As part of the report for the manuscript ***Bethe Ansatz inside Calogero-Sutherland models***
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A. Comments/Clarifications

1. I think one detail can potentially be confusing to someone with no experience in the fusion procedure. In appendix A, the authors presented the fused R-matrix using $\theta_{j+1} = \theta_j + i$ for fusion into triplet and $\theta_{j+1} = \theta_j - i$ for fusion into singlet, in all the rest of the paper though (starting on section (2.4.2)), they use $\theta_{j+1} = \theta_j + i$ for fusion into singlet instead. To obtain the results, it looks like in the appendix A, the authors used equation 2.36 as it is now, and in the rest of the paper they used it with $j \leftrightarrow j+1$. That is completely okay. But since the readers are referred to appendix A several times, I would suggest that the authors change the conventions there to match the rest of the manuscript, i.e. to have singlet always coming form $\theta_{j+1} = \theta_j + i$, for example.

2. In appendix A, if one does not know fusion, I believe it would not be clear that $\underline{R}^{(1/2,0)}(u)$ is nothing more than the lhs (or the rhs) of equation (A.1) with Π^+, using the basis $\{|1,1\rangle, |1,0\rangle, |1,-1\rangle\}$. I believe giving a name for one side of eq. (A.1) and then mentioning the basis, would make the whole explanation cleaner.

3. In appendix D, section D.1, I wasn't able to reproduce the results. Did the authors used the monodromy matrix as defined in equation (2.5) using the $\bar{R}(u)$ as in equation (2.4)? For me doing that generates the following matrices

$$\bar{A}(u) = \begin{pmatrix} \bar{Q}_\theta^+ & 0 & 0 & 0 \\ 0 & (u-\theta_1+\frac{i}{2})(u-\theta_2-\frac{i}{2}) & 0 & 0 \\ 0 & -1 & (u-\theta_1-\frac{i}{2})(u-\theta_2+\frac{i}{2}) & 0 \\ 0 & 0 & 0 & \bar{Q}_\theta^- \end{pmatrix} \tag{1}$$

$$\bar{B}(u) = i \begin{pmatrix} 0 & 0 & 0 & 0 \\ u-\theta_1+\frac{i}{2} & 0 & 0 & 0 \\ u-\theta_2-\frac{i}{2} & 0 & 0 & 0 \\ 0 & u-\theta_2+\frac{i}{2} & u-\theta_1-\frac{i}{2} & 0 \end{pmatrix} \tag{2}$$

$$\bar{C}(u) = i \begin{pmatrix} 0 & u-\theta_1-\frac{i}{2} & u-\theta_2+\frac{i}{2} & 0 \\ 0 & 0 & 0 & u-\theta_2-\frac{i}{2} \\ 0 & 0 & 0 & u-\theta_1+\frac{i}{2} \end{pmatrix} \tag{3}$$

$$\bar{D}(u) = \begin{pmatrix} \bar{Q}_\theta^- & 0 & 0 & 0 \\ 0 & (u-\theta_1-\frac{i}{2})(u-\theta_2+\frac{i}{2}) & -1 & 0 \\ 0 & 0 & (u-\theta_1+\frac{i}{2})(u-\theta_2-\frac{i}{2}) & 0 \\ 0 & 0 & 0 & \bar{Q}_\theta^+ \end{pmatrix}. \tag{4}$$

   I was unable to make these matrices match with eqs (D.1)-(D.4) even using the definitions of $\bar{Q}_\theta$, $\bar{Q}_\theta^\pm$ and u_0. Additionally, using the matrices above I can reproduce exactly the results in (D.5)-(D.7) and I agree with all the rest discussed in the section. But if I try to use (D.1)-(D.4) instead, I find something slightly different for (D.5) although most of what is discussed in the section still holds. Could the authors please double check (D.1)-(D.4) and indicate if indeed they are using eqs (2.4), (2.5) and (2.6) to obtain them, or perhaps some transformed version?

4. Going from eq. (2.20) to the Bethe equations as described in appendix B.1. was very clear. It was not clear however, how to construct eq.(2.20). Is there a procedure? Could the authors add a few comments.

5. Just after equation (2.22) "...reduce to the Hamiltonians of the rational Gaudin model". Would it be useful to add a reference for the rational Gaudin model?

B. Typos and minor suggestions/questions

1. Introduction, second paragraph: bona fide $\to$ *bona fide*.
2. After equation (1.1): Mention what are β and N.
3. Page 5, sixth line: representation $\to$ representation**s**.
4. Section 2.1, mention if κ is always real.
5. Before equation (2.10) $C(u)|0\rangle \to \bar{C}(u)|0\rangle$.
6. In equation 2.21 and afterwards, perhaps do $G \to \bar{G}$?
7. Last paragraph in page 9: "One obtains N inhomogeneous...", should be L instead of N?
8. Equation 2.33: $\alpha \to \bar{\alpha}$.
9. Page 12: when going from equation (2.34) to equation (2.36), the lhs in (2.34) goes to the rhs in (2.36) and vice-versa. A minor suggestion would be to swap them, so that lhs goes to lhs, etc. This would make it easier to a reader with less experience to follow the computation.
10. Figure 2: "M-paticle" $\to$ "M-pa**r**ticle".
11. Above equation (2.46): $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \to \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$
12. Eq. (2.47): should end in "." instead of comma.
13. Above eq. (2.60) "contain" $\to$ "contains".
14. Page 20, second line: "mondromy" $\to$ "monodromy"
15. Page 31, equation (4.27): should end in "." instead of comma.
16. Appendix A, just before equation (A.1): "intial" $\to$ "initial"
17. Eq. (A.3), are the numerator and denominator swapped in the expression for the qdet?