Dear Editor,

Thank you for the assignment to review the paper entitled "Non-standard neutrino spectra from annihilating neutralino dark matter" submitted by Melissa van Beekveld, Wim Beenakker, Sascha Caron, Jochem Kip, Roberto Ruiz de Austri and Zhongyi Zhang.

The manuscript concerns the phenomenology of neutralino dark matter in an extended version of the MSSM. This extended version encompasses the inverse seesaw mechanism that gives rise to neutrino masses and a $B-L$ symmetry, leading to the so-called BLSSMIS model. The sterile neutrino superfields needed for the inverse seesaw change both the neutrino and the sneutrino sector, while the new Z' boson and the Higgs doublet needed to give it mass enlarge the gauge boson, Higgs, chargino and neutralino sectors. The authors discuss the phenomenology of the neutralino as dark matter concerning the relic density constraints, the direction bounds and show the indirect detection thermally averaged annihilation cross-sections. The neutralino is shown to be a good dark matter candidate when is it has a large component of gaugino or higgsino coming from the $B-L$ extension. However this is not surprising as the model parameter space is so large that it can accommodate a very wide range of neutralino compositions which escape detection. The main feature of the BLSSMIS model is an unconventional neutrino energy spectrum, which is composed by a sharp line plus a separate plateau. This is interesting, however this feature is only presented at the end of the paper within a separate section and shows a couple of benchmark points, but is not exploited in the analysis in its full potential. This peculiar spectrum is the main point of the paper while reading the abstract, however when it comes to the analysis this is not the case.

This paper lacks off a proper analysis of the neutrino signals and just shows some conventional indirect detection bounds, which are not particularly suitable for the model itself, hence I believe it does not meet the SciPost standards for publications. I would reconsider reviewing this paper and reconsider it for publication after the issues described below would have been properly addressed.

Strengths

- The neutrino energy spectrum coming from neutralino annihilation is peculiar and can be an interesting study for neutrino telescopes, such as ICeCube, KM3NeT, etc.

Weaknesses

- The neutrino energy spectrum is very peculiar and can probably provide interesting bounds from current neutrino telescopes and the sensitivity to such signal can be enhanced in future telescopes such as KM3NeT. The authors only show the precomputed bound from IceCube for annihilation into $\mu^+\mu^-$ as reference case however this is not an interesting result. First, all annihilation cross-sections are very low, hence it would be difficult to probe the model parameter space via indirect detection. Secondly $\mu^-\mu^+$ produces a continuum of neutrino in the energy spectrum without any particular spectral shape. It would be interesting conversely to study the sensitivity of IceCube and KM3NeT to the specific neutrino signal of the BLSSMIS via a likelihood analysis for example. Or at least show the exclusion bound of neutrino telescopes in terms of neutrino final states, which are more sensitive than those into charged leptons because of the monochromatic neutrino line. There have been many studies in the near past that provide such bounds, see e.g. arXiv:1912.09486.

- An error of 0.03 on Ωh^2 is very large, values of the order of 0.15 have been ruled out many years ago by cosmological observations. Couldn't this error be reduced?
- The authors mention what are the bounds of the scan but do not give any further informations about how many points have been considered and how the scans have been performed. This is however very relevant when it comes to such a large number of free parameters, to be sure to have explored all the available parameter space.
- The introduction is very (maybe too) concise and probably do not consider all the relevant literature for supersymmetric inverse seesaw models, where neutralino or sneutrino are the dark matter candidates. I also find that a proper citation to the inverse seesaw mechanism is in order, at least to the seminal papers.

Requested changes

- It would relevant to show the bounds from current neutrino telescopes using the spectral shape information, as discussed in the first item of the weaknesses;
- There is an additional channel from neutralino annihilation that provides neutrino lines, the annihilation via t-channel exchange of sleptons. How is this channel performing in this model?
- Add more information on how the sampling of the model parameter space is performed;
- In figure 5 the $< \sigma v >$ displayed is the total one or only the fraction that goes into $b\bar{b}$ and/or $\mu^- \mu^+$?
- I am under the impression that to have a successful neutralino as dark matter it is not necessary to have as free parameters all the gaugino masses but splitting only M_{BL} would be enough. This would in my opinion single out more the feature of the new fields with respect to a standard MSSM; Also it is not clear to me what is the impact of changing μ_S rather than keeping it small as required but fixed to one value, to limit the number of free variables in the scan;
- I would like to see the impact of the LZ recent exclusion bound concerning direct detection;
- Page 5: why the radiative corrections on μ_S are small?

- Page 5: the authors mention that potential dark matter candidates can be the heavy neutrino or the sneutrino. They argue why neutrinos are not suitable dark matter after all, however they do not comment about the sneutrino.

With Best Regards