Report on 2004.12413v1 - "Symmetry restoration and the gluon mass in the Landau gauge"

The authors investigate whether a gluon mass is generated when Gribov ambiguities are taken into account. It must be said that some of the authors of the present manuscript worked on some foundational aspects of this work. However, they motivate the present work as a tentative to shed light on some formal manipulations used in previous works. The paper is clearly written. However, I have some questions that I judge relevant for the construction discussed in the paper and a general comment. My questions are:

- It is not clear to me how eq.(4) is defined for configurations that live on the so-called Gribov horizon. Such configurations represent zero modes of the operator \mathcal{F} and, therefore, the denominator seems to be ill-defined. The authors could provide a comment on that.
- The parameter ζ introduces a mass term for the Faddeev-Popov ghosts and the authors claim that it should not be taken as a problem. However, in the standard formulation of the Landau gauge, mass terms for the ghosts are forbidden by the Ward identities. In the standard (Refined) Gribov-Zwanziger construction this remains true. Can the authors comment about that and explain how to reconcile such things? Or am I missing something?
- The gluon-mass parameter is associated with the parameter β which is akin to a gauge parameter. This is very confusing to me. Are the authors claiming that the mass parameter that is generated by averaging over Gribov copies is gauge dependent and therefore can be taken to any value? Does it mean that β cannot enter correlation functions of gauge-invariant correlators?
- Can the authors connect the mass parameter that they obtain to the so-called Gribov parameter which is generated in the standard elimination of infinitesimal Gribov copies in the Landau gauge by the restriction of the path integral to the Gribov region?
- Below eq.(4), the authors say that \mathcal{F} represents the FP operator in the Landau gauge and write $\mathcal{F}[A, U] = \mathcal{F}[A^U]$. Can they explain what do they mean by this equality?
- It seems that this averaging method does not (strongly) rely on the choice of the Landau gauge and also on the "type" of copies, i.e., if they are infinitesimal or "large". This would be a strong advantage with respect to the

restriction to the Gribov region, which has a strong dependence of those aspects. Can the authors make comments about the extension to other gauges and if this is compatible with BRST invariance?

My general comment is that the authors focus on the averaging procedure to eliminate Gribov copies, but mostly do not make any reference to what has been done using the "restriction to the Gribov region" method. It would be beneficial for their work to connect their results with the other perspective in some way.