Referee report 'Quantum Bit threads' (2105.08072)

A sharp connection between information and geometry is through the Ryu-Takayanagi (RT) formula for holographic entanglement entropy, and its generalizations, in particular its extension to capture bulk quantum corrections. Headrick and Freedman's flow-based 'bit thread' reformulation of the RT formula deepens the connection between information, networks and geometry, whilst addressing important conceptual issues. In this article, the author takes the next step and offers a reformulation of the quantum generalization of the RT formula using 'quantum' bit threads. As such, this article (along with the jointly posted 2105.08063 [hep-th]) fills an important gap in the literature on holographic entanglement entropy and the program initialized by Headrick and Freedman. Furthermore, this article makes use of 'double holography' to bolster the prescription of quantum bit threads and make contact (though limited) to 'islands', which have had success at providing a providing a possible and calculable explanation of the unitary Page curve for black hole radiation.

Therefore, I am pleased to recommend this article be accepted for publication – once my minor comments/questions are addressed.

(1) While only intended to be motivation, it would be worthwhile for the author to more clearly explain double holography. This could be done describing where the end of the world brane is in Fig. 1, for example. Also, explain how the defect CFT_d fits into the picture, i.e., what role does it play.

I also have two questions about the doubly holographic set-up:

(i) Ignoring the defect CFT, such that we are only considering braneworld holography, one starts with an asymptotically AdS bulk spacetime which is assumed to have dual holographic CFT living at its boundary. One then introduces a brane inside of the bulk, which, roughly, can be thought of replacing a UV cutoff surface upon integrating out UV degrees of freedom of the dual CFT in a holographic renormalization scheme. Consequently, the brane has an induced gravity theory which to leading order is general relativity and subleading order has higher curvature corrections (and the CFT with a UV cutoff lives on the brane). *Do these higher curvature corrections play a role in determining the bound on the flux, from the intermediate brane perspective? That is, can we think of these corrections being the reason for the modified flux bound, owed to a change in the change in norm bound due to the presence of higher derivative terms? Or, since one only works to leading order, the higher curvature corrections don't play a role to this leading order?*

(ii) In braneworld holography, the brane can have flat, AdS or dS geometry, and even have (quantum) black holes living on them (see, for example, the recent articles 2007.15999 [hep-th] and 2207.03302 [hep-th]). *How is the argument modified when the brane as a flat or dS brane geometry? Or what about when there is a black hole on the brane?* It might be worth addressing these questions in the conclusion, as possible future directions.

(2) Since this article was first published, multiple important articles in the bit thread literature have been posted, e.g., gatelines/Lorentzian threads to describe complexity (2106.12585 [hep-th] and

2105.12735 [hep-th]) and covariant bit threads (2208.10507 [hep-th]). It might be worth mentioning these works, and how they relate to quantum bit thread prescription proposed here. For example, can a Lorentzian analog of quanutm bit threads (namely, the modified divergenceless condition) be used to propose a modification of the complexity=volume conjecture?

(3) Lastly, can the author comment on whether the quantum bit thread proposal provides an easier route to derive monogamy of mutual information? Does one need to assume the bulk entanglement entropy obeys MMI?

Once points (1) – (3) have been suitably addressed (no need for calculations or extensive writing), I would be happy to recommend this article for publication.