

Second report on "Precision magnetometry exploiting excited state quantum phase transitions", by Q.Wang and U.Marzolino

The authors have made some changes to the manuscript, amending some of the claims which I considered problematic, and improved the overall clarity of the paper. There are, however, some points which I still considered unresolved, and that need addressed before I can recommend this paper for publication.

The main problem, to me, is still the claim that the super-extensive scaling is a property of the ESQPT. As I argued, although the peak that we observe in the QFI is certainly a signature of a ESQPT, the fact that the maximum value $\mathcal{F}_{h,m}$ scales like $O(N^2)$ is not. To resolve this point, I believe necessary to re-plot Fig.4, by showing $\frac{\mathcal{F}_{h,m}}{N^2}$ instead of $\mathcal{F}_{h,m}$. If, as I believe, the curves so obtained all collapse on each other for different N , then we can conclude that the super-extensive scaling does *not* come from the ESQPT; if not, some of this scaling may indeed be a signature of the ESQPT. I would therefore need to see such a plot and discussion before going forward with publication.

Depending on this result, some important claims in the manuscript will still need to be changed. For instance, the sentence " \mathcal{F}_h exhibits a sharp peak close to the critical energy E_c , and its maximum value... increases with the system size N " would need to be replaced by something like: " \mathcal{F}_h/N^2 a sharp peak close to the critical energy E_c , which is a signature of the ESQPT" (dropping the reference to the scaling with the system size). The rewritten claim "We suggest that the superextensive peaks of the QFI... are a signature of the ESQPT" should be dropped, or at least the mention to superextensivity removed. To summarize: the claim that the ESQPT gives an improvement, described by the peak in the QFI, is, I believe correct; but not the claim that the superextensive scaling is related to the ESQPT.

To be clear, I believe this work is certainly worthy of publication; however, I think there is some serious confusion about the role played by the ESQPT, which can and must be lifted now.