**Report on the paper**
*Notes on symmetries in particles physics*
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These introductory notes deal with the application of symmetries in quantum field theory and particle physics.

The first part of these lectures is devoted to a short introduction on group theory. Section 1 is devoted to the description of finite groups and continuous (or Lie) groups. In the latter case, the notion of Lie algebra is introduced. Sections 3 and 4 are devoted to the description of unitary groups, central for the Standard Model of particles physics, whilst Section 4 to the description of spacetime symmetries, namely the Lorentz and the Poincaré algebras. These four sections could be improved and sometimes the introduced notions are either imprecise or contain several mistakes (see the remarks below).

The second part of these lecture is devoted to the realisation of symmetries in QFT or in particle physics. Section 6 deals with the implementation of symmetries in QFT by means of the Nœther theorem which is proved with some details (even for a non-canonical theory). The local symmetries or gauge symmetries are then considered. In Section 7 the Standard Model of particle physics based on the local Lie group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is introduced. This is certainly the most interesting part of these lectures. In particular the goal of Sections 6 and 7 is to give with a lot of details the Lagrangian of the SM. More precisely the author emphasises on the final Lagrangian obtained after symmetry breaking, including a nice discussion on flavor physics (CKM matrix). Finally, in Section 8 an interesting discussion on the discrete symmetries $P, T$ and $C$ and on the $CPT-$theorem is performed.

Since there are, as mentioned by the author, many excellent books or monographs on the subject (in the reviewer's opinion, the author should have included more of such references), this is a quite difficult task to write lectures on group theory with an original approach. In spite of some interesting (in particular in Sections 6 and 7) and illustrative examples, it is considered that the lectures do not present enough new pedagogical material in order to be published. Furthermore, I have seen too many misleading presentations or incorrect definitions (especially in the first part). Thus I cannot recommend the publication of these lectures in SciPost Physics Lecture Notes.

In order to improve the manuscript, I will make now my remarks section-by-section.

## Section 2.1

1. Eq. (2.6) is not very clear, in fact noting $K$ the generator of (2) we have

$$R(\theta) = \lim_{n\to\infty} (1 + \frac{\theta}{n} K)^n \ .$$

**Section 2.2**

1. p.8 l-2 (2) symmetry transformations are **associative**. The author has to specify why the operations are associative. This is due to the fact that in physics, symmetry operators are represented either by matrices or by differential operators.

**Section 2.3**

1. p. 10 in the section **Representations** the author defines a representation of a group $G$ as a mapping $D : G \to GL(n)$. This is imprecise, as this could be $GL(n, \mathbb{R}), GL(n, \mathbb{C})$. This is not precised. In fact at this point I suggest to the author to present the three different types of representations: (1) real, (2) complex and (3) pseudo-real.

2. p. 11 in the last bullet the authors should speak about the Wigner Theorem.

**Section 2.4**

This section in unclear and very confusing. Many points have to be clarified

1. In the paragraph **Generators and structure constants** the author should specify that the Lie algebra is finite dimensional.

2. Eq. (2.15) why there is an $\hbar$ factor? I understand for the Lie groups of $SO(3)$ or $SU(2)$ since the generators are related to the angular momentum and have the dimension of an action. But I do not understand for $SU(3)$ or other Lie groups.

3. Eq. (2.15) the author has to specify that the structure constants belong to the field $\mathbb{F}$.

4. Eq. (2.15) the index structure of the structure constants is $f_{ab}{}^c$ and not $f_{abc}$ (see also below).

5. Three lines before Eq. (2.15) the author writes $X = \sum_a \alpha_a T_a$: (1) the index structure is not correct; (2) a factor $i$ is missing (for instance for $\mathfrak{so}(2)$ the generators are purely imaginary −see Eq.[2.18]−, but an element of $\mathfrak{so}(2)$ is a real matrix); (3) with the conventions of the author the generators have the dimension of an action, but elements of the Lie algebra are dimensionless so we have to divide by $\hbar$. Thus with the conventions of the author an element of the Lie algebra writes:

$$X = \frac{i}{\hbar} \sum_{a=1}^{\dim \mathfrak{g}} \alpha^a T_a .$$

   Subsequently all expansions have to be corrected (2.24), (2.15), etc.

