

”Volume-to-Area Law Entanglement Transition in a non-Hermitian Free Fermionic Chain” by Youenn Le Gal, Xhek Turkeshi, Marco Schirò

While many studies on entanglement entropies in non-Hermitian systems have been carried out, and in particular for the Su-Schrieffer-Heeger model, the emphasis of this paper is original. The authors compare the transition from a volume to an area scaling law in comparison to the transition from the PT-symmetric to the spontaneously broken PT-symmetry regime. The overall conclusion is that the transitions are not directly related as the volume to an area scaling law takes place deep inside the broken PT-symmetry regime. In fact the transition occurs precisely at the onset of the gapless spectrum. The authors speculate that only when all the short lived quasi-particles are present the transition occurs. This is an interesting results that inspires further investigations to clarify the universality of this observation.

The presentation is in general clear, but I have a few, mostly minor, points the authors should address:

- I think it is important to properly define the norm used in equation (3). In the PT-symmetric regime one would expect the norm to be conserved in contrast to the general statement made after this equation. Is this not the case?
- Equation (4) is not what is usually referred to as the nonlinear Schrödinger equation. The authors should also say in which sense this equation is meant to be nonlinear.
- “Schrödinger equation” appears in all kinds of variants throughout the text, mostly as Schrodinger equation, but also as Schrödinger equation. This should be fixed.
- The labelling between letters in figures and the text should be made consistent, e.g. in figure 1 the authors use ϕ whereas in equation (8) the letter Φ is used.
- In figures 2 and 3 some circles, triangles and diamond appear that have no use.
- There should be a comment on how the contour in equation (16) is to be understood.
- The references need some tidying up, for instance [55] and [73] are identical.

- The following reference seems to be relevant: Ali, T., Bhattacharyya, A., Haque, S. S., Kim, E. H., Moynihan, N. (2020). Post-quench evolution of complexity and entanglement in a topological system. *Physics Letters B*, 811, 135919.

After these issues have been addressed properly I would like to recommend the manuscript for publication.