Report for the paper "High-frequency transport and zero-sound in an array of SYK quantum dots" by A. V. Lunkin and M. V. Feigel'man

August 8, 2022

1 Summary

The current paper is an extension of the work about the chain of SYK_4 models coupled by random quadratic hoppings (arXiv:1705.00117) where the ground state exhibits transition from the heavy Fermi liquid to strange metal phase as a function of temperature. The extension of the current paper to the before-mentioned work consists in:

1. Adding additional intra-dot quadratic coupling as a perturbation (being however bigger then the critical value [1]) to the $cSYK_{q\geq 4}$ quantum dot chain.

2. Considering general non-zero particle-hole assymetry parameter.

3. Considering non-equilibrium dynamics.

The paper studies the influence of the perturbation mentioned in point 1. on the dynamic response functions such as electric and thermal conductivity. The authors compare the particular case of the static dc response functions with those calculated for the same system without the quadratic intra-dot perturbations (where it is known to have a non-Fermi liquid regime for $T > T_{FL}$ where the static dc electric resistivity is linear in temperature and the Lorentz number [2] is a universal constant) and make conclusions about the influence of the qudratic intra-dot perturbations on these observables depending on the temperature and corresponding perturbation strength.

In particular, authors find that for temperatures much bigger as compared to the corresponding coupling strength the perturbations are irrelevant while for the intermediate temperatures they change the qualitative behaviour of the response funcitons in the system. The authors find that in such case the Lorentz ratio becomes non-universal and temperature-dependent. It is also found that the latter is rooted into the thermal rather then electric conductivity. For non-zero particle-hole asymmetry parameter, thermal conductivity contains both the "intrinsic" (unrelated to charge transport) term, and the term proportional to electric conductivity.

[1] It is known, that adding even an infinitesimally small quadratic term to the SYK model can destroy its non-trivial non-FL phase. It was studied before that the critical value of the strength of the quadratic perturbations that one can add to the SYK_4 system and not destroy its non-Fermi liquid ground state goes to zero with the number of fermions in the system N as N^{-1}.

[2] The ratio of the thermal to electric static conductivity times T^{-1}.

The authors also "distill" some universality still present in the system in this regime. Namely, they find that in the high frequency region ($\omega \gg T$) electric conductivity is independent of the intra-dot parameters.

The authors use Keldysh functional integral approach averaged over the disorder. In order to calculate response functions they introduce corresponding source terms in the action. The authors expand the full action of the whole system with source terms and quadratic terms (both inter and intra-dot) (Eq. 7) for finite N around the IR mean field ($N \gg 1$) conformal solution of bare $\mathrm{SYK}_{q\geq 4}$ obtaining the effective theory of the soft Goldstone mode (GM). Considering that the fluctuations of the GM around the mean field are small, authors arrive to the effective action for these fluctuations which is again treated in the $N \gg 1$ limit.

2 Comments to the paper

1. <u>Structural and motivational remark:</u> the paper is an extension to the work (arXiv:1705.00117) that uses SYK_4 chain to describe an actual physical system - strange metal. However, the authors do not provide a physical motivation for such an extension neither in the abstract nor in the introduction. I think it would be beneficial for the structure of the paper and it would increase its value if the authors would emphasise both in the abstract and in the introduction why their extension is physically interesting. Maybe it does one of the following things

   - helps to answer to some physical question that the previous work was not able to
   - is able to add additional physical aspect of the actual system that was not present in the previous toy model therefore making the extended model closer to a real system
   - is able to describe a new type of material or experimentally observed phenomena

   The authors do say in the conclusion section that their work may be used for the description of a real extremely strongly interacting Fermi system (like ^{3}He or cuprate under strong pressure). I think that such a motivation is not only extremely interesting and valuable but also much more clear and has to be mentioned in the introduction and maybe even in abstract which is now purely technical. I also think that it would be great if the authors could elaborate a bit more on why the extension that they added to the system as compared to the previous work (namely intra-dot quadratic coupling) is needed to explain these materials.

2. Page 2: It is unclear what is the connection of the Jakiw-Teitelboim gravitation to the current study. The authors do not provide and conclusions related to it and do not say how their study can give any insights in it. Therefore, it is unclear why it is mentioned in the introduction.

3. Page 2: The constant J is used in the introduction without being defined. The authors can, for example, add "... and the interaction matrix elements are random **with zero mean and finite variance defined by** J" to the corresponding sentence in the first paragraph. It will avoid the confusion that can arise in the third paragraph where the authors refer to J. Same goes for Γ introduced in the same paragraph.

4. Page 4: Authors are repetitive in saying "The first term in the first line represents the Hamiltonian of an individual dot" and "The Hamiltonian $H_{\mathbf{r}}$ describes fermions in each single dot $\mathbf{r}$". The second sentence seems redundant. Same goes for "The dots are labeled by the index $\mathbf{r}$.." and "...with coordinate $\mathbf{r}$ on the ith site inside the dot".

5. Page 4: Do authors use real or complex coupling constants t, J, Γ? Also, if authors do not add hermitean conjugate term to $H_{\mathbf{r}}$, they should impose some symmetry constraints on the constants $J_{ijkl;\mathbf{r}}$ and $\Gamma_{i,j;\mathbf{r}}$.

6. Page 4: It is unclear for me why the authors include symmetrization term $\mathbf{r} \to \mathbf{r} - \delta\mathbf{r}$ in Eq. 3 since $\delta\mathbf{r}$ is defined as " $\delta\mathbf{r}$ is used to label the neighbors of a dot $\mathbf{r}$", so all the neighbours are already included.

7. Page 5: It is a bit confusing when the authors say "we will omit terms with sources" and then do not do it in Eq. 5.

8. Page 5: If there is a chemical potential in the action (Eq. 5), it should be also present in the Hamiltonian.

9. Page 6: Solutions to the SYK_q in IR are well-known therefore Eqns. 11, 12 would benefit from the reference.

10. Page 10: in Eqns. 36, 37 conductivities depend on ω on the l.h.s while on the r.h.s. authors used reparametrized frequency Ω. When not following the paper carefully, it can be confusing for a reader. One suggestion may be to write $\sigma(\mathbf{p}, \omega = 2\pi T\Omega) = ..$

11. Authors conducted a very heavy analytical calculation, namely, they departed from the solvable IR limit for the bare $\mathrm{SYK}_{q\geq 4}$ and considered both quadratic terms and finite time corrections as perturbations to derive the effecting low energy theory around that point. Would that be possible to start from the action (7), derive mean-field Kadanoff-Baym equations and solve those numerically using iterative method? If yes, it would be very useful since the authors would not only be able to see that they did not make any mistake in their involved analytics, but also would be able to make sure that the fluctuations of the soft mode captures everything.

3 Typos

1. Page 2: the name is not **Jakiw** but **Jackiw**.

2. Page 2: "...basic energy scale J **leaves** intact" $\to$ "...basic energy scale J **leave** intact"

3. Page 2: "As a byproduct, we **demonstrate**" $\to$ "As a byproduct, we **demonstrated**"

4. Appendix A, page 15: "Particular, we have not **wrote** action with sources to simplify formulas." $\to$ "Particularly, we have not **written the** action with sources to simplify formulas."

4 Conclusion

I would recommend that the paper is accepted after the minor revisions according to the comments presented in this review.