The authors present an interesting study on the crossover from 2D to 1D that the system of strongly interacting bosons undergoes depending on the strength of applied continuous periodic potential in one direction and the temperature. The results are obtained using the state-of-the-art worm algorithm path-integral quantum Monte Carlo. The manuscript is clearly written and the results are sound, with direct implication to experiments. I therefore recommend its publication in SciPost Physics, and have following questions.

1.) Why is there no complex conjugate term present in Eq. 5 for the tunneling part (Eq. 6)?

2.) In the phase diagram (Fig. 1), the thermal regime is identified by the superfluid fractions being below 0.1%. I am wondering if this can be corroborated by the decay of the correlation function (which in 2D changes from power-law to exponential in the thermal regime)? One might also expect a characteristic change in 1D.

Minor remarks:

1.) K is the Luttinger parameter, stated on page 9, which instead should be stated on page 6, where it is first mentioned.

2.) In Appendix A.1, the parameter \Lambda is not defined.

3.) In Appendix A.2, there is typo 'To me more specific..'

4.) In Fig. 5 caption, instead of Eq. 2, it should be Eq. 7.

5.) In Fig. 7, figure numbering is missing for the panels.

6.) Update Ref. [60] to the published one.