Report

This paper describes a rather general analysis of the dynamics and stabilization of moduli in meta-stable de Sitter (dS) vacua and/or slow-rolling time-dependent solution generating accelerated 4D expansion, which are obtained in M-theory (in the limit of 11D supergravity coupled to M2-branes) compactified on compact hyperbolic (negatively curved) 7-manifolds $\mathbb{H}_7$ with magnetic 7-form flux containing small closed geodesics ('circles') supporting localized negative Casimir energy densities. The resulting 4D effective scalar potential exhibits a structure of 3 compactification volume dependent terms, each of them successively less fast falling in inverse volume with the middle terms of negative sign. This structure is known to be able to contain meta-stable dS vacua if the 3 coefficients satisfy a certain 'window' of conditions, very similar to the dS vacua in type IIB string theory compactifications on a product of 3 Riemann surfaces in arXiv:hep-th/0411271. The crucial advances over the 2004 results consist in:

- Generalizing the specific choice of a product of 3 Riemann surfaces to a very general class of asymmetric hyperbolic manifolds obtained by gluing together local hyperbolic 'cusp' regions (which together make up a significant fraction of the $\mathbb{H}_7$-volume) and taking covers of the resulting compact manifold.

- Replacement of the type IIB specific O7-planes negative tension source by the negative quantum vacuum Casimir energy density arising near small closed geodesics (circles); the ability to obtain these circles in explicit hyperbolic cusp examples with explicitly known fiducial metrics which are rendered smooth by generalized Dehn fillings resolving the cusp singularities into 6-tori with a smoothly shrinking 1-cycle; establishment of the genericity of these small Casimir energy carrying circles via leveraging the wide (and beautiful) structure of systolic geometry.

- Establishing the fact first discussed in arXiv:0911.3378, that nontrivial profiles of the warp factor in the warped hyperbolic compactification can both stabilize fluctuations of the conformal factor and keep the small systoles (circles) at the end of the hyperbolic cusps stabilized at small but finite size (effectively shielding the unwarped run-away instability of the Casimir effect towards vanishing circle size). This in turn allows for using the Casimir energy density as a tunable controlled negative energy density, in turn allowing for controlled tunable dS vacua, and on top allows the authors to provide a fully backreacted solution of a non-compact Dehn filled hyperbolic cusp region.

- Finally, using the rewriting of the e.o.m. into an off-shell effective potential subject to a constraint resembling a stationary Schroedinger equation (which in turn allows for the treatment of the nontrivial warp factor stabilizing the Casimir circles) to show that metric and p-form fluctuations, which are not already removed by hyperbolic spaces having far fewer moduli to start out with, acquire a positive Hessian (mass matrix) and small tadpoles (linear scalar potential perturbations driving them to non-zero fluctuation vev), thus keeping the full compactification under control.

Altogether these results allow for a derivation of a reverted Maldacena-Nunez theorem: that there are no local AdS minima, but only dS minima, possible once the negative Casimir energy density is present. The authors even identify a Chern-Simons-coupling extended sector of p-form field strength supporting the mechanism of axion monodromy inflation. Finally, the provide arguments that the system of coupled PDEs describing the meta-stable dS vacuum of a given asymmetric and inhomogeneously perturbed $\mathbb{H}_7$ M-theory compactification pose a well formed boundary condition problem for acquiring fast numerical solutions from PINNs (physics informed neural networks) capable of directly solving the PDE system by using squares of the PDEs and their boundary conditions themselves as a loss function. A boon of this numerical approach to find full backreacted global $\mathbb{H}_7$-solutions is that PINN results with nearly vanishing loss function gradients constitute effectively solutions with 4D accelerated expansion as the loss function gradients are proportional to the slow-roll expressions.

These results (including their significant progress in handling general hyperbolic spaces using their cusp-gluing and cover-taking constructions, and use of systolic geometry to treat the circles in the Dehn-filled cusp regions) constitute a major step forward in establishing the existence of very large classes of hyperbolic compactifications as producing viable meta-stable vacua. The paper clearly deserves publication, and it is important.

Despite the very clear and concise discussion including the introductory parts for hyperbolic and systolic geometry, I would like to ask the authors a few questions for the sake of clarity:

- Concerning the $B_\star$ dependence of sign of full Hessian (see Fig. 7: "net estimate"): How is $B_\star$ controlled? Is it possible to discuss/estimate how much tuning is needed to get the positive-Hessian region $B < B_\star$ to cover a large part of the whole range of possible $B_\star$-values?

- Getting a competitive (in the sense of the negative middle term of the 3-term structure) negative Casimir contribution requires the systolic circle in the Dehn-filled cusp to be stabilized at a small value. However, at such small circle size certain non-vanishing curvature invariants of the hyperbolic space may blow up as function of circle size $\to 0$. Is it possible to estimate these curvature invariants? Are we sure, that they stay small enough for the stabilized circle sizes needed to get competitive Casimir contribution? Some discussion here would be likely appreciated by the reader.

- Concerning the $|F_7 + C_3 \wedge F_4|^2$ flux axion monodromy source: The aim is to have F_7 large and F_4 small, to create a suppressed linear potential from the cross term in the square lasting for a sufficient number Planck units field displacement away from $c = \int_{\Sigma_3} C_3 = 0$. However, for such a choice of F_7 and F_4 the minimum of c is not at $c \simeq 0$ but at $c = -F_7/F_4 \ll -1$ so the actual vev at the minimum of the axion monodromy potential is very large and negative at negative potential energy contribution. Around $c = 0$ there is no quadratic minimum, and thus while slow-roll still breaks at $c > 0$ about a Planckian distance away from 0, there is no good post-slow-roll-breaking oscillatory phase at positive vacuum energy. How is this to be cured (e.g. by providing a hybrid/waterfall exit by coupling the axion to another sector/axion rolling off quickly to a zero-energy potential when the inflaton axion reaches slow-roll breaking distance from zero)? Again, it it might be useful to provide some comments here for the interested reader.

- For readers who care about getting chiral gauge theories and chiral matter besides getting dS vacua: Are there notions of well-defined 3-cycles in these $\mathbb{H}_7$-manifolds, on which to place intersecting M2-brane stacks to generate particle physics sectors (similar to how this is doable with 7-brane sectors wrapping the 1-cycles in products of Riemann surfaces in type IIB)? Or how else can such sectors arise on the $\mathbb{H}_7$-manifolds used here?

A few text typos found while traveling systolically:
(boldface: corrections)

- p 25: paragraph after eq. (5.17), 3rd line: "[...] We would like impose that this correspond to a good approximation [...]." → "[...] We would like impose that this correspond**s** to a good approximation [...]."
- p 46: 1st paragraph, 2nd line: "[...] sigh [...]" → "[...] **sign** [...]"
- p 46: 1st paragraph, 3rd line: "[...] we displayed expicit [...]" → "[...] we displayed exp**l**icit [...]"
- p 46: 2nd paragraph, 1st line: "It is convenient to work with a configuration with most of the fields h in (1.1) are not being turned on, [...]." → "It is convenient to work with a configuration **where** most of the fields h in (1.1) are not being turned on, [...]."