Referee report on the paper "Dynamical Instantons and Activated Processes in Mean-Field Glass Models"

by V. Ros, G. Biroli and C. Cammarota

This is an excellent and deep paper which sheds the new and constructive light on a long-standing problem in theory of disordered systems: description of an activated escape from local minimum via barrier-crossing in mean-field models with one-step replica symmetry breaking mechanism. The corresponding dynamics probes time-scales exponentially large in the size N of the system. The correct order parameter allowing to describe glassy dynamics on such scales is a two-time correlation function, which is a quite difficult and less explored object to study. Important role in building the theory is played by a clever combination of Kac-Rice framework of counting stationary points with a fixed overlap with a given minimum with dynamical equations describing the evolution of the system. The authors derive the dynamical mean-field equations corresponding to Langevin dynamics, with initial condition chosen to be precisely at one (out of many) index-one saddles in the vicinity of local minimum. They then find a convincing way of analysing them both numerically and (to an extent) analytically. As a result, they provide quite a detailed picture explaining how activated crossing happens through saddles at different overlap and energy density. Along the way the authors show how to obtain the dynamical instanton corresponding to the involved activated process, and the associated shape of the two-point correlation function (which turns out to be quite involved). Technically the analysis is quite involved, and performing it is a considerable achievement. One of conclusions of the paper is that escape through most numerous saddles with big overlap leads typically to minima at higher energy. this indirectly implies that descrease in energy occurs through less numerous sadles which have smaller overlap, and their studies is left for future work.

In summary, this is an important work essentially contributing to our understanding of thermal relaxation in mean-field glassy models, and I strongly recommend its publication.