SciPost Report (arXiv:2108.01489)

The interaction between axions (or axion-like particles) and photons provides a number of avenues for detecting the existence of ALPs, an active area of research. This paper proposes the existence of a positive feedback mechanism, by which a laser beam creates a population of axions, which then amplify the photon signal, thus causing the axion density to grow exponentially. A significant benefit to this kind of mechanism is that it does not require a pre-existing population of axions: that is, the ALP does not need to be dark matter. If some version of the conclusions of this paper can be shown to be correct, it could warrant publication in SciPost, as the results would "Open a new pathway in an existing or a new research direction."

However, I have fundamental reservations about the validity of the results in this paper, and it cannot be published in its current form. The principal assertion, that the feedback is positive, has not been justified in the text or equations, and may be incorrect. In some parts of the draft there are what I assume to be typos in equations or basic errors, which makes it difficult to evaluate the other factual claims. Many of the descriptions in the text of what is happening at the particle level are either wrong or confusing. The "colinear" limit invoked in Section II may be dangerously optimistic, if it has been applied to the population of axions that are produced by the laser. The proposed experimental setup with two lasers of matching frequencies does not produce any axions, or any kind of feedback.

Below, I list the primary questions that I would like the authors to address.

1. **Is There Exponential Growth?**

   On the top of page 3, the authors state that "we focus only on the positive root" for $\Gamma(k)$. How do you know that these are the roots that correspond to the physical forward-propagating modes? One way to show that there is a tachyonic instability would be to calculate the group velocity, $d\omega/dk$, and to show that the group velocity is superluminal. See for example [1]: a superluminal group velocity, combined with the requirement of causality, implies that the retarded Green's function grows exponentially in the future light cone.

   In fact, Eq.(18) provides an expression for the group velocity (referred to here as a "phase velocity"), and it is not superluminal. Where, then is the evidence of any instability that would drive exponential growth? (I think that Eqs.(15–18) are not correct, however. See below.)

2. If there is growth in the axion field, where does the energy come from? Depleting the laser pulse? If so, please explain why this is an example of positive rather than negative feedback.

3. What assumptions have been made about the axion mass in Section III? More precisely, why is the quantity Γ of Eq.(14) and Eq.(16) assumed to be real? In the limit of small couplings and photon intensities, $g_{a\gamma\gamma}A_0 \to 0$, the "growth rate" $\Gamma(k)$ of Eq.(16) is strictly imaginary. Γ is supposed to be real-valued, and its imaginary component should be included in ω_a.

4. **Axion production and Eq.(2):**

   According to the text above Eq.(2), a population of axions is being created from the electromagnetic fields by the "decay…of one photon …into an axion and secondary photon." Please elaborate: what is the underlying assumption about the background electromagnetic fields? A laser fired into a vacuum does not decay in this way, the $\gamma \to \gamma + a$ decay process is kinematically forbidden.

   The PVLAS and light-shining-through-walls experiments rely on strong external magnetic fields. Other proposed experiments rely on a pre-existing background axion field (ALP dark matter) to modify the propagation of light. An external static $\mathbf{B}$ (or $\mathbf{E}$) field is needed to produce (massive) axions from a beam of (massless) photons: the photons associated with the external field are not on-shell, but come instead from some distribution $A_\mu(\mathbf{k})$ that is not constrained by $\omega^2 = \mathbf{k}^2$.

   In this paper, on the other hand, the "seed pulse" and the "weaker probe" are both linearly polarized radiation. So, the axion production mechanism here at the particle level must be light-by-light scattering, $\gamma + \gamma \to a$, which is kinematically allowed only if the frequencies of the two lasers differ by an amount tuned to the axion mass. If this is the picture in mind, then Eq.(2) should be written as

   $$\omega_0 - \omega_\gamma = \omega_a, \qquad \mathbf{k}_0 - \mathbf{k}_\gamma = \mathbf{k}_a \tag{1}$$

   to more clearly show the "before" and "after" pictures.

   Taken at face value, this paper appears to claim that freely propagating photons are liable to decay into an axion and a redshifted photon, and that this photon decay occurs in the absence of any external $\mathbf{E}$, $\mathbf{B}$, or axion fields. The paper should be substantially revised so as not to give the reader this impression.

5. **Axion production and Eq.(19):**

   My interpretation of Eq.(8) is that the "main" pulse is a monochromatic plane wave with frequency ω_0, and that at $t = 0$ the axion energy density, $\rho_a \sim m_a^2 \widetilde{a}^2$, is zero everywhere. The functions $\widetilde{A}$ and $\widetilde{a}$ keep track of the induced fields, which are small compared to $\mathbf{A}_0$. This is fine. But, the modified Maxwell's equations with $a = 0$ are simply SM electrodynamics, so for anything interesting to happen we need to create some axions. To find out how axions are produced from a laser beam, I look to Section IV, which adds "a weaker probe with polarisation in $\hat{y}$."

   Nothing is said about the frequency of the weaker probe. Is it the same as the "pump pulse"? Does it propagate in the same direction? Equation (19) should provide an answer, but does not. Taken literally, Eq.(19) is insensible:

   $$\frac{\mathbf{A} \cdot \hat{x} - \mathbf{A} \cdot \hat{y}}{A_0} = \frac{\mathbf{A} \cdot (\hat{x} - \hat{y})}{A_0}. \tag{2}$$

   This is just linearly polarized light, along some new $\hat{x} - \hat{y}$ axis. Even if the two instances of $\mathbf{A}$ are supposed to differ in magnitude, this initial setup is still just linearly polarized light along $(\hat{x} - \delta\hat{y})$ propagating in a vacuum.

