting, enjoyable read that exemplifies fundamental phenomena of current interest in Statistical Physics; in my opinion this is suitable for publication in SPLN after a revision.

2 Comments

- Page 7 Line 4: maybe emphasize which algorithms work only for specific boundary conditions.
- Eq. (11): the integration domain should exclude the diagonal
- below Eq. (17): "a solution": here and elsewhere discuss uniqueness of solutions of variational problems
- above Eq. (21): how does one rule out that the frozen region has e.g. 3 intervals ?
- P1L11: coulomb: Coulomb
- P13L8: "integers": discuss somewhere the different conventions on heights (RSOS or otherwise)

- above Figure 6: the color convention was discussed earlier.
- P14L12: not immediately clear what the figure means
- above Eq (26): maybe mention that there is no loss of generality in choosing $a, 1/a, b, 1/b$ as weights (gauge equivalence).
- Figure 7 caption and elsewhere: "hop around": wrap around
- Eq. (28): say something about the existence of the limit
- three lines above (32): "figure."
- P17L2: "zero mode": not sure this is an accurate terminology here (with boundary conditions)
- below (38) and throughout: "euclidean": Euclidean (similarly, Gaussian, Hermitian, etc.)
- footnote 4: GMC is not synonymous with GFF; $c = 1$ CFT is also dubious (especially in the dimer context, which is also closely related to $c = -2$ CFTs).
- paragraph below (39), last sentence: "essentially the same": what, if any, distinction is there ?
- P19L1: "BKT": expand acronyms on first occurrence.
- P18 first bullet point: the discussion here is unclear and perhaps dubious.
- Exercise 2.2.5: "odd case": what is the odd case ? The heuristics in this exercise are not terribly clear, in particular the position of K in the last displayed equation
- Figure 10 caption: "occupation": occupancy
- below Figure 10: "thiner": spelling
- discussion around (58) is unclear
- below (65): explain what antiperiodic conditions are
- below (67): "integrant": sp.
- below (75) and elsewhere: "explicitely": sp.
- below (84): independent of r and s: comment on whether this is a general or model-specific fact.
- below (90): "in section 4". this is section 4 ??
- below (92): "finding from": something missing

- two lines below (103): is there something missing in the LHS ?
- six lines below (106): “point”: vertex ?
- P40L-7: resembles: the resemblance is rather superficial (e.g. different tails etc.)
- P43L4: “In turns out”
- P50L10: “nontriaval”
- below (150): explain the notation $(-1)^{(matrix)}$
- A.2 line 1: “droping”: sp. ; “good physicists”: this can also be couched in terms of distinguishing between an algebra and a representation of said algebra.
- below (159): define $\{,\}$
- References: some capitalization issues (ising, harnack, burgers etc), some missing journal information.