The role of fluctuations in quantum and classical time crystals

by Toni L. Heugel, Alexander Eichler, R. Chitra, Oded Zilberberg

The paper discusses time crystals based on coupled parametric oscillators. The problem of time crystals in periodically driven systems of oscillators has been attracting much attention, both on the classical and quantum sides. There is some confusion in the literature, which makes the paper timely, besides the importance of the topic. The paper emphasizes the similarity between the classical and quantum dynamics in the presence of dissipation and fluctuations that invariably come along with dissipation. It is important that the effective temperature of quantum fluctuations of driven oscillators is generically nonzero even where the temperature of the thermal bath responsible for the dissipation is zero. One of the questions is whether a dissipative system can display broken time-translation symmetry. The authors analyze finite-size systems and indicate that, in this case, the answer is “no”: fluctuations ultimately lead to symmetry breaking.

The authors make an important and valuable point about the distinction of strongly and weakly coupled oscillators. Indeed, if the coupling is very strong, like the atomic coupling in a nanoresonator, parametrically excited vibrations display a long-lasting symmetry breaking where all atoms vibrate in phase. For weaker coupling there are regimes with different numbers of coexisting stable states. The analysis here is nice and very clear. Fluctuations in the regime with multiple coexisting stable states is a central point of the theoretical analysis and of the experiment. In particular, the experimental results show nonzero populations of states with different vibrational phases and amplitudes, which indicates breaking of the time crystal phase in the stationary regime. I also like the observations of the fluctuation spectra.

I have several comments.

I would downplay references to prethermalization. The term is used in the meaning that is somewhat different from what is frequently implied when referring to prethermalization. In particular, in paragraph 2 below Eq.(8), it is not clear what is meant by the prethermal regime. It is a cosmetic change, but worth clarifying.

Two paragraphs below Eq.(8) the reference to “suppression of activation” could be read as if in the classical regime activation is suppressed, whereas it is meant that activation is suppressed in the regime of large vibration amplitudes. This refers also to the statement “classical systems have diverging prethermal timescales, making them superior to their quantum counterparts as DTCs.”: what is meant here, I think, is large vibration amplitudes rather than classical fluctuations, but the text is somewhat ambiguous.

I would word more carefully the discussion of the possibility of a time crystal in the coherent regime for a closed system in the thermodynamic limit. A quantum phase transition to the corresponding state has been described in the literature. I believe the authors mean that their condition (ii) does not hold, since the concept of “stability” does not apply in a coherent regime.

Another term to straighten out is “fluctuations forming normal modes”: the authors are talking about fluctuations of normal modes, whereas the modes themselves are formed dynamically.

It would be good to explain the advantageous feature of studying fluctuations of symmetric and antisymmetric combinations of the displacements from the stable states of two oscillators rather than studying such fluctuations for each oscillator separately.

The paper would benefit if a more careful comparison with the previous work was done. In particular, the switching between coexisting vibrational states is well-understood for a single oscillator and also for weakly coupled multiple oscillators in the quantum and classical regime, see Ref. 11 and papers cited therein. I would also compare Eq. (7) with Eq.(11) in this paper. The problem of the spectra for single oscillators was addressed in PRA 83, 052115 (2011).

To conclude: the paper addresses important questions in the area of active research. It also poses new nontrivial questions, like the scaling of the lifetime of the broken-symmetry state with the size of the system, and thus it suggests new directions of research. I recommend publishing the paper after the comments above will have been considered.