In order to produce any axions through light-by-light scattering, the frequencies of the two pulses must differ by a precise amount tuned to the axion mass. Given that there is no discussion of any of this in the text, I am forced to conclude that there is never any axion particle production in the setup suggested in Section IV, and that all equations from Eq.(20) onward are incorrect.

6. **Colinearity:**

   Disregarding my previous objections, let us assume that some axions are indeed produced by the laser, following Eq.(2). For there to be positive feedback, this population of axions must stay put on the timescales associated with the laser pulse length, τ.

   In Section II, an assumption is made: the calculation will be done in a "*colinear* limit where all momenta, are parallel." For some parts of the calculation this is fine: the experimentalist can ensure, for example, that the laser pulses will all be parallel. For the axion production described in Eq.(2), however, this is suspicious. If the axions do indeed come from "photon decay," why must $\mathbf{k_a}$ be parallel to $\mathbf{k_0}$? Unfortunately, this assumption seems to be important for the posited feedback loop: without a population of axions, there is no positive feedback on the photon source of axion production. Instead, the posited photon-to-axion conversion merely depletes the photon source.

7. **Spatial homogeneity:**

   There seems to be an unspoken further assumption of spatial homogeneity: or at least, no comment is made about the spatial distributions of either $\mathbf{A}(\mathbf{x}, t)$ or $a(\mathbf{x}, t)$ after Eq.(9), except to say that the laser pulse is of finite duration. A lack of spatial gradients is a problem: the modified Maxwell equations depend entirely on the spatial and temporal derivatives of $a(\mathbf{x}, t)$. Without some nonzero $\partial_t a$ or ∇a, the laser will propagate trivially through the vacuum.

8. **What happens to the produced axions?**

   If the axions are produced with some nonzero $\mathbf{k}$, then they will tend to leave the path of the laser. If the axions are produced at relativistic speeds, as suggested in Section III, then this depletion of the axion density will be prompt (assuming that the laser beam is not several meters wide).

   Axions produced by this laser pulse would not be spatially homogeneous along the direction of photon propagation, and yet there is no discussion of what $\widetilde{a}(\mathbf{x}, t)$ is supposed to look like. The spatial profile of $\widetilde{a}$ is very important, if there is to be any positive feedback loop: the presence of axions is supposed to be altering the behavior of the parts of the laser pulse that pass through the space after the axions have been created. Without any gradients in $a(\mathbf{x}, t)$, there is no modification to Maxwell's equations.

9. **Coherence of the axion field:**

   To apply Eq.(3) to this problem, we need to treat the axions as a coherent classical field. This is well motivated if axions are the dark matter, because the velocity dispersion $v \lesssim 10^{-3}$ is small, so the axions remain coherent on relatively long length and time scales, large enough to encompass the experimental setup if the axion is sufficiently light.

It is not clear that the axion production rate here is large enough to justify treating the axions as a coherently oscillating field. What is the axion energy density $\rho_a(\mathbf{x}, t)$ in this scenario (for the axions created by a strong laser pulse)? For what values of the axion mass is the number density ρ_a/m_a large enough to take the classical limit?

These are my most pressing concerns, as any of them would invalidate the entire paper. If the authors can demonstrate that their principal conclusions are not incorrect, the analysis in Section IV would be improved by comparing the power of this "strong laser plus weaker laser" approach to the previously studied applications of axion electrodynamics. For example, given such a powerful laser, why use it in conjunction with "a weaker probe?" Why not fire it into an extremely strong $\mathbf{B}$ field, to create axions as in light-shining-through-walls experiments?

Similarly, how does the ALP density ρ_a compare to the dark matter density? There is existing work in the literature about the effect of a coherent axion background on propagating light. If this ALP happened to be dark matter, would the detection mechanism proposed in this paper be competitive with the existing proposals, or much weaker? Comparing the ρ_a generated by the laser in this work to the $\mathcal{O}(\mathrm{GeV/cm^3})$ energy density of dark matter would be a useful proxy.

Recommendations

Some of my concerns can be addressed directly by a calculation of the retarded Green's function for the photons:

1. This would show whether a local disturbance (e.g. the injection of a strong laser beam) causes exponential growth in any modes in the forward timelike direction. My claim is that the photon ω_γ shown in the right-hand side of Eq.(2) propagates in the backward timelike direction, so that the physical process is really $\gamma + \gamma \to a$.

2. Given the Green's functions for polarized light, it will be possible to analyze the spatial profiles of $\widetilde{a}(\mathbf{x}, t)$ and $\widetilde{A}(\mathbf{x}, t)$ that are created by some laser pulse. From $\widetilde{a}$, it will be possible to calculate the energy density $\rho_a(\mathbf{x}, t)$.

3. I also suggest finding expressions for $\Gamma(k)$ and $\omega_a(k)$ that are valid in the limit of small $g_{a\gamma\gamma} A_0$. As $g_{a\gamma\gamma} \to 0$, the results should recover SM electrodynamics. The axion dispersion relation is that of a decoupled massive scalar.

4. Once $\rho_a(\mathbf{x}, t)$ is known, it will be easier to address my concerns about "what happens to the produced axions." If the axions are actually produced via light-by-light scattering, $\gamma + \gamma \to a$, then the "colinear limit" can be invoked to explain why the produced axions remain in the path of the beam. Otherwise, the dissipation of the axion field is a substantial obstacle to the purported positive feedback system.

If this paper really does rely on the decay of photons in the vacuum ($\gamma \to \gamma + a$, in the absence of any external $\mathbf{E}$, $\mathbf{B}$ or a fields), then it is entirely incorrect and should be withdrawn.

References

[1] Y. Aharonov, A. Komar, and L. Susskind, "Superluminal behavior, causality, and instability," Phys. Rev. **182** (1969) 1400–1403.