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Title: Statistical Mechanics of Exponentially Many Low Lying States

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The premise of the paper revolves around understanding the classical gravity prediction for the large entropy of extremal non-supersymmetric black holes. It is not clear which symmetry protects the ground state degeneracy. Based on semiclassical thermodynamics, Page suggested that the gravity picture actually computes the number of exponentially many low lying states near the ground state. However, these states can only be probed at very small temperatures. Recent works have given reasonable proof that the thermodynamics of near-extreme black holes receives large quantum corrections that can be connected to their effective description in terms of an emergent Schwarzian theory. The author tries to argue that exponentially many equi-spaced low lying states can capture these quantum corrections (in a particular temperature regime), extending Page's idea. The author also suggests that any microscopic description of these black holes should have this feature and in particular shows that certain brane systems do satisfy this condition. The problem is interesting and the author's suggestion sheds new light on the statistical behavior of near-extremal black holes. However, I think there are certain crucial points that need more detailed explanations before accepting the manuscript for publication. I am listing these below:

1. In the description of near-extremal black holes in section 2, it is mentioned that

the slow variation of the sphere breaks the near-horizon AdS_2 isometries. It is worth mentioning that even the AdS_2 factor receives temperature corrections at the same order and both these corrections are responsible for the symmetry breaking.

2. In section 2, in the context of the SYK model, a connection between the lack of quasiparticles and exponentially small level spacing of low lying states was dicsussed. Also, a non-commutative feature of large system size and zero temperature limits was discussed. These statements are nontrivial and important for the context of the paper. Thus, it would be good if the author could elaborate and clarify these ideas with some useful references.

3. In the second para of section 3, the word 'discreet' should be corrected to 'discrete'.

4. Since the detail of the brane system in section 3 is out of scope of the paper, the author should put some references in the discussion of the superfield system (starting on page 7).

5. The paper tries to support the idea of a band of states near the ground state separated by exponentially small energy gaps. Section 3.1 tries to argue that extremal non-supersymmetric brane systems might have this low energy structure. At the end of section 3.1, it is found that there can be a 'band of ground states' of $\mathcal{O}(\hbar^2)$ energies, separated from excited states with $\mathcal{O}(\hbar)$ energies. I think the author should stress on why (or how) these ground states are non-degenerate? In appendix A, a similar example is presented where the degenerate vacua are slightly lifted due to soft breaking of supersymmetry. It is not clear how these ground states will still be separated by 'exponentially small' energy gaps even for the non-supersymmetric case. This point needs to be addressed.

6. Section 3.2 argues that exponentially low lying states is expected to be equispaced for a large number of states. Why is it expected that a generic spectrum is supposed to be featureless? I think this argument should be more refined.

7. The statistical mechanical computations in a low but not too low temperature regime $\frac{\Delta}{\Omega} \ll T \ll \Delta$ of section 3.3, correctly captures the results coming from Schwarzian theory. The statistical system has equispaced low lying states near the vacuum separated from the excited states by a gap E_{gap}. The parameter Δ is identified with the thermodynamic breakdown scale M_{gap} of the gravity analysis and $\log \Omega = S_0$ is the extremal entropy. However, from gravity side, it does not seem that there should be a gap beyond the scale $\Delta \sim M_{\text{gap}}$. Can the author comment on what the scale E_{gap} should correspond to in the gravity side? And how these two statements are consistent?

I think if the author addresses the above points, the manuscript can be accepted for publication in SciPost.