Supersymmetric Ground States of 3d $\mathcal{N} = 4$ Gauge Theories on a Riemann Surface : Referee Report

Synopsis

This paper computes the supersymmetric ground states for a class of A-twisted and B-twisted 3d $\mathcal{N} = 4$ theories on $\Sigma \times \mathbb{R}$, where Σ is a Riemann surface of genus g. The technical tools for the computation have been developed by the authors in a series of earlier papers. The current paper uses these tools to explicitly work out the supersymmetric ground states for a large class of Lagrangian field theories (under certain assumptions) and checks the consistency of the computation under 3d mirror symmetry.

The basic idea behind the computation is to first write the 3d theory as an effective gauged supersymmetric quantum mechanics (SQM) with an infinite-dimensional gauge group and appropriate matter content. The SQM can be viewed as a sigma model with a target space which is generically infinite-dimensional. In the next step, one uses supersymmetric localization to arrive at a SQM with a smaller target space. If the target space is smooth and compact, one can use the standard technique of realizing the ground states as harmonic (p, q) forms on the target space. If the target space is not compact, one turns on real masses in the Cartan subalgebra of Higgs branch global symmetry. The fixed locus of the resultant $\mathbb{C}^*_m$ (or $U(1)_m$) action gives the final target space, which is compact.

For the A-twisted case, the effective SQM is a (2,2) SQM with a target space M, where M is the moduli space of twisted quasimaps from Σ to the Higgs branch X. Then, localization and turning on of mass parameters reduces the target space to the locus of quasimaps from Σ to a set of isolated points on the Higgs branch. This locus has a simple realization in terms of symmetric products of the curve Σ and their tensor products. The ground states can then be computed from the de Rham cohomology of these symmetric products.

For the B-twisted case, the effective SQM is a (0,4) SQM with a target space X (i.e. the Higgs branch) and a hyperholomorphic line bundle $\mathcal{E}$, for a generic choice of the FI parameters. Localization and turning on of real masses, in this case, leads to a fixed locus which is a set of isolated points. In the limit where these real masses are infinitely large, the supersymmetric ground states are given by the perturbative ground states attached to each point on the Higgs branch, up to instanton corrections. The authors argue that in the specific case of these (0,4) SQMs, there exists no suitable instantons. Therefore, the perturbative answer is exact.

The authors work out several examples and confirm that the space of A-twisted ground states of a given theory agrees with the space of B-twisted ground states and vice-versa. They also confirm that their result agrees with the appropriate limits of the A-twisted and the B-twisted indices in each case.

Comments and Questions

The paper addresses a very interesting physics question (i.e. finding supersymmetric ground states) for a 3d $\mathcal{N} = 4$ theory and gives a very nice answer in terms of the Higgs branch geometry. I think the paper easily meets the acceptance criteria for this journal and I recommend its publication. However, I have a few questions/comments that I would like the authors to address, prior to publication.

- **The simplest mirror symmetry:** Comparing the computations in Section 5 and Section 7, one observes that the space of ground states for the A-twisted $U(1)$ theory with a single hyper agrees with that of the B-twisted free hyper. However, it seems that the ground states for the A-twisted free hyper agrees with that of the B-twisted mirror only when the degree of the background holomorphic line bundle $d = 0$ – comparing equation (5.9) with the statement following equation (7.9). Why is this reasonable?
- **The limit of large mass in the B-twisted case:** For the A-twisted computation, one justifies the limit $|\widetilde{\zeta}| \to \infty$ with $\widetilde{\zeta} = \frac{e^2\, vol(\Sigma)\, \zeta}{2\pi}$, as the IR/strong-coupling limit. In contrast, taking the large mass limit looks like a mere computational trick in the B-twisted case. Can one give a physical justification for taking this large

real mass limit? For example, is such a limit dictated by mirror symmetry once you have taken $|\widetilde{\zeta}| \to \infty$ in the A-twisted computation?

- **The limit of large FI parameter in the A-twisted case:** The authors take the IR limit as $|\widetilde{\zeta}| \to \infty$ where $\widetilde{\zeta} = \frac{e^2\, vol(\Sigma)\, \zeta}{2\pi}$ and $e^2 \to \infty$ with ζ held fixed, and comment that this limit is important for matching the ground states under mirror symmetry. Naively, one could also take the IR limit as $e^2 \to \infty$ with $\zeta \to 0$, such that $\widetilde{\zeta}$ is held fixed. This would mean approaching a wall of the FI chamber, and generically one would have certain extra states to account for. Is the first limit a simplifying computational trick or is the second limit inconsistent with mirror symmetry for some reason? I think a comment clarifying this point will be helpful.

Typos

- Equation 2.21: I think there is a missing r' on the RHS.
- Page 20, first and second sentences : missing references.
- Page 25, last sentence: I think the equation number should be (2.13) instead of (2.14).

There are quite a few (less significant) typos in various parts of the paper.