

REPORT

The manuscript under review studies a novel class of 2d non-unitary RCFTs constructed from a topological twisting of some 3d $N = 4$ rank-0 SCFTs. It contains many interesting new results and is well presented. In particular, it makes connections to Haagerup TQFTs and Haagerup CFTs which draws lots of attentions in recent years. **I strongly recommend the manuscript to be published on SciPost** after considering the following suggestions.

- (1) Nahm sums already have precise definition in mathematics, e.g. by Don Zagier. The general half-indices formula the author present is not really Nahm sum, because of the $(q)_{2m_1}$ in the denominator. To not cause confusion, I would suggest the authors not to call these as Nahm sums, e.g. in the bottom of page 15. Note that the name "generalized Nahm sums" is also taken to describe the situation when the K matrix is not symmetric.

On the other hand, it looks to me that some characters of the orbifold Haagerup RCFT are indeed Nahm sums after scaling m_1 . If it's true, I suggest the authors to mention this.

- (2) I was under the impression that all non-unitary Haagerup RCFTs constructed in this way have effective central charge $c_{\text{eff}} = 1$. If this is correct, I suggest the authors to add this clear statement into the manuscript.
- (3) It would be helpful to readers if the authors can simply write down how many characters there are for each k . For example, in pages 8, the authors can say the S -matrix is $(2k + 2) \times (2k + 2)$. Same for page 11.
- (4) I would like to know if (and how) the authors prove the S and T matrices in page 8 satisfy the relation $S^2 = (ST)^3 = I$ for **arbitrary** k .
- (5) Below eq (2.32), the author use \bar{p} . It would be better to write down the conductor for which the \bar{p} is defined and relation between the conductor and the n in Hg_{2n+1} in eq (2.32).
- (6) The authors discuss the Haagerup RCFT $\mathcal{R}_{k=4}$ in details, which has 10 characters. It looks to me that this theory is exactly the bosonic theory of $N = 1$ supersymmetric minimal model $SM(12, 2)$, which is non-unitary as expected. The effective $SM(12, 2)$ has NS conformal weights $0, \frac{1}{6}, \frac{1}{2}$. I checked the NS characters are related to the $\mathcal{R}_{k=4}$ characters given in eq (3.52) by

$$\begin{aligned}\chi_0^{\text{NS}} &= \chi_2 + \chi_3, \\ \chi_{1/6}^{\text{NS}} &= \chi_6 + \chi_8, \\ \chi_{1/2}^{\text{NS}} &= \chi_0 + \chi_1.\end{aligned}$$

Similar for the three R characters. The $\chi_9 - \chi_5 = 1$ is the constant \tilde{R} character. The authors can write about this if they would like to.

- (7) For orbifold Haagerup RCFT $\tilde{\mathcal{R}}_{k=6}$, I tried eqs (3.22) and (3.23) with the K matrix given in eq (3.24) and $m_1 \in \mathbb{N}/2$ and $m_{2,3,4,5} \in \mathbb{N}$. Unfortunately, I cannot recover the authors'

q -series. For both eqs (3.22) and (3.23), I get $q^{1/2}$ series. I would like to know whether the formulas (3.22), (3.23) and (3.24) are precisely presented as the authors compute them.

Besides, it seems that (3.22) and (3.23) give the same q -series. I would like to know whether this is a coincidence or there is a reason.

- (8) For Haagerup RCFT $\mathcal{R}_{k \geq 5}$, the authors present the χ matrices which I feel are not directly useful to most readers. It would be better if the authors can present the characters they find even in a mathematica attachment, such that readers can easily follow their work and understand these Haagerup RCFTs.