

Referee's report on
“Bosonic and fermionic Gaussian states from Kähler structures”
by L.Hackl and E.Bianchi.

The paper is devoted to a unified presentation of bosonic and fermionic Gaussian states. As the authors rightly point out, this is an important topic with many applications. It is also quite complicated, with a relatively big number of various mathematical structures involved. One can approach it from several angles, using distinct, even if equivalent, concepts, different terminology, etc. It appears that many authors rediscover the same concepts and identities again and again. The authors decided to write a comprehensive and well-organized review of this subject.

In general, I find the article well structured and readable. The terminology and notation used by the authors is often not my favorite, but I understand that there are various communities with different traditions.

The authors claim that their approach has some elements of novelty—treating the Kähler structure as the starting point. I agree with the authors that this is a very good organizing principle. However, it is contained in the literature almost in the same (or equivalent) form. In particular, I would like to draw the author's attention to “Mathematics of Quantization and Quantum Fields” by J.Derezinski and C.Gerard, which will denote [DerGer]. Essentially all elements of the refereed manuscript can be found there. Let me make a more detailed comparison with that monograph.

1. The triangle of Kähler structures is described in Def. 1.85 and 1.88 of [DerGer].
2. (26) can be compared to Def. 1.83 of [DerGer].
3. Δ of (38) coincides with k of Thm 11.20 and 16.29 of [DerGer].
4. The Cartan decomposition is known as the polar decomposition in a space with a scalar product, and is discussed in [DerGer]. In the context of a Kähler space, in [DerGer] it is called the j -polar decomposition and is discussed in Thm 16.13.
5. One can compare the discussion on how to find the Gaussian (or Bogoliubov) transformation from one Gaussian state to another with implementers of positive symplectic transformation Subsect. 11.3.3 and j -positive orthogonal transformations Subsect. 16.3.7 of [DerGer].

6. “Mixed Gaussian states” also known as “quasi-free states”, are discussed (in a very similar way) in Sects 17.1 and 17.2 of [DerGer]
7. Compare the formula for the ground state energy of a quadratic Hamiltonian with Thm 10.17, 11.27 and 14.13, 16.33 of [DerGer]

Here are more remarks

1. In footnote 1 on page 2 there should be $\langle \dots \rangle = 0$.
2. What is the meaning of $\{T, J\} = 0$ at the beginning of Subsect.5 Cartan decomposition, p.13? Is it the anticommutator?
3. What is a “homogeneous Gaussian state”? Is it a “centered Gaussian vector”?
4. “Mixed complex structure” (p. 17) is not a good name.
5. Instead of “Two Fock spaces are unitarily equivalent” I would write “Two Fock representations are unitarily equivalent” (p. 31).
6. Personally, in (250) I would write

$$u^* \hat{a} + u \hat{a}^\dagger$$

and not $u \hat{a} + u^* \hat{a}^\dagger$.

To sum up. I encourage the authors to have a look at some other treatments of the subject, e.g. at [DerGer]. Nevertheless, I agree that the manuscript, possibly after a revision, can be useful to many readers.