Universal geometry of two-neutron halos and Borromean Efimov states close to dissociation

The author addresses the problem of the geometry of Borromean trimer states consisting of a nucleus core interacting with two neutrons, the two latter being close and away from resonance. Employing the Faddeev equations, he rederives analytical expressions for the matter and core mean-square radii close to the three-body dissociation threshold, when the two neutrons are resonantly interacting.

Subsequently, the author checks numerically the analytical expressions for the first three trimer states, allowing for variations of the neutron-neutron scattering length at different (finite) values of the neutron-core scattering length. When the neutron-neutron scattering length is such that the system is close to the three-body dissociation threshold, very good agreement is found between the analytical expressions and numerical computations. When the neutron-neutron interaction is resonant, the agreement is good only for values of the neutron-core scattering length such that the system is close to the three-body dissociation threshold. This agreement applies not only to the ground trimer state, but also to excited ones.

The hyperspherical formalism is finally employed to explain the universality of the geometry for all trimer states, stemming from the decoupling of hyperradial and hyperangular degrees of freedom.

The three-body calculations carried out in this work provide more insights regarding the universal geometry of these systems, treated in a previous publication. There, analytical expressions were derived within non-relativistic effective field theory for the radii, in the case of resonant neutron-neutron scattering close to the three-body dissociation threshold. I recommend revisions to clarify certain points that are confusing and to improve the structure of the paper, so that the distinction with the previous work becomes clearer. Below, there is the list of these points:

1. The author refers in the title to Borromean Efimov states. To my understanding Efimov states are always Borromean, so this statement seems to be redundant. Is there some particular distinction for Borromean Efimov states ?

2. To my understanding, the analytical relations found in the work based on effective field theory [Ref. 24 in the manuscript] apply only to the ground trimer state. If that is the case, I would suggest to include this statement at the last sentence of the first paragraph of the Introduction.

3. It would help the reader if the range of the interactions Lambda was introduced in Equation (1), instead of introducing it later in the text [Section 4].

4. Regarding the three pairwise interactions, the author assumes later in the text (beginning of chapter 5) that Lambda is the same both for neutron-neutron and neutron-core interactions. Are the two interactions of the same form, or is there some reason why the range of interactions can be treated to be the same ?

5. Right before Eq. (10), where the Jacobi vectors Rij,k are introduced, it would be good if there was a reference to Fig. 1.

6. How is the low-energy expression for the T-matrix elements derived ? If the steps have been already carried out in a different publication, I suggest to cite these works.

7. I think there is a typo in the inline formula after Eq. (13). I think it should read, $\sqrt{2 \mu_{12} z_{12}/ \hbar^2} << a^{-1}_{12}<< a^{-1}_{23},a^{-1}_{31}$.

8. The author argues before Eq. (14) that the Faddeev component associated to the neutron-neutron dimer is the dominant one. Is that the case due to its dependence on the inverse square root of the two-body neutron-neutron energy, which is assumed to be very small ?

9. Do the results in Section 4 apply only to the ground state ?

10. In Section 4, the author distinguishes between two limiting cases for the matter over core mean-square radii, providing results depending solely on A, the mass ratio. How do these limits affect the geometry of the three-body system, and what are the imposed relations on the Jacobi vectors Rij,k ?

11. Why does the author consider only the case A=10 as a mass ratio between the core and the neutrons ? Does it refer to a particular system, or is it a prototype system, and larger or smaller mass ratios lead to the same phenomenology ?

12. In Fig. 2, at any fixed inverse neutron-neutron scattering length, an infinity of trimer states appears due to the Efimov effect, as the neutron-core length is tuned to larger values. Why only five states appear in the leftmost corner of both panels ? Is it due to numerical difficulties as the energy of the states becomes smaller and smaller, or does the scaling factor become very large ?

13. Is it true that for highly excited trimers [upper panel and leftmost part of Fig. 2], the scaling factor for the dissociation thresholds becomes the same as for three identical particles ?

14. In the caption of Fig. 3, the last sentence should refer to $|a^{-1}_{12}|$ and not $|a^{-1}_{23}|$?

15. In Fig. 4, the good agreement with the analytic formula for beta go to zero, applies only for neutron-core scattering lengths such that the system is close to the three-body dissociation threshold. The author states “Therefore, it appears that the analytical formula does require a fine tuning of the core-particle interaction”. In PRL 128, 212501 (2022) however, it seems that the formula

applies only close to the three-body dissociation threshold and neutron-neutron two-body resonance. Where does the fine tuning come from ? It appears that in the current work, the author tests the range of applicability of the formula derived in PRL 128, 212501 (2022), providing bounds for its validity.

16. It would be less confusing if the horizontal axis in Fig. 4, was just the neutron-neutron scattering length. My understanding is that both the a12 scattering length as well as the binding energy |E| vary as one follows the horizontal dashed lines sketched in Fig. 3. Is therefore the matter-to-core radii depicted with respect to beta out of convenience ? So that a comparison with the analytical formulas is straightforward ?

17. Do the curves for |a23| away from the three-body threshold come closer to the 2*A/3 value in Fig. 4, when one considers smaller values of beta (10^-7) ? Or do they already saturate as suggested by the presented values ?

18. How does the choice of separable interactions or the gaussian form factors affect the limit of beta go to zero in the numerical calculations presented in Fig. 4 ? Are there small deviations if one chooses other form factors ?

19. Regarding Eqs. (23) and (24) for the hypespherical formalism, I suggest to provide a few references regarding the method.

20. I suggest to move Section 6 at the beginning, after the introduction of the model. In that regard, a neat explanation is provided for the universal relations of the radii for the ground state, along with an extension to excited states. I think it would be better to present these results directly, to make the distinction with PRL 128, 212501 (2022) more clear.

21. The results obtained in Section 6 apply close to the zero energy three-body threshold ? If yes, it would be helpful for the reader to state that explicitly.

22. The sentence at the end of Section 6 is a bit confusing. From the results obtained within the hyperspherical formalism, it is shown that the limiting cases of the mean-square radii apply to all states. To which lack of Efimov universality does the author refer to ?