

Referee's comments on  
on the submission to SciPost Physics Lecture Notes by  
Saiei-Jaeyeong Matsubara-Heo, Sebastian Mizera, Simon Telen  
titled "Four Lectures on Euler Integrals"

This is an excellent review article which includes a number of results and perspectives which -- in my view -- are highly original. The topic is the study of Euler integrals, i.e. integrals over powers of products of polynomials. Here the exponents can be allowed to vary over complex numbers, and also the coefficients of the polynomials might vary. For fixed integral cycle (or twisted cycle), one thus obtains a function of those exponents or coefficients, and the aim is to represent and evaluate such a function as efficiently and explicitly as possible. This problem arises in many parts of physics, notably in the study of Feynman integrals and scattering amplitudes, but also in cosmology and string theory, and in many other settings (e.g. Bayesian integrals in statistics).

The article is concise and well-written, and it will be very useful for both students and experienced researchers. There are many well-chosen examples. With the possible exception of Chapter 3, the text is very accessible for beginners. It shows that one of the authors used this material in the teaching of a graduate class, and that he incorporated feedback from attendees.

I tried hard to look for typos and other bugs, but found almost none. Here are a few comments which the authors might consider when preparing their final version:

- page 2, line 18: "Bayesian statistics". Maybe give a reference for this?
- page 3, line -6: "Following [8]". This is a rather long paper. Can you please give a specific reference to where in [8] this is found.
- page 6, line -5: " $-1, P_{\pi}$ ". It would be clearer to insert the word "then".  
In fact, general it is best to use "then" in an "If ..., then ...." sentence.
- page 6, line -1 : Perhaps replace  $\subset$  by  $\subseteq$ , since the case of equality is particularly important here?
- page 8, line 3: Also "If ..., then ..." here please
- page 9, line -3: In the evaluation of the integral, I was initially a bit confused. Perhaps replace the upper and lower margins "0" and " $\infty$ " by " $z_i = 0$ " and " $z_i = \infty$ " for extra clarity.
- page 13: One question about "field theory limit" and "high energy limit":  
Is this the same as what some physicists call "infrared" and "ultraviolet"?
- page 14, line -9: Need " $= 0$ " before "consists of"
- page 22: Section 3 would benefit from a bit more of friendly motivation at the beginning. Why are now changing perspective, and why are we introducing twisted cycles? Will it help with anything that was seen in Sections 1 and 2?
- page 23, line 5: In this display your  $t$  ranges in the interval  $[0, 1]$ .  
What what about  $a$  and  $\theta$ ? Say that  $a$  is 0 or 1 and that  $\theta$  is 0 or  $\pi$ ?
- pages 25 to 32: As I said: Not easy to read. Can you make it a little bit more friendly?
- Section 4: Is the work of Schneider at RISC-Linz relevant here?
- Section 5: The list of open problems is very nice and useful.

In summary, this is an excellent article, and I recommend it very strongly for SciPost Physics Lecture Notes.