6. Eq.[2.16] many points to be corrected

   (a) In the Lie algebra $\mathfrak{g}$ the product of two elements is not defined, thus $T_a T_b$ has no meaning (only the Lie bracket $[T_a, T_b]$, which *is not the commutator* has a meaning). The metric is then defined through a given representation, say the adjoint representation (see p. 17). Thus we have

$$g_{ab} = \mathrm{Tr}(T_a^{\mathrm{adj}} T_b^{\mathrm{adj}}) = -f_{ac}{}^{d} f_{bd}{}^{c} \ .$$

This is the definition of the Killing form. We also see that the Lie algebra must be finite dimensional in order that this definition makes sense, since the dimension of the adjoint representation is the dimension of the Lie algebra itself.

(b) If the Lie algebra is not semisimple (as for example the Poincaré algebra) the Killing form is degenerate and in this case $g^{-1}$ does not exist.

(c) If the Lie algebra is semisimple the Killing form is non-degenerate (or invertible), but not definite positive, for instance for the Lorentz group in four-dimensions the metric has signature $(+++---)$. Only for compact Lie algebra we have

$$\mathrm{Tr}(T_a^{\mathrm{adj}} T_b^{\mathrm{adj}}) = C\delta_{ab}, C > 0$$

(d) When the algebra is compact, since the space is Euclidean, we do not have to care about the position of the indices, but in general for a simple Lie algebra we have to raise and lower indices with the metric and its inverse. Thus

$$f_{abc} = f_{ab}{}^{d} g_{cd} \ .$$

(e) Page 16: The Lie algebra $\mathfrak{u}(n)$ is not semisimple, but reductive. In this sense, the Footnote 2 is false, as a semisimple algebra is always non Abelian. Otherwise it would conflict with the semisimplicity criterion of Cartan, that establishes the non-degeneracy of the Killing form.

(f) p. 16 In the paragraph **Lie algebra representation** point (2) the author defines a representation as satisfying $D([X, Y]) = [D(X), D(Y)]$. This is incomplete. Indeed if $V$ is the vector space on which $D(X)$ acts then $D(X)$ is an endomorphism of $V$. Since the composition of two endomorphisms makes sense the correct definition is

$$D([X, Y]) = [D(X), D(Y)] = D(X)D(Y) - D(Y)D(X) \ .$$

**Section 2.5**

1. p. 17, when the author defines a Lie group by its set of parameters this is correct, but may be he should make some relations with differentiable manifolds. For instance we have that the Lie group $SU(2) \cong \mathbb{S}^3$ (the three-sphere).

2. The definition of Lie subgroup is incorrect. Either the subgroup is closed as topological space, in which case it is indeed a Lie subgroup inheriting the differentiable structure of $G$, or it is not closed, in which case it is called an immersed Lie subgroup. Moreover, in the definition of simple Lie group, the assumption on connectivity is not necessary, but again, that the group is not Abelian.

3. Eq.[2.33] is correct only for compact Lie groups since in this case any element of the Lie group can be obtained exponentiating an element of the Lie algebra. This is wrong for non-compact Lie groups. For instance if we consider $SL(2,\mathbb{R})$ which is non-compact then

$$\begin{pmatrix} -e^{\sigma} & 0 \\ 0 & -e^{-\sigma} \end{pmatrix} \ , \sigma \in \mathbb{R}$$

belongs to $SL(2,\mathbb{R})$. However, for any matrix of $\mathfrak{sl}(2,\mathbb{R})$

$$x = \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \ , a,b,c \in \mathbb{R}$$

we have

$$X \neq e^x \ .$$

The correct result is the following: any element of a Lie group $G$ can be obtained as a finite product of exponentials of elements of the corresponding Lie algebra.

4. P. 19 in the examples given, the author should define more precisely the various Lie groups and Lie algebras, and in particular should explain the relationship between a Lie group $G$ and its Lie algebra $\mathfrak{g}$, *i.e.* for $X \in G$ if we write $X = \mathbb{1} + ix$ with $x \sim 0$ how the conditions upon $X$ translate into appropriate conditions for $x$. For instance

$$SO(n) = \{R \in \mathcal{M}_n(\mathbb{R}\}, \det R = 1, \ R^t R = \mathbb{1}\}$$

and (with $R = \mathbb{1} + ir$)

$$\mathfrak{so}(n) = \{ir \in \mathcal{M}_n(\mathbb{R}\}, \ r^t + r = 0\}$$

Indeed in physicists literature there is often a confusion between Lie algebras and Lie groups (confusion not made by the author). The difference between Lie algebras and Lie groups has to be emphasised in a pedagogical text.

