Report on "Multi species asymmetric simple exclusion process with impurity activated defects"

The paper considers a multi-species variant of the well-studied totally asymmetric exclusion process, on a periodic lattice. The model contains 'defect' particles which activate switches of species when particles become adjacent to them. The key finding is that the model admits a matrix product solution of the stationary state. This adds to the range of models that have matrix product solutions.

Although the model is of intrinsic interest, due to its matrix product stationary state, various further motivations for the model are given in the introduction. I find the mapping to a multi-lane TASEP of most interest.

The proof of the matrix product solution follows the usual construction outlined in equations (4)-(6). The auxiliary 'tilde' matrices turn out to be scalars which is a crucial simplification.

It is noted that the model is non-ergodic, meaning that the configuration space breaks up into different sectors corresponding to different intial conditions and for each sector of initial conditions the partition function has to be computed separately. The partition function is computed for a special class of initial conditions and observables such as density profiles and currents are computed using the grand canonical ensemble in the large system limit.

I find the results interesting and worthy of eventual publication in SciPost. First, the authors should consider the following points.

1) The first matrix product solution for a two species ASEP was given in Derrida, B., Janowsky, S.A., Lebowitz, J.L. , Speer E.R.. Exact solution of the totally asymmetric simple exclusion process: Shock profiles. J Stat Phys 73, 813–842 (1993). https://doi.org/10.1007/BF01052811. (I will refer to this paper as [DJLS] below.)

The work [DJLS] should certainly be cited. It also showed how the stationary state factorises about the defect (a 'second-class' particle in that case) which implies a projector form for the matrix A, the same as in equations (10) and (13) of the present work. This factorisation property due to the projector form of A should be acknowledged in the current paper.

2) I am not sure I understand the last sentence of Section 2 'However, we should mention that.. this is not the general exression for α ..'. Does this mean that generally α would appear as α_I in equation (15)? Perhaps this point can be clarified.

3) Second sentence of Section 3. I would put transfer matrix in inverted commas as this is not the same as a usual equilibrium transfer matrix. i.e. 'Here the "transfer matrix"

T refers to ..'

4) In section 3 the grand canonical ensemble is used , which results in a fugacity z_0 which is fixed by the density. It would be helpful to have the solution for z_0 appear somewhere, perhaps in an appendix if it is really very complicated. Does the solution for z_0 simplify in some limits?

5) Section 5.4 considers Negative Differential Mobility (NDM). Some references and discussion of specific, related models, exhibiting NDM would be appropriate e.g.

Cividini J, Mukamel D and Posch H A "Driven tracer with absolute negative mobility" (2018) J. Phys. A: Math. Theor. 51 085001

6) Some typos: p.3 paragraph 2 'with variety' → 'with a variety'
p.4. paragraph 2 'For specific choice of' → 'For a specific choice of'
p.5 last line 'itlaics' → 'italics'
p.7 after equation (10) 'resembles to that of the defect or second class' → 'resembles that of the defect of second class'. Here reference [DJLS] (see point 1. above) should be cited.
p.36 reference [59] author is J. Szavits-Nossan