Report

This paper is relevant for the community working on supersymmetric Wilson loops and, in particular, explores the construction of such operators in 4d to include not only scalars, but also fermions. As mentioned in the introduction, it goes along recent developments in the 3 dimensional setting and extends is to 4 dimensional theories.

To my knowledge it is the first time that fermionic operators are considered in 4d and I believe this is not sufficiently highlighted. Even though the authors do not make any particular application of the operators derived, I believe it is important to stress that their nature is different from the ones previously considered in the 4d literature and this could possibly open new venues to explore. This is outlined in the Conclusions, but I would suggest mentioning it also in the Abstract and the Introduction.

I would suggest an overall revision of the text, aiming at filling in the current step-by-step structure with comments and physical interpretations whenever possible. This would make the content more fluid and consequently more appealing to the reader. I also find typos, which is also an indicative that the text needs revision, and list them below.

Particular comments for each section are listed separately.

Abstract

Typos

- "the connections of these fermionic BPS Wilson **loops** have a supermatrix structure";
- "and **circular** BPS Wilson loops in Euclidean space".

Introduction

- Footnote in page 3: WLs preserving only Poincare supercharges appear in 3d only if matter has non-canonical dimensions, right? Why not mention that?

- To a casual reader it is not clear what is meant by "we should employ a supertrace in the old approach". It is left implicit that there is a more recent approach and you should mention it. May be worth commenting about the definition in terms of a supertrace at the price of adding a twist matrix [arXiv:1209.4032] versus supertrace at the price of adding constant shifts, originally proposed in Chapter 2 of [arXiv:1910.00588].

- There is a clear intention of explaining the state of the art in 3d CSM theories, which is contained in the fourth paragraph. Then the fifth paragraph is intended to consider 4d superconformal gauge theories. However, it ends up going back and forth to 3d. To a casual reader this may lead to confusion so I would suggest introducing everything concerning 3d properly, either shortening the fourth paragraph and moving the 3d information from paragraph five to it, or introducing a new paragraph about 3d between the fourth and fifth. I particularly find the part concerning 3d too long given the fact that it is not the type of theory considered in the paper. However you choose to proceed, the idea would be to make each part, 3d and 4d, self-contained.

- I believe you should mention that your construction is based on the idea of deforming bosonic loops to obtain fermionic ones. This is glimpsed in the beginning of Section 2.2 but I believe should be properly stressed in the Introduction, in particular, mentioning the fact that it was originally proposed in Chapter 2 of [arXiv:1910.00588] (later explored in [arXiv:2004.11393, arXiv:2012.07096]).

Typos

- "earlier days of this **holographic** duality";

- "General BPS Wilson loops in $\mathcal{N} \geq 2$ super-Chern-Simons-matter theories **were** constructed";

- "straight line are not **scale** invariant";

- "involving **multiple** copies has been employed";

Section 2

Section 2.1

Typos

- "as we **have** just mentioned";
- "The **definitions** of the covariant derivatives are";
- "invariant under the $\mathcal{N} = 2$ superconformal **transformations**";

Section 2.2

Typos

- "timelike infinite straight line ~~straight line~~";
- "~~persevered~~ **preserved** supersymmetries";
- eq. (17) isn't charge conjugation of ζ missing?
- above eq. (29) "$Q_u L = \mathcal{D}_{\mathbf{0}} G_u$";

Section 2.3

- While solving the form of L you do not mention how the bosonic part B is fixed. Even if the process is the same as in Section 2.2, you should mention how the G_s^2 piece is obtained in (48).
- I am sorry, but I could not follow the supersymmetry enhancement conclusions. By construction all loops are at least 1/16 BPS and preserve Q_s. Then what confuses me in each of the bullet points is the following
  - susy enhancement to 3/16 would mean that there are two different Q_u's that are also preserved, but you only list one in (54)?

  - susy enhancement to 1/8 would mean one extra Q_u in addition to Q_s, then why do you refer to (55) in the plural? Isn't it a single supercharge?
  - what is the extra Q_u in this case?
  - maybe restate that 1/16 BPS means that only Q_s preserved?

  Overall, why there are no explicit realizations of each case to clarify the results?

- Maybe above (44) you meant "QF takes the form" and not δF, since G_s is in principle a bosonic matrix.

Typos

- In eq. (47) there is a r factor floating around in the second equality;

Conclusion

What would be the consequences of scale invariance breaking due to fermions in the WL? Any words about implications? For instance, a dCFT scenario would not be possible, right?

Appendix B

Appendices B.1 and B.2 appear in the body of the paper in reverse ordering. Why not present B.2 before B.1?