SciPost-2109.09706: Color mixed QCD/QCD evolution

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In this paper the author discuss a parton shower algorithm that combines the QED and QCD effect in the DIRE framework. More specifically they are interested in the interference contributions of the non-trivial QCD colour structures and the similar QED amplitudes as it is nicely illustrated in Eq. (1). In order to do this they have to deal with the colour evolution in the parton shower algorithm. This is very complicated task since the colour basis is not orthogonal and a dimensionality of the problem is enormous. It scales with the number of external legs as $\sim (n!)^2$. The QED part of their shower implementation looks reasonable in a simple shower framework where the spins of the particles are averaged.

The major part of this project is to deal with colour in the parton shower. This procedure is based on the algorithm that is defined in of Ref.[14]. That is a published paper in Phys.Rev.D. and in principle this would make my job easier. I should simply accept this paper, but I cannot recommend it without major revision since I have several concerns.

QED contributions

In Sec. II.A there is review of the QED part of the Dire parton shower. In the introduction the author pointed out that the interference terms between the QED and QCD radiations has to be understood and treated very carefully. In Section II.A there are only two type of QED vertices are discussed, the photon emission one $(q \to q + \gamma)$, $P_{\mathrm{q},\gamma,k}$, and the photon splitting vertex $(\gamma \to q + \bar{q})$, $P_{\gamma,\mathrm{q},k}$. These vertices are leading contributions in α_{em} and they can describe the α_{em}^2 terms in Eq (1) after two steps of the shower.

The parton showers algorithms evolves diagonal states in the momentum and flavour space. In a first order shower, which is discussed here, in every step of the shower procedure we can generate only one extra parton, thus the interference diagrams of Eq. (1) (the last two graphs) are completely unreachable by the first order shower. With first order shower we cannot generate a gluon on the bra side and a photon on the ket side of the matrix element square in the first step as it is suggested in the last two terms of Eq. (1). That requires higher order contributions in the parton shower algorithm. Namely a splitting kernel that is proportional to $\alpha_s \alpha_{em}$,

$$P^{(2)} \sim \alpha_s \alpha_{em} \langle \boldsymbol{V}^\dagger(q \to q + (g \to q' + \bar{q}')) \cdot \boldsymbol{V}(q \to q + (\gamma \to q' + \bar{q}')) \rangle,$$

where $\boldsymbol{V}$ is the Feynman graph that describes the given partonic process. The $\langle \cdots \rangle$ is the spin averaging. This is the territory of the NLO shower algorithms with real and virtual QCD and QED corrections to splitting kernels.

These higher order contributions are clearly not in this shower algorithm, thus it cannot deal with the QED/QCD interference terms. Could you please clarify this in the text?

The colour evolution, Sec. II.B

The main idea here and in Ref.[14] is to go beyond the usual leading colour approximation in a systematic way and sum up some subleading colour effect ($1/N_c^2$) correctly. As far as I know that requires amplitude level evolution in the colour space.

The authors in this paper use the colour flow basis and in this case any m-parton amplitude can be written as

$$\big|\mathcal{M}_m\big\rangle = \sum_\sigma |\sigma\rangle M_m(\sigma) \ ,$$

where $M_m(\sigma)$ are the colour subamplitude in the colour flow basis.

If we want to do colour evolution on this basis, for the shower evolution equation we have something like[1]

$$\mathbb{S}_{\mu^2_{\rm cut},\mu^2_m}\big[|\mathcal{M}_m\rangle\langle\mathcal{M}_m|\big] = \Delta(\mu^2_{\rm cut},\mu^2_m)|\mathcal{M}_m\rangle\langle\mathcal{M}_m|\Delta^\dagger(\mu^2_{\rm cut},\mu^2_m)$$
$$+\int_{\mu^2_{\rm cut}}^{\mu^2_m}\frac{d\mu^2}{\mu^2}\sum_{\substack{i,k=1\\ i\neq k}}^m \mathbb{S}_{\mu^2_{\rm cut},\mu^2}\Big[\boldsymbol{T}_i\Delta(\mu^2,\mu^2_m)|\mathcal{M}_m\rangle\frac{P_{i,m+1,k}(\mu^2)}{\boldsymbol{T}_i^2}\langle\mathcal{M}_m|\Delta^\dagger(\mu^2,\mu^2_m)\boldsymbol{T}_k^\dagger\Big].$$

Here $\mathbb{S}_{\mu_1^2,\mu_2^2}[\cdots]$ represents shower evolution between two scales. In the first term we have evolution between the scale of the M_m state and the final cutoff scale without any real emission. While in the second term we have no emission from μ_m^2 to μ^2 and then we have a real emission. After this we can continue the shower with more emission by $\mathbb{S}_{\mu^2_{\rm cut},\mu^2}[\cdots]$. Surely, all the amplitudes $|\mathcal{M}_m\rangle$ has to be expanded on the colour basis.