**Section 3.**

For clarity the author should specify that $U(1) = \{z \in \mathbb{C}, |z|^2 = 1\}$.

**Section 4.1**

1. p. 22, l. 8 the author should specify that $\mathfrak{su}(2)$ is a **real** vector space.

2. after Eq.[4.6] there is a problem of label and (2.29) should be (2.30).

**Section 4.2**

1. I think that equation (4.11) necessitates more details, for instance specifying that $(n-1)-$order fully antisymmetric tensors are isomorphic to the anti-fundamental representation through

$$\bar{\psi}_i = \epsilon_{ij_1\cdots j_{n-1}} \psi^{(j_1\cdots j_{n-1})}$$

because of (4.10).

2. I think that the section on Young tableaux (a very important topic) necessitates more details. For instance in the rules 1-4 given p. 24 there is an ambiguity since it seems that a Young Tableaux is symmetric in its rows whilst is antisymmetric in its columns. In fact it is not the case. Indeed if we consider a Young tableaux $Y$ we define the Young projector as follows: $P = AS$, where $S$ symmetrises first the row and then $A$ antisymmetrises the columns. Thus given a tensor T, its corresponding irreducible part associated to $Y$ is given by
$$T_Y = AST \ .$$

3. In the tensor decomposition given p. 25, for clarity I suggest to the author to substitute $a_1, a_2, \cdots$ by $b_1, b_2, \cdots$ since these numbers are associated to the row of $B$.

**Section 4.3**

1. As for the $U(1)$ section, I suggest to the author to specify that
$$SU(2) = \left\{ \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}, a, b \in \mathbb{C}, |a|^2 + |b|^2 = 1 \right\} .$$

2. Some important part which is missing in this text is the following. Given a representation of a Lie algebra $D(T_a) = M_a$, satisfying
$$[M_a, M_b] = i f_{ab}{}^c M_c$$
then we have three other possible representations: the dual representation $-M_a^t$, the complex conjugate representation $-M_a^*$ and the dual of the complex conjugate representation $M_a^\dagger$. If the representation is unitary then there is possibly only one other representation since $-M_a^* = -M_a^t$ and $M_a^\dagger = M_a$
$$[-M_a^*, -M_b^*] = i f_{ab}{}^c (-M_c)^*$$
If $M_a \neq -M_a^*$ and there exists a matrix $P$ such that
$$-M_a^* = P M_a P^{-1}$$
the two representations are equivalent, and the representation is said to be pseudo-real. In the case of the fundamental representation of $\mathfrak{su}(2)$ we have
$$-(\sigma^i)^* = \sigma^2 \sigma^i (\sigma^2)^{-1}.$$
Thus the fundamental and anti-fundamental representation are isomorphic. May be some words in this direction should be said.

**Section 4.4**

1. The definition of unitary representation is crucially missing.

2. Some important part which also missing is the concept of Cartan subalgebra (which is mentioned in a subliminal way by the author). The algebra $\mathfrak{su}(3)$ is of rank two, that is there exist two self-normalised commuting operators, say $T_3$ and $Y$.

   Let me replace these operators in a more general context: Let $\mathfrak{g}$ be a compact semisimple Lie algebra of rank $\mathrm{rk}\mathfrak{g}$ and dimension $\dim \mathfrak{g}$.

   (a) The minimal set of operators to characterise unambiguously all vectors in an arbitrary representation is given by

   $$\frac{1}{2}(\dim \mathfrak{g} + \mathrm{rk}\mathfrak{g}) = \mathrm{rk}\mathfrak{g} + \frac{1}{2}(\dim \mathfrak{g} - \mathrm{rk}\mathfrak{g})$$

   (b) Any representation are characterised by the eigenvalues of the $\mathrm{rk}\mathfrak{g}$ Casimir operator. In the case of $\mathfrak{su}(3)$ representations are classified by the eigenvalues of the quadratic and cubic Casimir operators given after equation (4.26). This should be emphasised by the author. Moreover, it should be welcome that the author explains (even succinctly) formulæ (4.28).

   (c) Given a representation specified by the eigenvalues of the Casimir operators, any states are specified by the eigenvalues of the $\mathrm{rk}\mathfrak{g}$ generators of the Cartan subalgebra of $\mathfrak{g}$, but also internal label operators. In the case of $\mathfrak{su}(3)$ since $\frac{1}{2}(\dim \mathfrak{g} - \mathrm{rk}\mathfrak{g}) = 3$ one need beyond $T_3$ and $Y$ an additional operator called $I^2$ by the author (see p. 28, before Eq.[4.27]). Again the author should emphasise this point in a pedagogical lecture. If not we wonder why $I^2$ is considered.

