Report for 2311.07367v1

Dear Editor,

Please find my report for the aforementioned manuscript.

Recommendation

Accept.

Remarks

The article's main result is an explicit parametrization of the quartic $SL(2,\mathbb{C})$ vertex for bosonic closed string field theory (SFT). The main result of the paper is quite technical, but in my opinion it is an important step nonetheless. SFT has recently seen a major revival with the first complete construction of the BV action of superstring field theory that has rigorously established several key aspects of superstring perturbation theory, such as unitarity, background independence, and explicit computations of mass renormalization in string theory that were previously unknown. Sen's $SL(2,\mathbb{C})$ vertex formulation of superstring field theory has in particular been central to all of these major breakthroughs. Despite these striking results, a lack of explicit parametrization of the region of the moduli space covered by these $SL(2,\mathbb{C})$ vertices, and a rather unfortunate construction algorithm which is recurrent in nature, means performing explicit calculations with these vertices are rather challenging.

This paper makes an important step towards such a parametrization for the quartic vertex for the bosonic case. While the authors fall short of finding an explicit set of local co-ordinates, I still think their result is remarkable and significant for the following reasons.

- By nature the vertex construction for bosonic case should readily extend to the superstring case (with some subtleties, all of which are very well understood and in control), so the scope of the main result of this paper is actually wider than just closed bosonic strings.
- Furthermore, it has been known for sometime now that the classical closed bosonic string field theory is described by the L_∞ homotopy algebra. The minimal model theorem for L_∞ algebras guarantees that there are a set of quasi-isomorphisms that would allow a rewriting of the classical closed bosonic string action which would involve ***only*** cubic vertices. One of the authors of this manuscript has been actively making progress from multiple directions to arrive at such a purely cubic action. The result of this paper is a significant benchmark in finding a purely cubic action for closed bosonic SFT as well. Any possible cubic vertices in terms of new (auxiliary) string fields should at the bare minimum manage to cover the same region of the moduli space as the quartic vertex, to begin with.

The paper is very well-written. Despite a very technical focus, the paper is remarkably self-contained, with sufficient details and impeccable referencing that will allow a much broader audience to engage with the results. I only have two suggestions for the authors. One regarding the content and one minor stylistic suggestion.

1. I would like the authors to comment on what they think regarding a possible connection to the light-cone formulation of SFT. I am curious if the authors have any thoughts or plans regarding trying to find a map between $SL(2,\mathbb{C})$ vertices and the light-cone vertices. This is quite interesting since the light-cone action is already polynomial and can potentially

provide insight into how to construct a minimal model for the covariant action. Perhaps, an easier task is to start with the open strings (and therefore $SL(2,\mathbb{R})$ vertices) since a map between Witten's covariant action and light-cone SFT was recently put forward in [1] and very recently even connected to the hyperbolic vertices in [2]. Determining a direct connection between $SL(2,\mathbb{C})$ (or $SL(2,\mathbb{R})$ for open strings) and light-cone formulation might be worth investigating. I understand such an investigation is beyond the scope of the present manuscript, but if the authors agree with this possibility, I think it is worthwhile including their thoughts on such a possible connection in the discussion section.

2. The equations, and symbols with subscripts in the various figures are in quite a small font. While this should not be an issue for readers reading on a big screen, it can present a significant challenge for readers reading on a small screen or reading a print out version. I do not know if this can be fixed during the publication process by the journal. If so then the authors can disregard this comment. If not, I will request the authors to consider enlarging the figures and/or the small font equations and symbols with subscripts in the figures to make them more legible.

Bibliography

[1] T. Erler and H. Matsunaga, "Mapping between Witten and lightcone string field theories," *JHEP*, vol. 11, p. 208–209, 2021, doi: 10.1007/JHEP11(2021)208.

[2] V. Bernardes and U. Portugal, "A two parameter family of lightcone-like hyperbolic string vertices," *arxiv:2404.17268*, 2024, doi: https://doi.org/10.48550/arXiv.2404.17268.