It would be nice to have a precise definition of the shower evolution operator/equation in the paper, especially there are extra complications from the QED part.

Question I: What represents the partonic state? If we want to do colour evolution it has to be the density matrix in the colour space,

$$\big|\mathcal{M}_m\big\rangle\big\langle\mathcal{M}_m\big| = \sum_{\sigma,\sigma'}|\sigma\rangle\langle\sigma'|\, M_m(\sigma)M_m(\sigma')^* \ ,$$

where $|\sigma\rangle$ is the colour basis vector as it is given in Eq. (7). The paper suggests that the object that is evolved by the shower evolution is the matrix element square,

$$\big\langle\mathcal{M}_m\big|\mathcal{M}_m\big\rangle \ ,$$

but in this case a consistent colour evolution is impossible. Without the precise definition of the shower evolution equation it is hard see how it can happen.

[1] I hope this equation is clear enough, I tried to stick to the notation of the paper. For the sake of simplicity the z and azimuth integrals and the momentum mappings are included in $P_{i,j,k}(\mu^2)$ kernels and I ignored the $g\to q+\bar{q}$ vertex as well as the QED vertices.

Question II: What is the real splitting operator? If we take seriously the colour evolution the real splitting operator should be something like

$$P_{i,m+1,k}(\mu^2)\frac{\boldsymbol{T}_i|\sigma\rangle\langle\sigma'|\boldsymbol{T}_k}{\boldsymbol{T}_i^2}$$

when it is act on a $|\sigma\rangle\langle\sigma'|$ color state. One can expand the $\boldsymbol{T}_i$ operator on the colour flow basis and express this operator in terms of $\boldsymbol{t}_c\otimes\boldsymbol{t}_{c'}$, $\boldsymbol{t}_c\otimes\bar{\boldsymbol{t}}_{c'}$, $\boldsymbol{t}_c\otimes\boldsymbol{s}$,.... In Eq.(10) the splitting operator depends on the colour operators only linearly. This is not clear at all. Since we have bra and ket states and they have to be evolved independently in the colour space.

Question III: What is the Sudakov operator? The Sudakov operator is a timed ordered exponential of the splitting kernel of the virtual and unresolvable radiation, thus qualitatively we should have something like this:

$$\Delta(\mu^2,\mu_m^2) = \mathbb{T}\exp\left(-\frac{1}{2}\int_{\mu^2}^{\mu_m^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\sum_{\substack{i,k=1\\ i\neq k}}^{m}\frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_k}{\boldsymbol{T}_i^2}P_{i,m+1,k}(\bar{\mu}^2)\right) \quad .$$

The Sudakov operator is a nontrivial exponentiated operator in the colour space and it sums up virtual and unresolvable radiation at all order level. Since the colour basis is not orthogonal and dimension of the colour space is enormous, this exponentiation is rather hopeless. Thus, it important to have a precise definition of the Sudakov operator. Furthermore, the QED part of the Sudakov operator is not even mentioned in this manuscript.

The unitarity is an important property of the parton shower algorithms and that relies on the consistent definition of the real emission operator and the Sudakov operator.

As far as I understood their Sudakov operator is something like

$$\begin{aligned}&\Delta(\mu^2,\mu_m^2)|\sigma\rangle\langle\sigma'|\Delta^\dagger(\mu^2,\mu_m^2)\\ &\quad= |\sigma\rangle\exp\left(-\int_{\mu^2}^{\mu_m^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\sum_{\substack{i,k=1\\ i\neq k}}^{m}\frac{\langle\sigma'|\boldsymbol{T}_i\cdot\boldsymbol{T}_k|\sigma\rangle}{\langle\sigma'|\boldsymbol{T}_i^2|\sigma\rangle}P_{i,m+1,k}(\bar{\mu}^2)\right)\langle\sigma'| \quad .\end{aligned}$$

I think it is obvious why it is wrong. One can achieve unitarity by this, but most important and primary role of the Sudakov operator is to sums up virtual and unresolvable emissions at all order level. Expanding this Sudakov in α_s the generated term don't correspond to any Feynman graphs in the strongly ordered soft and collinear limits. This Sudakov is not even the first approximation of the proper Sudakov operator. It is kinda acceptable only in the strict leading colour limit, beyond that it completely ad-hoc and messes up completely the $1/N_c^2$ subleading colour effects.