3. I think that the discussion above, not often given in physical literature should be given in order to put the results in perspective.

4. I think that the explicit expressions (4.31) are interesting. May be the author should add some word in order to explains these relations, and surely give some references. But again in order to put these results in perspective I also suggest to relate the representation $D(p, q)$ with the highest weight representation introduced by H. Weyl and in particular explain (even in few words) the algorithm to construct the whole representation from the highest weight state. This is mentioned very briefly without any details just after eq.[4.31]. See for instance the Ref. [9] of the author where highest weigh representations are studied with many details and examples.

**Section 5.1**

1. The author takes two different definitions for $\sigma_\mu$ (see p. 35, l. 1 and p.37 after Eq.[5.28]). He should take a uniform definition.

2. In Footnote 4 the author introduces $\bar{\sigma}_\mu$, not defined at this stage and does not explain how he raises indices for a matrix.

3. After Eq.[5.16] the author writes

$$SL(2, \mathbb{C}) = SU(2) \oplus SU(2) \quad \text{or} \quad \mathfrak{sl}(2, \mathbb{C}) = \mathfrak{su}(2) \oplus \mathfrak{su}(2)$$

This is a **very big mistake** often stated in physical literature. The confusion comes from real form of complex Lie algebras (an important concept note presented in these lectures). For instance, if we consider the three-dimensional complex Lie algebra $\mathfrak{sl}(2,\mathbb{C})$ with Lie brackets

$$[H, X_\pm] = \pm X_\pm \ , \quad [X_+, X_-] = 2H$$

it has two real forms

(a) the compact real form or $\mathfrak{su}(2) \cong \mathfrak{so}(3)$, generated by $T_1 = 1/2(X_+ + X_-), T_2 = -i/2(X_+ - X_1), X_3 = H$ with Lie brackets

$$[T_a, T_b] = i\epsilon_{ab}{}^c T_c$$

(b) the split real form or $\mathfrak{sl}(2,\mathbb{R}) \cong \mathfrak{so}(1,2) \cong \mathfrak{su}(1,1)$, generated by $V_1 = i/2(X_+ + X_-), V_2 = -i/2(X_+ - X_1), V_3 = iH$ with Lie brackets

$$[V_3, V_1] = -iV_2 \ , \quad [V_1, V_2] = iV_3 \ , \quad [V_2, V_3] = iV_1$$

4. The fact that the equality written in Item 3 above is incorrect is very simple to understand. Indeed for all compact Lie groups unitary representations are finite dimensional whereas all unitary representations of non-compact Lie groups are infinite dimensional. Since $SU(2) \oplus SU(2)$ is compact unitary representations are finite dimensional and classified by two half-integer numbers. Since $SL(2,\mathbb{C})$ is non-compact unitary representations are infinite dimensional, the study of unitary representations of $SL(2,\mathbb{C})$ is a difficult task. **This mistake has to be corrected throughout the lectures**.

5. The discussion after Eq.[5.16] (Taking $\cdots$) is confusing.

**Section 5.2**

1. Eq.[5.23] are representations of $\mathfrak{sl}(2,\mathbb{C})$, but these representations are **non-unitary** (see item above). This has to be specified.

2. In eq.[5.24] $\epsilon^{\mu\nu\rho\sigma}$ is not defined. It has to be defined.

3. $\Gamma$ is a Casimir operator. The Lorentz algebra admits a second Casimir operator. May be it should be given too.

4. Usually left-handed spinors have undoted indices whilst right-handed spinors doted indices, the author takes the opposite convention, this might lead to some confusions.

5. When the author raises or lowers spinor indices he takes the following conventions $\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta$ and $\psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta$. The second relation should be explicitly written and seems to be in contradiction with the last equation p. 37.

6. We have to be careful with $\delta^\alpha_\beta$ since
$$\delta_\alpha{}^\beta = \epsilon_{\alpha\gamma}\epsilon^{\gamma\beta}$$
implies
$$\epsilon^{\gamma\alpha}\delta_\alpha{}^\beta\epsilon_{\delta\beta} = -\delta^\gamma{}_\delta$$
For this reason I specify $\delta_\alpha{}^\beta$ or $\delta^\beta{}_\alpha$ and never write $\delta^\beta_\alpha$.

