Comments on the manuscript at SciPost

The manuscript discusses the issue of defining multipole moment in arbitrary spacetime. The paper is interesting, but there are a few serious issues, which need to be addressed. I elaborate on these issues below.

- The definition of the improvement twist vector $\omega^{\rm I}_\mu$ is arbitrary. In the context of Einstein-Maxwell theory, neither ρ, nor $\tilde{\rho}$ are unique, since they can differ considerably by gauge transformations. In particular, one of them can be set to zero by an appropriate gauge transformation, in which case the improvement twist vector vanishes. Then the twist vector is not at all well-defined. The same applies to other situations well. For example, in the context of supergravity as well, such gauge choices exist and they can affect the multipole moments. In the general case, the improvement twist vector is defined by using the result that $dA = 0$, implies $A = dB$. Again the choice of B is not unique, since $B + dC$ will also satisfy the same equation. Thus the definition of multipole moment, using the potential B, is not unique. However, the multipole moment is a unique quantity (since, it is an observable) and therefore, cannot depend on the gauge choice. This needs to be addressed, since all the conclusions are based on the existence of improvement twist vector.

- The analysis only holds for asymptotically flat spacetime, for which the integral of W over a two-sphere at infinity vanishes. However, there are ways to generalize the multipole moments to asymptotically dS spacetimes, using the Noether charge formalism (Ref. [10]). This needs to be mentioned in the manuscript.

- The Noether charge formalism holds in the non-stationary spacetimes as well. Therefore, rather than comparing the results with Geroch-Hansen or, the Thorne's formalism, for which the improvement twist vector identically vanishes, the author should also compare with the Noether charge formalism (Ref. [10]). Further, there have been earlier attempts to define the multipole moments of generic spacetime, see e.g., 1008.1278. Possible connection of the present paper with that of the reference mentioned above will be very helpful.

- Finally, the author has not provided any example of the formalism presented in the manuscript. For example, the author can consider the Kerr-Vaidya spacetime, which does not fall under the purview of the Geroch-Hansen or, the Thorne formalism, in general. But when the mass function becomes constant, it does reduce to the Kerr spacetime. Therefore, it would be interesting to see how the multipole moments for Kerr-Vaidya spacetime reduces to that of the Kerr spacetime under appropriate limits. Any other example would also work, e.g., with a non-trivial scalar field. In brief, a scenario, where multipole moment cannot be determined using earlier formalism, but can be derived using the formalism presented in the manuscript would be very helpful.

After the authors have addressed these queries, I will be happy to reconsider the manuscript for publication.