Review of "Composing topological domain walls and anyon mobility'' by Peter Huston, Fiona Burnell, Corey Jones and David Penneys

This work contains some interesting results:

1. A construction of the boundaries of Walker-Wang models based on $\mathcal{A}$-enriched fusion categories, together with a dictionary between physical notions and mathematical notions, generalizing an old dictionary in Levin-Wen models.

2. Generalizing an earlier result in [KK12] (including only non-chiral cases), the authors show that computing the fusion of two parallel domain walls among 2+1D topological orders (including all chiral/non-chiral cases) amounts to compute the relative tensor product of bimodule categories (over certain enriched fusion categories). How to compute this relative tensor product is a highly non-trivial and important question. These authors developed necessary mathematical tools to compute it. The main result is summarized in Theorem III.22.

I think that the result is new, interesting and important in many physical applications. In particular, it provides a systematic tool to compute the fusion rules of non-invertible symmetries in 2+1D.

The paper is very well written. So I recommend it for the publication in SciPost under the conditions that the following minor problems are taken care.

1. The notation "2D bulk" in Fig. 1 is confusing. This "2D" must be space dimension, right? but 2+1D is clearly a spacetime dimension. Maybe a declaration of the convention is needed.

2. Page 2, the mathematical term "Morita 4-category UBFC of unitary braided fusion categories" appears suddenly. I think that it is better to explain its definition. Otherwise, I do not see why "objects in individual UMTCs, do not appear as higher morphisms in the 4-category UBFC."

3. Page 2, the notion of a module tensor category over a braided tensor category also appeared in Definition 2.6.1 in [KZ18a].

4. Page 7, what does the term "Ω tensor" mean? Is it the Ω in Eq. (3)? If so, it is better to refer to (3) here.

5. Page 8, is $\mathcal{C}$ in Remark II.5 a typo?

6. Page 11, "also that Witt equivalent UMTCs can appear as surface topological orders of the same invertible bulk." I have a question here. Is that true that Witt inequivalent UMTCs can not realized as surface topological

orders the same invertible bulk? If so, the invertible 3+1d TQFT are classified by the Witt group of UMTC, right? Two surface topological orders associated to two Witt inequivalent UMTCs can be connected by 1+1D gapless domain wall. Does this gapless wall produces a 2+1D 'relative bulk' in 3+1D?

7. Page 12. is the "UBFC" in Definition II.7 a typo for UMTC? Appendix B only discuss UMTC.

8. Page 13, the "$\mathcal{M}$" at the bottom of the left column should be a typo.

9. Page 14, $\overline{A}$ in the footnote is a typo.

10. Page 15, "we now turn to the question of central interest: namely, what can happen when we compose two or more domain walls by stacking them? This question is very important, since any domain wall can be obtained by such compositions." The reason provided here does not sound very convincing. In addition to the reasons from condensed matter physics, maybe the authors can also add that computing the "fusion rules" of topological defects in various dimensions is becoming a fashion especially after the new wave of studying the non-invertible symmetries from the high energy community.

11. Page 20, I am not sure if the notation $(\mathcal{M} \boxtimes \mathcal{N})_{S_y}$ has been explained anywhere in the paper. Is it the category of left or right S_y-modules in $(\mathcal{M} \boxtimes \mathcal{N})$? If so, this notion is not so obvious to physics oriented journal. Maybe an explanation is needed.

12. Page 20, "In this alternative definition, a y string emanating from the left (X,Y) boundary cannot end in the bulk, and must cross to the right (Y, Z) boundary." This sentence is hard to visualize without a picture.

13. Page 22, Example III.12 and more examples can be found in Table 1 in arXiv:2205.05565.

14. Page 41-42, some mathematical details in the discussion of double Ising might have appeared in arXiv:1903.12334.