**Section 5.3**

This section has to be completely rewritten.

1. First sentence of this section: Poincaré transformations are an **extension** $\cdots$. I would not use the word extension, since an algebraic extension has a totally different meaning.

2. This section misses an extremely important feature of the Poincaré algebra. Indeed, since the Poincaré algebra has a semi-direct structure (as says the author), the study of its representations follows the method of induced representations of Wigner established in the seminal paper [V. Bargmann and E. P. Wigner, *Group theoretical discussion of relativistic wave equations*, Proc. Nat. Acad. Sci. **34** (1948) 211.] (see also the first tome of the excellent book of S. Weinberg on Quantum field Theory).

   (a) One of the Casimir operator is $P_\mu P^\mu$ as said the author. But three cases must be considered: (i) $P_\mu P^\mu > 0$, (2) $P_\mu P^\mu = 0$ and $P_\mu P^\mu < 0$. The first case corresponding to massive particles, the second case to massless particles and the last case to the unphysical tachyons.

   (b) Then the so-called little group or little algebra has to be introduced. To obtain unitary representations of the Poincaré algebra: (1) study representations of the little algebra in the so-called standard frame, (2) boost these representations to any frame.

   (c) In the massive case since the little group is $SO(3)$ the degrees of freedom are the mass $m$ (eigenvalue of $P_\mu P^\mu$) and the spin $s$ (related to the eigenvalue of $W_\mu W^\mu$) and are associated with the two Casimir operators of the Poincaré algebra. The representation is of dimension either $(2s+1)$ (particle = anti-particle) or $2(2s+1)$ (particle and anti-particle).

   (d) In the massless case since the little group is $E_2$, considering $SO(2) \subset E_2$, the eigenvalues of the two Casimir operators are not enough and we have to introduce the so-called helicity. Massless states have two degrees of freedom corresponding to the two possible values of the helicity $\pm h, h = 0, 1/2, 1, \cdots$.

3. P. 42, do not understand when the author says **For massless $\cdots$ close to the speed of light, such that $p_\mu \to 0$.**

**Section 6.1**

Just one remark. P.47 the author mentions that Weyl fermions forbid a mass term, what he calls a *curious feature*. In fact this observation can be deduced directly from representation theory of the Poincaré group (method of induced representations) since massive and massless particles behave differently.

**Section 6.2**

1. In this section the author considers a general symmetry group. He has to specify that in fact it is a compact Lie group since only in this case finite dimensional representations are unitary.

2. Up to now the author only considers compact unitary groups, *i.e.*, $SU(n)$, and nothing is said about the other series or the exceptional groups. In particular he does not explain how to obtain representations in these cases. Either this has to be clarified or the author has to specify that $G_{\rm int} = SU(n)$.

3. Considering a Lie algebra $\mathfrak{g}$ with generators $T_a$, given an $n-$dimensional representation the author notes $D(T_a)$ the corresponding $n \times n$ matrix representation. However, in this section, the author notes sometimes $T_a$ the matrix representation instead of $D(T_a)$. This has to be unified.

4. The author has to recall that $f_{ab}{}^c$ are the structure constants of the Lie algebra $\mathfrak{g}_{\rm int}$ (he should not write them $f_{abc}$, see below).

5. The index structure has to be uniformed. Even if the Lie group is compact and the Killing form reduces to $\delta_{ab}$ meaning that covariant and contravariant indices are the sames, I think it would be more clear, if the author respects the usual conventions of summation of repeated indices (see for instance Eq.[6.35, 6.42]). Note that sometimes the convention is respected (see between eq.[6.35] and [6.35]).

6. Footnote 11 (and also the discussion on p. 86) are not correct. In general relativity, as said the author, the symmetry group is the diffeomorphism group, but these transformations correspond to a gauging of spacetime translations and not of Lorentz transformations. Indeed, because of the principle of equivalence in any point of the spacetime we can find a frame where gravitation has been eliminated. It is in this tangent flat spacetime that we have local Lorentz invariance (a Lorentz transformation in a curved space has no meaning).

7. P. 50 the authors speaks of "curious" byproduct. This is in fact not a curious observation but more precisely a consequence of the Nœther procedure which implies in what way matter couples to gauge fields. This is a general property of gauges theories.

