

## Review result

In the manuscript, the authors thoroughly examined the Lie group symmetries and their gauge in 2d topologically ordered phases. The symmetry gauging for both the finite group case and the  $U(1)$  case has been systematically investigated. In this study, the exploration of the more general scenario in which the symmetry group is a Lie group was provided.

The gauging process are divided into two steps: the first step is to using the Lie group extension of  $G$  ( $G$  is the extension of  $K$  by  $G$ ) to obtain a bigger Lie group  $G$  and gauge the  $G$  by coupling the topological phase  $\mathcal{C}$  to a one-form  $G$  gauge field. Then in the second step, the resulting phase  $\mathcal{C} \boxtimes G_{-\sigma_H}$  in the first step has a  $K$  symmetry, this  $K$  symmetry can be gauged by anyon condensation.

The authors thoroughly examined the gauging process by considering various examples and both algebraic approach and field theoretic approach are carefully examined.

I think the results are very interesting and of great importance, the paper is well-organized. **I recommend the publication of the manuscript in SciPost Physics after some minor revisions.**

I think that it would be helpful if the authors can add an appendix to discuss the mathematical theory of Lie group symmetry of a TQFT and their gauging in a more rigorous way, including their obstructions, their algebraic structure, etc.

There are also some typos and incorrect statements that should be corrected

- Page 3, line 2 of Sec II. Paragraph 2, " $G$  action on  $\mathcal{C}$  are" should be " $G$  action on  $\mathcal{C}$  is"
- Page 4, line 6, "for the a stand-alone" should be "for the stand-alone".
- Page 4, line 9, "have not be systematically" should be "have not been systematically".
- Page 4, at the end of Paragraph 2, "Detailed example" should be "Detailed examples".
- Page 4, line 2 of Sec II. A., "character" should be "characterize".
- Page 4, line 4 of Sec II. A. Paragraph 3, "carried" should be "carries".
- Page 5, line 2 of Sec. II. B. Paragraph 1, "fractionalization" should be "fractionalization".

- Page 5, line 4 of Sec. II. B. Paragraph 1, "the exact same" should be "the same" or "exactly the same".
- Page 5, line 3 of Sec. II. C., "In first step" should be "In the first step".
- Page 7, line 3 after Eq. (11), "corresponds" should be "corresponding".
- Page 7, the line after Eq. (12), "in" should be "is".
- Page 7, line 2 after Eq. (12), "form  $K = \mathbb{Z}_2$ " should be "from  $K = \mathbb{Z}_2$ ".
- Page 8, line 5 in Paragraph 2, "a electric" should be "an electric".
- Page 9, line after Eq. (16), "will explained" should be "will be explained".
- Page 12, line -3 before Sec. II. K., "Similar with" should be "Similar to".
- Page 13, line 2, "in in Table" should be "in the Table". And "these is" should be "there is".
- Page 14, line -2 before Sec III. E., "there is an 't Hooft anomaly present this system" should be "there is an 't Hooft anomaly present in this system".
- Page 16, line 1 after Eq. (48), "respects" should be "respect".
- Page 17, line 2, "global ymmetry" should be "global symmetry".
- Page 17, footnote [41], "flows" should be "flow".
- Page 25, line 1 after Eq. (102), "paramterizations" should be "parameterizations".
- Page 26, line 1 after Eq. (107), "Here, we've have" should be "Here, we've". And "short hand" should be "shorthand".
- Page 27, line 5 after Eq. (114), "will be be" should be "will be".
- Page 27, line 1 after Eq. (115), "a the CS term" should be "a CS term".
- Page 31, line 2 of Appendix A, "obey Abelian fusion rule" should be "obey the Abelian fusion rule".
- Page 32, line -1 before Eq. (B3), "They satisfies" should be "They satisfy".
- Page 33, line 1 after Eq. (C1), " $\sum_a$  represent" should be " $\sum_a$  represents".