8. In Eq.[6.44] and the following, the author has to recall his notations concerning left, right and Dirac spinors.

9. In Eq[6.45] the author has to specify that $q$ is the charge of the various fields.

10. P. 52, the author says that the kinetic term of gauge fields is introduced by hand. In fact we can show that if we don't introduce such a kinetic term, energy is not conserved. Thus conservation of the energy implies such kinetic terms

11. Between Eqs[6.55] and [6.56], when we do not speak of representation $\mathrm{tr}(T_a T_b)$ has no meaning (see also my remark related to Section 2.6).

12. P. 53 $\epsilon^{\mu\nu\rho\sigma}$ has to be defined.

## Section 6.3

1. P. 54 when the author gives the vertex interactions, may be he should give also the corresponding Feynman rules. For instance for the first vertex for a Dirac fermion it will be $-ie\gamma^\mu$.

## Section 6.4

1. In this section the author wants to be general, *i.e.*, doesn't specify the gauge group. This is a little bit confusing and the distinction between Higgs space and Goldstone space is not very clear. [I suggest the very good reference K. Huang, Quarks Leptons and Gauge Fields.]

2. In all scalar potentials the author has to specify $\lambda > 0$, in order that potentials are bounded from below. Similarly he has to say $\mu^2 > 0$ (in (6.73) and (6.68), but in (6.68) the potential has the term $+\hbar\mu^2$ and not $-\hbar\mu^2$ (or equivalently $-\mu^2 < 0$). This misprint should be corrected.

3. When the field $\phi$ develops the vev (7.74) the author does not specify under which group $G_{\text{int}}$ is broken too. For $SU(n)$ it would be down to $SU(n-1)$.

4. Eq.[6.75] is not very clear $\hat{\phi}_0$ is an $n-$dimensional vector, the exponential term an $n\times n$ matrix, $v$ is a number, but what is $\eta$ (I suppose that it is an excitation around the minimum). This has to be specified.

5. In [6.75] again the count of the degrees of freedom is also confusing. If the gauge group is $SU(n)$, since the unbroken group is $SU(n-1)$, in the exponential factors only the $D(T_a)$ corresponding to $SU(n)/SU(n-1)$ act non-trivially on $\hat{\phi}_0$. Since $SU(n)/SU(n-1)$ is of dimension $2n-1$ and $\eta$ is a real scalar the field $\phi$ has $2n$ real degrees of freedom.

6. P. 58, the author explains in which way Higgs mechanism gives a mass to the gauge bosons. He should explain that it is the only way to have a renormalisable theory with massive gauge bosons.

## Section 6.5

1. May be recall that $\chi$ is a left-handed spinors whereas $\psi$ is a right-handed spinor throughout.

2. In Eq.[6.89] precise that $\phi_0 = \begin{pmatrix} 0 & 1 \end{pmatrix}^t$.

3. After relations (6.94-6.97), may be recall the Gell-Mann-Nishijima formula.

4. In the "Mexican-hat" potential (6.88) precise $\lambda > 0, \mu^2 > 0$, when he set $v = \mu/\sqrt{\lambda}$ what is the sign of $\mu = \sqrt{\mu^2}$?

5. In the vertices of interactions p. 64 and p. 65, may be it should be interesting to precise the Feynman rules.

6. What about propagators?

7. What about ghost sector?

**Section 7**

1. When considering a compact Lie group as the unitary group, $SU(n)$, any representations of the Lie algebra $\mathfrak{su}(n)$ can be exponentiate to a representation of the Lie group $SU(n)$. Conversely, to any representation of the Lie group $SU(n)$, by differentiation we can associate a representation of the Lie algebra $\mathfrak{su}(n)$. Thus, the discussion in the first paragraph of Section 7.1 is confusing. For instance for $\mathfrak{su}(2)$, I wonder why the author makes the distinction between $\mathfrak{su}(2)$ and $SU(2)$ representations. I think that the author should say that $A, B, \cdots = 1, 2, 3$ and $I, J, \cdots, = 1, 2$ are indices in respectively the adjoint or fundamental representation.

2. In Eq.[7.17] we can identify easily the part which leaves $\hat{\phi}_0$ invariant, *i.e.*, when $\pi = \pi_0$, could the author make some conclusion of this observation? In fact I wonder why the coupling constants are not present (see Eq. [6.89]). See also Footnote 1 of Section 6.4 above.

**Section 8**

1. P. 80, in **Symmetry operators algebra** the author writes $P^2, T^2, C^2 \propto \mathbb{1}$, but in fact we have $P^2, T^2, C^2 = \mathbb{1}$ since the group is $\mathbb{Z}_2^3$.